2024

AP

AP® Calculus AB

Free-Response Questions

CALCULUS AB

SECTION II, Part A

Time—30 minutes
2 Questions

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

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t (minutes)	0	3	7	12
$C(t)$ (degrees Celsius)	100	85	69	55

1. The temperature of coffee in a cup at time t minutes is modeled by a decreasing differentiable function C, where $C(t)$ is measured in degrees Celsius. For $0 \le t \le 12$, selected values of $C(t)$ are given in the table shown.

(a) Approximate $C'(5)$ using the average rate of change of C over the interval $3 \le t \le 7$. Show the work that leads to your answer and include units of measure.

(b) Use a left Riemann sum with the three subintervals indicated by the data in the table to approximate the value of $\int_0^{12} C(t)\, dt$. Interpret the meaning of $\frac{1}{12}\int_0^{12} C(t)\, dt$ in the context of the problem.

(c) For $12 \le t \le 20$, the rate of change of the temperature of the coffee is modeled by $C'(t) = \frac{-24.55e^{0.01t}}{t}$, where $C'(t)$ is measured in degrees Celsius per minute. Find the temperature of the coffee at time $t = 20$. Show the setup for your calculations.

(d) For the model defined in part (c), it can be shown that $C''(t) = \frac{0.2455e^{0.01t}(100 - t)}{t^2}$. For $12 < t < 20$, determine whether the temperature of the coffee is changing at a decreasing rate or at an increasing rate. Give a reason for your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

2. A particle moves along the x-axis so that its velocity at time $t \geq 0$ is given by $v(t) = \ln\left(t^2 - 4t + 5\right) - 0.2t$.

(a) There is one time, $t = t_R$, in the interval $0 < t < 2$ when the particle is at rest (not moving). Find t_R. For $0 < t < t_R$, is the particle moving to the right or to the left? Give a reason for your answer.

(b) Find the acceleration of the particle at time $t = 1.5$. Show the setup for your calculations. Is the speed of the particle increasing or decreasing at time $t = 1.5$? Explain your reasoning.

(c) The position of the particle at time t is $x(t)$, and its position at time $t = 1$ is $x(1) = -3$. Find the position of the particle at time $t = 4$. Show the setup for your calculations.

(d) Find the total distance traveled by the particle over the interval $1 \leq t \leq 4$. Show the setup for your calculations.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

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END OF PART A

CALCULUS AB

SECTION II, Part B

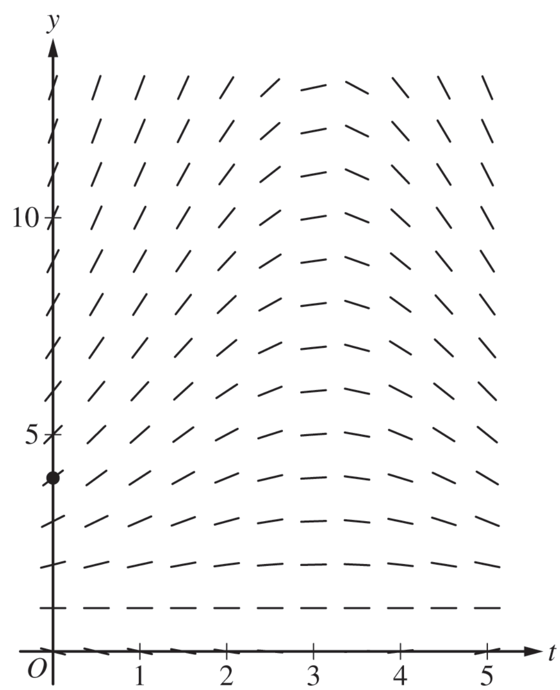
Time—1 hour

4 Questions

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.

3. The depth of seawater at a location can be modeled by the function H that satisfies the differential equation $\frac{dH}{dt} = \frac{1}{2}(H - 1)\cos\left(\frac{t}{2}\right)$, where $H(t)$ is measured in feet and t is measured in hours after noon ($t = 0$). It is known that $H(0) = 4$.

(a) A portion of the slope field for the differential equation is provided. Sketch the solution curve, $y = H(t)$, through the point $(0, 4)$.



(b) For $0 < t < 5$, it can be shown that $H(t) > 1$. Find the value of t, for $0 < t < 5$, at which H has a critical point. Determine whether the critical point corresponds to a relative minimum, a relative maximum, or neither a relative minimum nor a relative maximum of the depth of seawater at the location. Justify your answer.

(c) Use separation of variables to find $y = H(t)$, the particular solution to the differential equation $\frac{dH}{dt} = \frac{1}{2}(H - 1)\cos\left(\frac{t}{2}\right)$ with initial condition $H(0) = 4$.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

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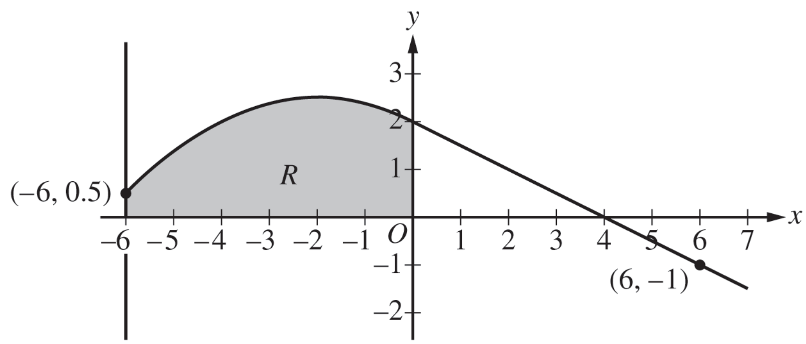


Graph of f

4. The graph of the differentiable function f, shown for $-6 \le x \le 7$, has a horizontal tangent at $x = -2$ and is linear for $0 \le x \le 7$. Let R be the region in the second quadrant bounded by the graph of f, the vertical line $x = -6$, and the x- and y-axes. Region R has area 12.

(a) The function g is defined by $g(x) = \int_0^x f(t)\,dt$. Find the values of $g(-6)$, $g(4)$, and $g(6)$.

(b) For the function g defined in part (a), find all values of x in the interval $0 \le x \le 6$ at which the graph of g has a critical point. Give a reason for your answer.

(c) The function h is defined by $h(x) = \int_{-6}^x f'(t)\,dt$. Find the values of $h(6)$, $h'(6)$, and $h''(6)$. Show the work that leads to your answers.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

5. Consider the curve defined by the equation $x^2 + 3y + 2y^2 = 48$. It can be shown that $\dfrac{dy}{dx} = \dfrac{-2x}{3 + 4y}$.

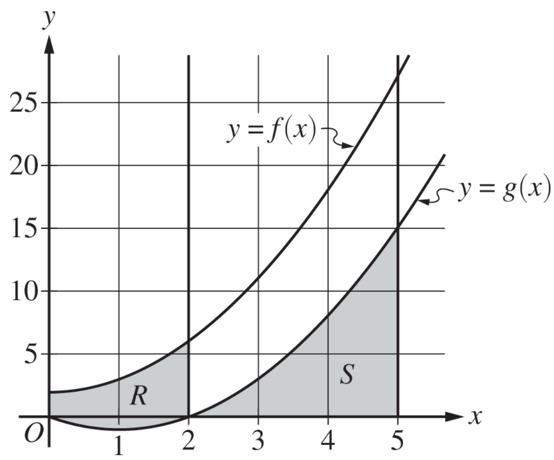
(a) There is a point on the curve near $(2, 4)$ with x-coordinate 3. Use the line tangent to the curve at $(2, 4)$ to approximate the y-coordinate of this point.

(b) Is the horizontal line $y = 1$ tangent to the curve? Give a reason for your answer.

(c) The curve intersects the positive x-axis at the point $(\sqrt{48}, 0)$. Is the line tangent to the curve at this point vertical? Give a reason for your answer.

(d) For time $t \geq 0$, a particle is moving along another curve defined by the equation $y^3 + 2xy = 24$. At the instant the particle is at the point $(4, 2)$, the y-coordinate of the particle's position is decreasing at a rate of 2 units per second. At that instant, what is the rate of change of the x-coordinate of the particle's position with respect to time?

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.



6. The functions f and g are defined by $f(x) = x^2 + 2$ and $g(x) = x^2 - 2x$, as shown in the graph.

(a) Let R be the region bounded by the graphs of f and g, from $x = 0$ to $x = 2$, as shown in the graph. Write, but do not evaluate, an integral expression that gives the area of region R.

(b) Let S be the region bounded by the graph of g and the x-axis, from $x = 2$ to $x = 5$, as shown in the graph. Region S is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis is a rectangle with height equal to half its base in region S. Find the volume of the solid. Show the work that leads to your answer.

(c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when region S, as described in part (b), is rotated about the horizontal line $y = 20$.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

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STOP

END OF EXAM

2023

AP® Calculus AB

Free-Response Questions

CALCULUS AB

SECTION II, Part A

Time—30 minutes
2 Questions

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

t (seconds)	0	60	90	120	135	150
$f(t)$ (gallons per second)	0	0.1	0.15	0.1	0.05	0

1. A customer at a gas station is pumping gasoline into a gas tank. The rate of flow of gasoline is modeled by a differentiable function f, where $f(t)$ is measured in gallons per second and t is measured in seconds since pumping began. Selected values of $f(t)$ are given in the table.

(a) Using correct units, interpret the meaning of $\int_{60}^{135} f(t)\,dt$ in the context of the problem. Use a right Riemann sum with the three subintervals $[60, 90]$, $[90, 120]$, and $[120, 135]$ to approximate the value of $\int_{60}^{135} f(t)\,dt$.

(b) Must there exist a value of c, for $60 < c < 120$, such that $f'(c) = 0$? Justify your answer.

(c) The rate of flow of gasoline, in gallons per second, can also be modeled by $g(t) = \left(\frac{t}{500}\right)\cos\left(\left(\frac{t}{120}\right)^2\right)$ for $0 \le t \le 150$. Using this model, find the average rate of flow of gasoline over the time interval $0 \le t \le 150$.

Show the setup for your calculations.

(d) Using the model g defined in part (c), find the value of $g'(140)$. Interpret the meaning of your answer in the context of the problem.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

2. Stephen swims back and forth along a straight path in a 50-meter-long pool for 90 seconds. Stephen's velocity is modeled by $v(t) = 2.38e^{-0.02t}\sin\left(\frac{\pi}{56}t\right)$, where t is measured in seconds and $v(t)$ is measured in meters per second.

(a) Find all times t in the interval $0 < t < 90$ at which Stephen changes direction. Give a reason for your answer.

(b) Find Stephen's acceleration at time $t = 60$ seconds. Show the setup for your calculations, and indicate units of measure. Is Stephen speeding up or slowing down at time $t = 60$ seconds? Give a reason for your answer.

(c) Find the distance between Stephen's position at time $t = 20$ seconds and his position at time $t = 80$ seconds. Show the setup for your calculations.

(d) Find the total distance Stephen swims over the time interval $0 \le t \le 90$ seconds. Show the setup for your calculations.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

END OF PART A

CALCULUS AB

SECTION II, Part B

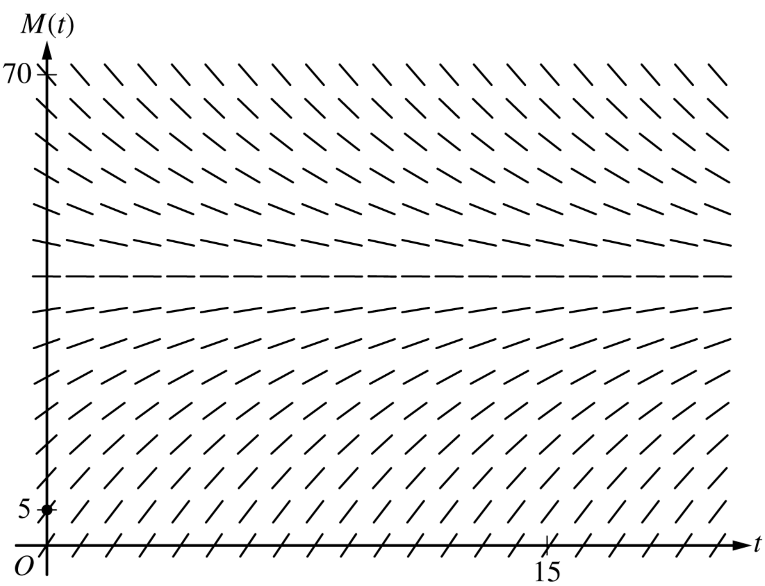
Time—1 hour

4 Questions

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.

3. A bottle of milk is taken out of a refrigerator and placed in a pan of hot water to be warmed. The increasing function M models the temperature of the milk at time t, where $M(t)$ is measured in degrees Celsius (°C) and t is the number of minutes since the bottle was placed in the pan. M satisfies the differential equation $\frac{dM}{dt} = \frac{1}{4}(40 - M)$. At time $t = 0$, the temperature of the milk is 5°C. It can be shown that $M(t) < 40$ for all values of t.

(a) A slope field for the differential equation $\frac{dM}{dt} = \frac{1}{4}(40 - M)$ is shown. Sketch the solution curve through the point $(0, 5)$.

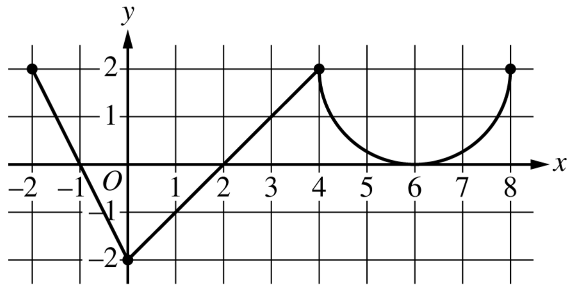


(b) Use the line tangent to the graph of M at $t = 0$ to approximate $M(2)$, the temperature of the milk at time $t = 2$ minutes.

(c) Write an expression for $\frac{d^2M}{dt^2}$ in terms of M. Use $\frac{d^2M}{dt^2}$ to determine whether the approximation from part (b) is an underestimate or an overestimate for the actual value of $M(2)$. Give a reason for your answer.

(d) Use separation of variables to find an expression for $M(t)$, the particular solution to the differential equation $\frac{dM}{dt} = \frac{1}{4}(40 - M)$ with initial condition $M(0) = 5$.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.



Graph of f'

4. The function f is defined on the closed interval $[-2, 8]$ and satisfies $f(2) = 1$. The graph of f', the derivative of f, consists of two line segments and a semicircle, as shown in the figure.

(a) Does f have a relative minimum, a relative maximum, or neither at $x = 6$? Give a reason for your answer.

(b) On what open intervals, if any, is the graph of f concave down? Give a reason for your answer.

(c) Find the value of $\lim\limits_{x \to 2} \dfrac{6f(x) - 3x}{x^2 - 5x + 6}$, or show that it does not exist. Justify your answer.

(d) Find the absolute minimum value of f on the closed interval $[-2, 8]$. Justify your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

x	0	2	4	7
$f(x)$	10	7	4	5
$f'(x)$	$\frac{3}{2}$	-8	3	6
$g(x)$	1	2	-3	0
$g'(x)$	5	4	2	8

5. The functions f and g are twice differentiable. The table shown gives values of the functions and their first derivatives at selected values of x.

(a) Let h be the function defined by $h(x) = f(g(x))$. Find $h'(7)$. Show the work that leads to your answer.

(b) Let k be a differentiable function such that $k'(x) = (f(x))^2 \cdot g(x)$. Is the graph of k concave up or concave down at the point where $x = 4$? Give a reason for your answer.

(c) Let m be the function defined by $m(x) = 5x^3 + \int_0^x f'(t)\, dt$. Find $m(2)$. Show the work that leads to your answer.

(d) Is the function m defined in part (c) increasing, decreasing, or neither at $x = 2$? Justify your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

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6. Consider the curve given by the equation $6xy = 2 + y^3$.

   (a) Show that $\dfrac{dy}{dx} = \dfrac{2y}{y^2 - 2x}$.

   (b) Find the coordinates of a point on the curve at which the line tangent to the curve is horizontal, or explain why no such point exists.

   (c) Find the coordinates of a point on the curve at which the line tangent to the curve is vertical, or explain why no such point exists.

   (d) A particle is moving along the curve. At the instant when the particle is at the point $\left(\dfrac{1}{2}, -2\right)$, its horizontal position is increasing at a rate of $\dfrac{dx}{dt} = \dfrac{2}{3}$ unit per second. What is the value of $\dfrac{dy}{dt}$, the rate of change of the particle's vertical position, at that instant?

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

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END OF EXAM

2022

AP® CollegeBoard

AP® Calculus AB

Free-Response Questions

CALCULUS AB

SECTION II, Part A

Time—30 minutes
2 Questions

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

1. From 5 A.M. to 10 A.M., the rate at which vehicles arrive at a certain toll plaza is given by $A(t) = 450\sqrt{\sin(0.62t)}$, where t is the number of hours after 5 A.M. and $A(t)$ is measured in vehicles per hour. Traffic is flowing smoothly at 5 A.M. with no vehicles waiting in line.

   (a) Write, but do not evaluate, an integral expression that gives the total number of vehicles that arrive at the toll plaza from 6 A.M. ($t = 1$) to 10 A.M. ($t = 5$).

   (b) Find the average value of the rate, in vehicles per hour, at which vehicles arrive at the toll plaza from 6 A.M. ($t = 1$) to 10 A.M. ($t = 5$).

   (c) Is the rate at which vehicles arrive at the toll plaza at 6 A.M. ($t = 1$) increasing or decreasing? Give a reason for your answer.

   (d) A line forms whenever $A(t) \geq 400$. The number of vehicles in line at time t, for $a \leq t \leq 4$, is given by $N(t) = \int_a^t (A(x) - 400)\,dx$, where a is the time when a line first begins to form. To the nearest whole number, find the greatest number of vehicles in line at the toll plaza in the time interval $a \leq t \leq 4$. Justify your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

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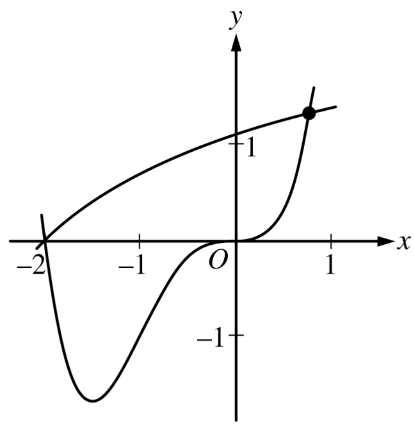

2. Let f and g be the functions defined by $f(x) = \ln(x + 3)$ and $g(x) = x^4 + 2x^3$. The graphs of f and g, shown in the figure above, intersect at $x = -2$ and $x = B$, where $B > 0$.

(a) Find the area of the region enclosed by the graphs of f and g.

(b) For $-2 \le x \le B$, let $h(x)$ be the vertical distance between the graphs of f and g. Is h increasing or decreasing at $x = -0.5$? Give a reason for your answer.

(c) The region enclosed by the graphs of f and g is the base of a solid. Cross sections of the solid taken perpendicular to the x-axis are squares. Find the volume of the solid.

(d) A vertical line in the xy-plane travels from left to right along the base of the solid described in part (c). The vertical line is moving at a constant rate of 7 units per second. Find the rate of change of the area of the cross section above the vertical line with respect to time when the vertical line is at position $x = -0.5$.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

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END OF PART A

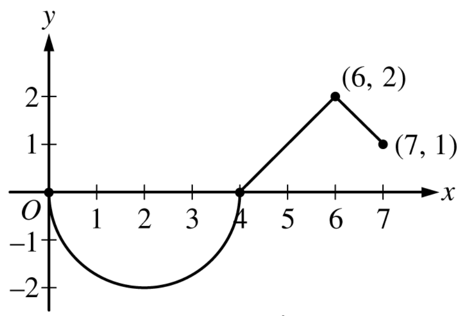
CALCULUS AB

SECTION II, Part B

Time—1 hour

4 Questions

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.



Graph of f'

3. Let f be a differentiable function with $f(4) = 3$. On the interval $0 \le x \le 7$, the graph of f', the derivative of f, consists of a semicircle and two line segments, as shown in the figure above.

   (a) Find $f(0)$ and $f(5)$.

   (b) Find the x-coordinates of all points of inflection of the graph of f for $0 < x < 7$. Justify your answer.

   (c) Let g be the function defined by $g(x) = f(x) - x$. On what intervals, if any, is g decreasing for $0 \le x \le 7$? Show the analysis that leads to your answer.

   (d) For the function g defined in part (c), find the absolute minimum value on the interval $0 \le x \le 7$. Justify your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

t (days)	0	3	7	10	12
$r'(t)$ (centimeters per day)	-6.1	-5.0	-4.4	-3.8	-3.5

4. An ice sculpture melts in such a way that it can be modeled as a cone that maintains a conical shape as it decreases in size. The radius of the base of the cone is given by a twice-differentiable function r, where $r(t)$ is measured in centimeters and t is measured in days. The table above gives selected values of $r'(t)$, the rate of change of the radius, over the time interval $0 \le t \le 12$.

(a) Approximate $r''(8.5)$ using the average rate of change of r' over the interval $7 \le t \le 10$. Show the computations that lead to your answer, and indicate units of measure.

(b) Is there a time t, $0 \le t \le 3$, for which $r'(t) = -6$? Justify your answer.

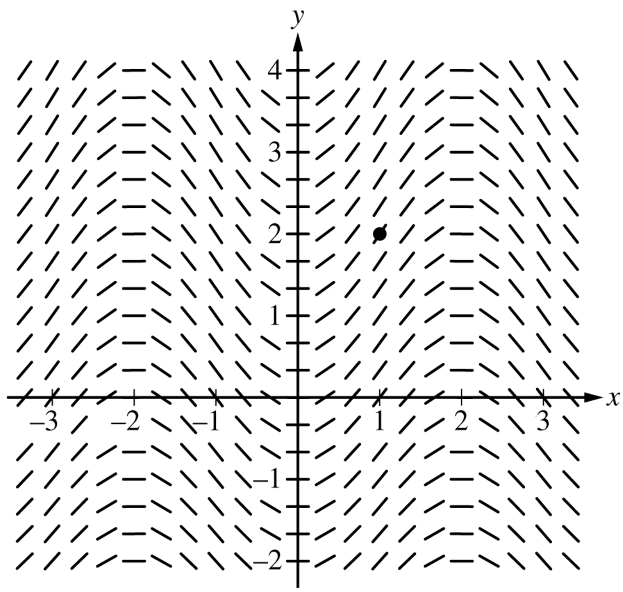
(c) Use a right Riemann sum with the four subintervals indicated in the table to approximate the value of $\int_0^{12} r'(t)\,dt$.

(d) The height of the cone decreases at a rate of 2 centimeters per day. At time $t = 3$ days, the radius is 100 centimeters and the height is 50 centimeters. Find the rate of change of the volume of the cone with respect to time, in cubic centimeters per day, at time $t = 3$ days. (The volume V of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.)

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

5. Consider the differential equation $\frac{dy}{dx} = \frac{1}{2}\sin\left(\frac{\pi}{2}x\right)\sqrt{y+7}$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = 2$. The function f is defined for all real numbers.

(a) A portion of the slope field for the differential equation is given below. Sketch the solution curve through the point $(1, 2)$.



(b) Write an equation for the line tangent to the solution curve in part (a) at the point $(1, 2)$. Use the equation to approximate $f(0.8)$.

(c) It is known that $f''(x) > 0$ for $-1 \le x \le 1$. Is the approximation found in part (b) an overestimate or an underestimate for $f(0.8)$? Give a reason for your answer.

(d) Use separation of variables to find $y = f(x)$, the particular solution to the differential equation $\frac{dy}{dx} = \frac{1}{2}\sin\left(\frac{\pi}{2}x\right)\sqrt{y+7}$ with the initial condition $f(1) = 2$.

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6. Particle P moves along the x-axis such that, for time $t > 0$, its position is given by $x_P(t) = 6 - 4e^{-t}$.

   Particle Q moves along the y-axis such that, for time $t > 0$, its velocity is given by $v_Q(t) = \frac{1}{t^2}$. At time $t = 1$, the position of particle Q is $y_Q(1) = 2$.

   (a) Find $v_P(t)$, the velocity of particle P at time t.

   (b) Find $a_Q(t)$, the acceleration of particle Q at time t. Find all times t, for $t > 0$, when the speed of particle Q is decreasing. Justify your answer.

   (c) Find $y_Q(t)$, the position of particle Q at time t.

   (d) As $t \to \infty$, which particle will eventually be farther from the origin? Give a reason for your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

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STOP

END OF EXAM

2021

AP® CollegeBoard

AP® Calculus AB

Free-Response Questions

CALCULUS AB

SECTION II, Part A

Time—30 minutes

2 Questions

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

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r (centimeters)	0	1	2	2.5	4
$f(r)$ (milligrams per square centimeter)	1	2	6	10	18

1. The density of a bacteria population in a circular petri dish at a distance r centimeters from the center of the dish is given by an increasing, differentiable function f, where $f(r)$ is measured in milligrams per square centimeter. Values of $f(r)$ for selected values of r are given in the table above.

(a) Use the data in the table to estimate $f'(2.25)$. Using correct units, interpret the meaning of your answer in the context of this problem.

(b) The total mass, in milligrams, of bacteria in the petri dish is given by the integral expression $2\pi \int_0^4 r f(r)\, dr$. Approximate the value of $2\pi \int_0^4 r f(r)\, dr$ using a right Riemann sum with the four subintervals indicated by the data in the table.

(c) Is the approximation found in part (b) an overestimate or underestimate of the total mass of bacteria in the petri dish? Explain your reasoning.

(d) The density of bacteria in the petri dish, for $1 \le r \le 4$, is modeled by the function g defined by $g(r) = 2 - 16(\cos(1.57\sqrt{r}))^3$. For what value of k, $1 < k < 4$, is $g(k)$ equal to the average value of $g(r)$ on the interval $1 \le r \le 4$?

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

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2. A particle, P, is moving along the x-axis. The velocity of particle P at time t is given by $v_P(t) = \sin\left(t^{1.5}\right)$ for $0 \le t \le \pi$. At time $t = 0$, particle P is at position $x = 5$.

A second particle, Q, also moves along the x-axis. The velocity of particle Q at time t is given by $v_Q(t) = (t - 1.8) \cdot 1.25^t$ for $0 \le t \le \pi$. At time $t = 0$, particle Q is at position $x = 10$.

(a) Find the positions of particles P and Q at time $t = 1$.

(b) Are particles P and Q moving toward each other or away from each other at time $t = 1$? Explain your reasoning.

(c) Find the acceleration of particle Q at time $t = 1$. Is the speed of particle Q increasing or decreasing at time $t = 1$? Explain your reasoning.

(d) Find the total distance traveled by particle P over the time interval $0 \le t \le \pi$.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

END OF PART A

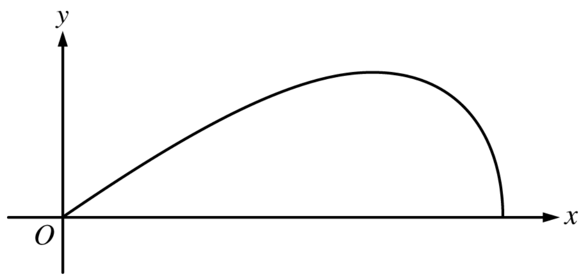
CALCULUS AB

SECTION II, Part B

Time—1 hour

4 Questions

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.



3. A company designs spinning toys using the family of functions $y = cx\sqrt{4 - x^2}$, where c is a positive constant. The figure above shows the region in the first quadrant bounded by the x-axis and the graph of $y = cx\sqrt{4 - x^2}$, for some c. Each spinning toy is in the shape of the solid generated when such a region is revolved about the x-axis. Both x and y are measured in inches.

(a) Find the area of the region in the first quadrant bounded by the x-axis and the graph of $y = cx\sqrt{4 - x^2}$ for $c = 6$.

(b) It is known that, for $y = cx\sqrt{4 - x^2}$, $\dfrac{dy}{dx} = \dfrac{c\left(4 - 2x^2\right)}{\sqrt{4 - x^2}}$. For a particular spinning toy, the radius of the largest cross-sectional circular slice is 1.2 inches. What is the value of c for this spinning toy?

(c) For another spinning toy, the volume is 2π cubic inches. What is the value of c for this spinning toy?

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

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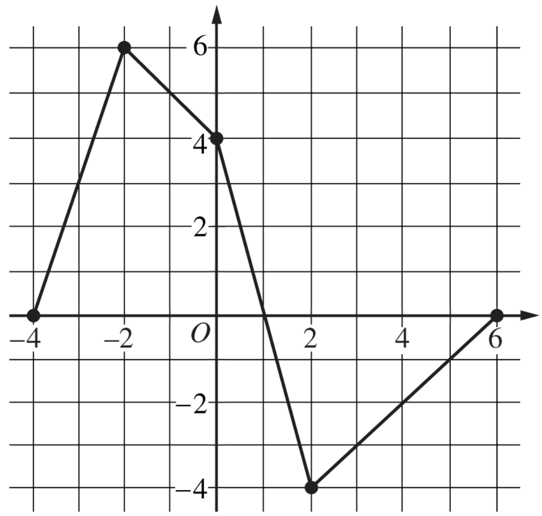

Graph of f

4. Let f be a continuous function defined on the closed interval $-4 \le x \le 6$. The graph of f, consisting of four line segments, is shown above. Let G be the function defined by $G(x) = \int_0^x f(t)\,dt$.

(a) On what open intervals is the graph of G concave up? Give a reason for your answer.

(b) Let P be the function defined by $P(x) = G(x) \cdot f(x)$. Find $P'(3)$.

(c) Find $\lim_{x \to 2} \dfrac{G(x)}{x^2 - 2x}$.

(d) Find the average rate of change of G on the interval $[-4, 2]$. Does the Mean Value Theorem guarantee a value c, $-4 < c < 2$, for which $G'(c)$ is equal to this average rate of change? Justify your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

GO ON TO THE NEXT PAGE.

5. Consider the function $y = f(x)$ whose curve is given by the equation $2y^2 - 6 = y \sin x$ for $y > 0$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{y \cos x}{4y - \sin x}$.

(b) Write an equation for the line tangent to the curve at the point $(0, \sqrt{3})$.

(c) For $0 \le x \le \pi$ and $y > 0$, find the coordinates of the point where the line tangent to the curve is horizontal.

(d) Determine whether f has a relative minimum, a relative maximum, or neither at the point found in part (c). Justify your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

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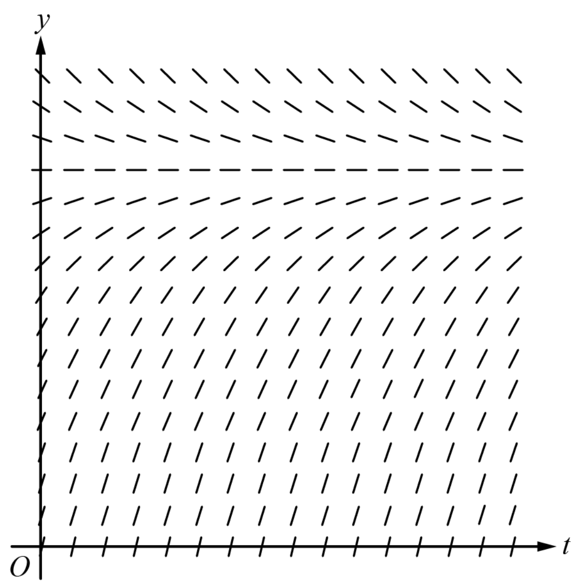
6. A medication is administered to a patient. The amount, in milligrams, of the medication in the patient at time t hours is modeled by a function $y = A(t)$ that satisfies the differential equation $\frac{dy}{dt} = \frac{12 - y}{3}$. At time $t = 0$ hours, there are 0 milligrams of the medication in the patient.

(a) A portion of the slope field for the differential equation $\frac{dy}{dt} = \frac{12 - y}{3}$ is given below. Sketch the solution curve through the point $(0, 0)$.



(b) Using correct units, interpret the statement $\lim_{t \to \infty} A(t) = 12$ in the context of this problem.

(c) Use separation of variables to find $y = A(t)$, the particular solution to the differential equation $\frac{dy}{dt} = \frac{12 - y}{3}$ with initial condition $A(0) = 0$.

(d) A different procedure is used to administer the medication to a second patient. The amount, in milligrams, of the medication in the second patient at time t hours is modeled by a function $y = B(t)$ that satisfies the differential equation $\dfrac{dy}{dt} = 3 - \dfrac{y}{t+2}$. At time $t = 1$ hour, there are 2.5 milligrams of the medication in the second patient. Is the rate of change of the amount of medication in the second patient increasing or decreasing at time $t = 1$? Give a reason for your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

GO ON TO THE NEXT PAGE.

STOP

END OF EXAM

2019

AP® | CollegeBoard

AP® Calculus AB

Free-Response Questions

CALCULUS AB

SECTION II, Part A

Time—30 minutes

Number of questions—2

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

1. Fish enter a lake at a rate modeled by the function E given by $E(t) = 20 + 15\sin\left(\frac{\pi t}{6}\right)$. Fish leave the lake at a rate modeled by the function L given by $L(t) = 4 + 2^{0.1t^2}$. Both $E(t)$ and $L(t)$ are measured in fish per hour, and t is measured in hours since midnight ($t = 0$).

   (a) How many fish enter the lake over the 5-hour period from midnight ($t = 0$) to 5 A.M. ($t = 5$) ? Give your answer to the nearest whole number.

   (b) What is the average number of fish that leave the lake per hour over the 5-hour period from midnight ($t = 0$) to 5 A.M. ($t = 5$) ?

   (c) At what time t, for $0 \le t \le 8$, is the greatest number of fish in the lake? Justify your answer.

   (d) Is the rate of change in the number of fish in the lake increasing or decreasing at 5 A.M. ($t = 5$) ? Explain your reasoning.

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t (hours)	0	0.3	1.7	2.8	4
$v_P(t)$ (meters per hour)	0	55	–29	55	48

2. The velocity of a particle, P, moving along the x-axis is given by the differentiable function v_P, where $v_P(t)$ is measured in meters per hour and t is measured in hours. Selected values of $v_P(t)$ are shown in the table above. Particle P is at the origin at time $t = 0$.

(a) Justify why there must be at least one time t, for $0.3 \le t \le 2.8$, at which $v_P'(t)$, the acceleration of particle P, equals 0 meters per hour per hour.

(b) Use a trapezoidal sum with the three subintervals [0, 0.3], [0.3, 1.7], and [1.7, 2.8] to approximate the value of $\int_0^{2.8} v_P(t)\,dt$.

(c) A second particle, Q, also moves along the x-axis so that its velocity for $0 \le t \le 4$ is given by $v_Q(t) = 45\sqrt{t}\cos\left(0.063t^2\right)$ meters per hour. Find the time interval during which the velocity of particle Q is at least 60 meters per hour. Find the distance traveled by particle Q during the interval when the velocity of particle Q is at least 60 meters per hour.

(d) At time $t = 0$, particle Q is at position $x = -90$. Using the result from part (b) and the function v_Q from part (c), approximate the distance between particles P and Q at time $t = 2.8$.

END OF PART A OF SECTION II

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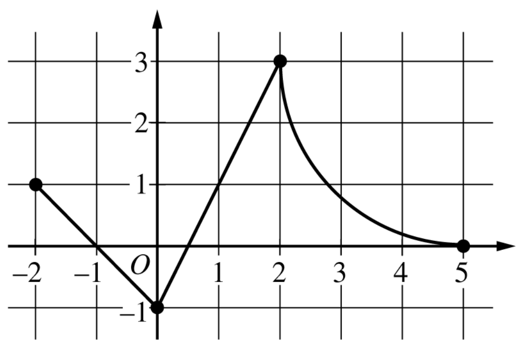
CALCULUS AB
SECTION II, Part B
Time—1 hour
Number of questions—4

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.



Graph of f

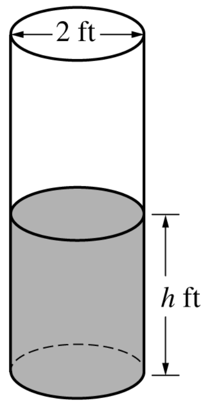
3. The continuous function f is defined on the closed interval $-6 \le x \le 5$. The figure above shows a portion of the graph of f, consisting of two line segments and a quarter of a circle centered at the point $(5, 3)$. It is known that the point $(3, 3 - \sqrt{5})$ is on the graph of f.

   (a) If $\int_{-6}^{5} f(x)\,dx = 7$, find the value of $\int_{-6}^{-2} f(x)\,dx$. Show the work that leads to your answer.

   (b) Evaluate $\int_{3}^{5} (2f'(x) + 4)\,dx$.

   (c) The function g is given by $g(x) = \int_{-2}^{x} f(t)\,dt$. Find the absolute maximum value of g on the interval $-2 \le x \le 5$. Justify your answer.

   (d) Find $\lim\limits_{x \to 1} \dfrac{10^x - 3f'(x)}{f(x) - \arctan x}$.

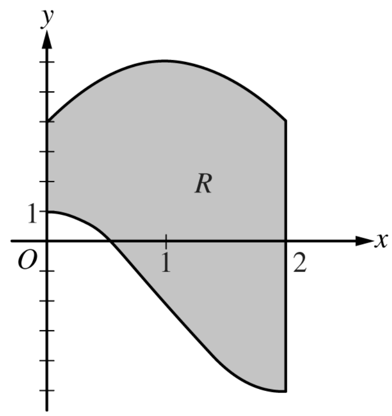


4. A cylindrical barrel with a diameter of 2 feet contains collected rainwater, as shown in the figure above. The water drains out through a valve (not shown) at the bottom of the barrel. The rate of change of the height h of the water in the barrel with respect to time t is modeled by $\frac{dh}{dt} = -\frac{1}{10}\sqrt{h}$, where h is measured in feet and t is measured in seconds. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)

(a) Find the rate of change of the volume of water in the barrel with respect to time when the height of the water is 4 feet. Indicate units of measure.

(b) When the height of the water is 3 feet, is the rate of change of the height of the water with respect to time increasing or decreasing? Explain your reasoning.

(c) At time $t = 0$ seconds, the height of the water is 5 feet. Use separation of variables to find an expression for h in terms of t.



5. Let R be the region enclosed by the graphs of $g(x) = -2 + 3\cos\left(\frac{\pi}{2}x\right)$ and $h(x) = 6 - 2(x - 1)^2$, the y-axis, and the vertical line $x = 2$, as shown in the figure above.

   (a) Find the area of R.

   (b) Region R is the base of a solid. For the solid, at each x the cross section perpendicular to the x-axis has area $A(x) = \dfrac{1}{x + 3}$. Find the volume of the solid.

   (c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 6$.

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6. Functions f, g, and h are twice-differentiable functions with $g(2) = h(2) = 4$. The line $y = 4 + \frac{2}{3}(x - 2)$ is tangent to both the graph of g at $x = 2$ and the graph of h at $x = 2$.

   (a) Find $h'(2)$.

   (b) Let a be the function given by $a(x) = 3x^3h(x)$. Write an expression for $a'(x)$. Find $a'(2)$.

   (c) The function h satisfies $h(x) = \dfrac{x^2 - 4}{1 - (f(x))^3}$ for $x \neq 2$. It is known that $\lim\limits_{x \to 2} h(x)$ can be evaluated using L'Hospital's Rule. Use $\lim\limits_{x \to 2} h(x)$ to find $f(2)$ and $f'(2)$. Show the work that leads to your answers.

   (d) It is known that $g(x) \leq h(x)$ for $1 < x < 3$. Let k be a function satisfying $g(x) \leq k(x) \leq h(x)$ for $1 < x < 3$. Is k continuous at $x = 2$? Justify your answer.

STOP

END OF EXAM

2018

AP® CollegeBoard

AP Calculus AB

Free-Response Questions

CALCULUS AB

SECTION II, Part A

Time—30 minutes

Number of questions—2

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

1. People enter a line for an escalator at a rate modeled by the function r given by

$$r(t) = \begin{cases} 44\left(\dfrac{t}{100}\right)^3\left(1 - \dfrac{t}{300}\right)^7 & \text{for } 0 \le t \le 300 \\ 0 & \text{for } t > 300, \end{cases}$$

where $r(t)$ is measured in people per second and t is measured in seconds. As people get on the escalator, they exit the line at a constant rate of 0.7 person per second. There are 20 people in line at time $t = 0$.

(a) How many people enter the line for the escalator during the time interval $0 \le t \le 300$?

(b) During the time interval $0 \le t \le 300$, there are always people in line for the escalator. How many people are in line at time $t = 300$?

(c) For $t > 300$, what is the first time t that there are no people in line for the escalator?

(d) For $0 \le t \le 300$, at what time t is the number of people in line a minimum? To the nearest whole number, find the number of people in line at this time. Justify your answer.

GO ON TO THE NEXT PAGE.

2. A particle moves along the x-axis with velocity given by $v(t) = \dfrac{10\sin\left(0.4t^2\right)}{t^2 - t + 3}$ for time $0 \le t \le 3.5$.

   The particle is at position $x = -5$ at time $t = 0$.

   (a) Find the acceleration of the particle at time $t = 3$.

   (b) Find the position of the particle at time $t = 3$.

   (c) Evaluate $\int_0^{3.5} v(t)\,dt$, and evaluate $\int_0^{3.5} \left|v(t)\right|\,dt$. Interpret the meaning of each integral in the context of the problem.

   (d) A second particle moves along the x-axis with position given by $x_2(t) = t^2 - t$ for $0 \le t \le 3.5$. At what time t are the two particles moving with the same velocity?

END OF PART A OF SECTION II

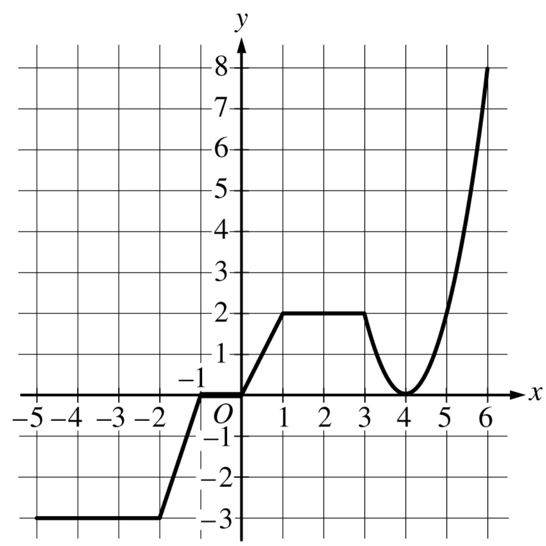
CALCULUS AB

SECTION II, Part B

Time—1 hour

Number of questions—4

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.



Graph of g

3. The graph of the continuous function g, the derivative of the function f, is shown above. The function g is piecewise linear for $-5 \le x < 3$, and $g(x) = 2(x - 4)^2$ for $3 \le x \le 6$.

   (a) If $f(1) = 3$, what is the value of $f(-5)$?

   (b) Evaluate $\int_1^6 g(x)\, dx$.

   (c) For $-5 < x < 6$, on what open intervals, if any, is the graph of f both increasing and concave up? Give a reason for your answer.

   (d) Find the x-coordinate of each point of inflection of the graph of f. Give a reason for your answer.

t (years)	2	3	5	7	10
$H(t)$ (meters)	1.5	2	6	11	15

4. The height of a tree at time t is given by a twice-differentiable function H, where $H(t)$ is measured in meters and t is measured in years. Selected values of $H(t)$ are given in the table above.

(a) Use the data in the table to estimate $H'(6)$. Using correct units, interpret the meaning of $H'(6)$ in the context of the problem.

(b) Explain why there must be at least one time t, for $2 < t < 10$, such that $H'(t) = 2$.

(c) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the average height of the tree over the time interval $2 \le t \le 10$.

(d) The height of the tree, in meters, can also be modeled by the function G, given by $G(x) = \dfrac{100x}{1 + x}$, where x is the diameter of the base of the tree, in meters. When the tree is 50 meters tall, the diameter of the base of the tree is increasing at a rate of 0.03 meter per year. According to this model, what is the rate of change of the height of the tree with respect to time, in meters per year, at the time when the tree is 50 meters tall?

GO ON TO THE NEXT PAGE.

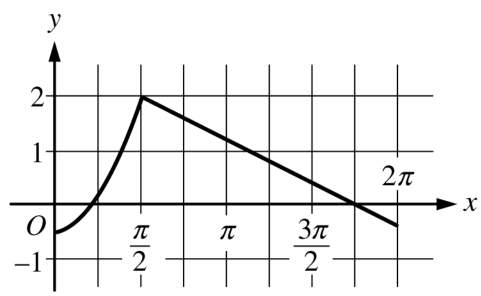
5. Let f be the function defined by $f(x) = e^x \cos x$.

(a) Find the average rate of change of f on the interval $0 \le x \le \pi$.

(b) What is the slope of the line tangent to the graph of f at $x = \dfrac{3\pi}{2}$?

(c) Find the absolute minimum value of f on the interval $0 \le x \le 2\pi$. Justify your answer.

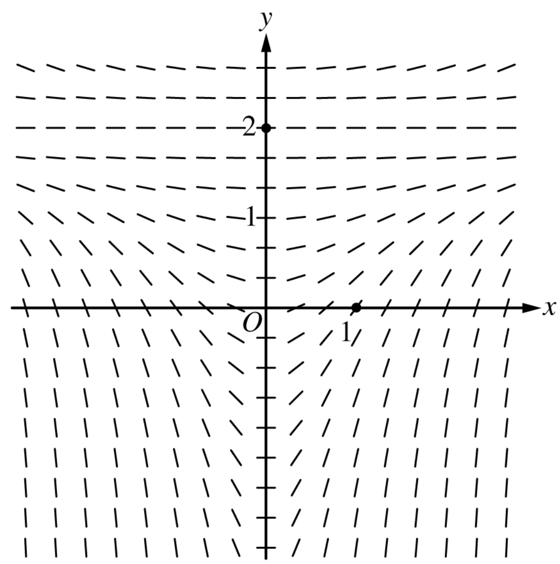
(d) Let g be a differentiable function such that $g\left(\dfrac{\pi}{2}\right) = 0$. The graph of g', the derivative of g, is shown below. Find the value of $\displaystyle\lim_{x \to \pi/2} \frac{f(x)}{g(x)}$ or state that it does not exist. Justify your answer.



Graph of g'

6. Consider the differential equation $\frac{dy}{dx} = \frac{1}{3}x(y-2)^2$.

(a) A slope field for the given differential equation is shown below. Sketch the solution curve that passes through the point $(0, 2)$, and sketch the solution curve that passes through the point $(1, 0)$.



(b) Let $y = f(x)$ be the particular solution to the given differential equation with initial condition $f(1) = 0$. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$. Use your equation to approximate $f(0.7)$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with initial condition $f(1) = 0$.

STOP

END OF EXAM

2017

AP® CollegeBoard

AP Calculus AB

Free-Response Questions

CALCULUS AB

SECTION II, Part A

Time—30 minutes

Number of questions—2

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

h (feet)	0	2	5	10
$A(h)$ (square feet)	50.3	14.4	6.5	2.9

1. A tank has a height of 10 feet. The area of the horizontal cross section of the tank at height h feet is given by the function A, where $A(h)$ is measured in square feet. The function A is continuous and decreases as h increases. Selected values for $A(h)$ are given in the table above.

(a) Use a left Riemann sum with the three subintervals indicated by the data in the table to approximate the volume of the tank. Indicate units of measure.

(b) Does the approximation in part (a) overestimate or underestimate the volume of the tank? Explain your reasoning.

(c) The area, in square feet, of the horizontal cross section at height h feet is modeled by the function f given by $f(h) = \dfrac{50.3}{e^{0.2h} + h}$. Based on this model, find the volume of the tank. Indicate units of measure.

(d) Water is pumped into the tank. When the height of the water is 5 feet, the height is increasing at the rate of 0.26 foot per minute. Using the model from part (c), find the rate at which the volume of water is changing with respect to time when the height of the water is 5 feet. Indicate units of measure.

2. When a certain grocery store opens, it has 50 pounds of bananas on a display table. Customers remove bananas from the display table at a rate modeled by

$$f(t) = 10 + (0.8t)\sin\left(\frac{t^3}{100}\right) \text{ for } 0 < t \le 12,$$

where $f(t)$ is measured in pounds per hour and t is the number of hours after the store opened. After the store has been open for three hours, store employees add bananas to the display table at a rate modeled by

$$g(t) = 3 + 2.4\ln\left(t^2 + 2t\right) \text{ for } 3 < t \le 12,$$

where $g(t)$ is measured in pounds per hour and t is the number of hours after the store opened.

(a) How many pounds of bananas are removed from the display table during the first 2 hours the store is open?

(b) Find $f'(7)$. Using correct units, explain the meaning of $f'(7)$ in the context of the problem.

(c) Is the number of pounds of bananas on the display table increasing or decreasing at time $t = 5$? Give a reason for your answer.

(d) How many pounds of bananas are on the display table at time $t = 8$?

END OF PART A OF SECTION II

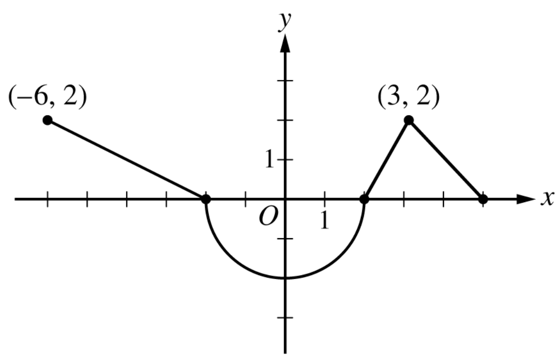
CALCULUS AB

SECTION II, Part B

Time—1 hour

Number of questions—4

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.



Graph of f'

3. The function f is differentiable on the closed interval $[-6, 5]$ and satisfies $f(-2) = 7$. The graph of f', the derivative of f, consists of a semicircle and three line segments, as shown in the figure above.

(a) Find the values of $f(-6)$ and $f(5)$.

(b) On what intervals is f increasing? Justify your answer.

(c) Find the absolute minimum value of f on the closed interval $[-6, 5]$. Justify your answer.

(d) For each of $f''(-5)$ and $f''(3)$, find the value or explain why it does not exist.

4. At time $t = 0$, a boiled potato is taken from a pot on a stove and left to cool in a kitchen. The internal temperature of the potato is 91 degrees Celsius (°C) at time $t = 0$, and the internal temperature of the potato is greater than 27°C for all times $t > 0$. The internal temperature of the potato at time t minutes can be modeled by the function H that satisfies the differential equation $\frac{dH}{dt} = -\frac{1}{4}(H - 27)$, where $H(t)$ is measured in degrees Celsius and $H(0) = 91$.

(a) Write an equation for the line tangent to the graph of H at $t = 0$. Use this equation to approximate the internal temperature of the potato at time $t = 3$.

(b) Use $\frac{d^2H}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the internal temperature of the potato at time $t = 3$.

(c) For $t < 10$, an alternate model for the internal temperature of the potato at time t minutes is the function G that satisfies the differential equation $\frac{dG}{dt} = -(G - 27)^{2/3}$, where $G(t)$ is measured in degrees Celsius and $G(0) = 91$. Find an expression for $G(t)$. Based on this model, what is the internal temperature of the potato at time $t = 3$?

5. Two particles move along the x-axis. For $0 \le t \le 8$, the position of particle P at time t is given by $x_P(t) = \ln\left(t^2 - 2t + 10\right)$, while the velocity of particle Q at time t is given by $v_Q(t) = t^2 - 8t + 15$. Particle Q is at position $x = 5$ at time $t = 0$.

   (a) For $0 \le t \le 8$, when is particle P moving to the left?

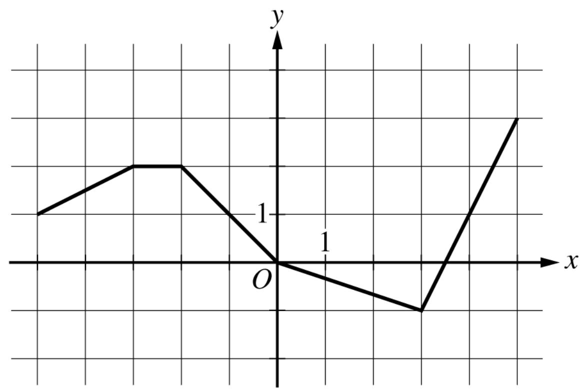
   (b) For $0 \le t \le 8$, find all times t during which the two particles travel in the same direction.

   (c) Find the acceleration of particle Q at time $t = 2$. Is the speed of particle Q increasing, decreasing, or neither at time $t = 2$? Explain your reasoning.

   (d) Find the position of particle Q the first time it changes direction.

---

x	$g(x)$	$g'(x)$
–5	10	–3
–4	5	–1
–3	2	4
–2	3	1
–1	1	–2
0	0	–3



Graph of h

6. Let f be the function defined by $f(x) = \cos(2x) + e^{\sin x}$.

   Let g be a differentiable function. The table above gives values of g and its derivative g' at selected values of x.

   Let h be the function whose graph, consisting of five line segments, is shown in the figure above.

   (a) Find the slope of the line tangent to the graph of f at $x = \pi$.

   (b) Let k be the function defined by $k(x) = h(f(x))$. Find $k'(\pi)$.

   (c) Let m be the function defined by $m(x) = g(-2x) \cdot h(x)$. Find $m'(2)$.

   (d) Is there a number c in the closed interval $[-5, -3]$ such that $g'(c) = -4$? Justify your answer.

STOP

END OF EXAM

AP® Calculus AB
2016 Free-Response Questions

CALCULUS AB

SECTION II, Part A

Time—30 minutes

Number of problems—2

A graphing calculator is required for these problems.

t (hours)	0	1	3	6	8
$R(t)$ (liters / hour)	1340	1190	950	740	700

1. Water is pumped into a tank at a rate modeled by $W(t) = 2000e^{-t^2/20}$ liters per hour for $0 \le t \le 8,$ where t is measured in hours. Water is removed from the tank at a rate modeled by $R(t)$ liters per hour, where R is differentiable and decreasing on $0 \le t \le 8.$ Selected values of $R(t)$ are shown in the table above. At time $t = 0,$ there are 50,000 liters of water in the tank.

   (a) Estimate $R'(2).$ Show the work that leads to your answer. Indicate units of measure.

   (b) Use a left Riemann sum with the four subintervals indicated by the table to estimate the total amount of water removed from the tank during the 8 hours. Is this an overestimate or an underestimate of the total amount of water removed? Give a reason for your answer.

   (c) Use your answer from part (b) to find an estimate of the total amount of water in the tank, to the nearest liter, at the end of 8 hours.

   (d) For $0 \le t \le 8,$ is there a time t when the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank? Explain why or why not.

---

2. For $t \geq 0,$ a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = 1 + 2\sin\left(\frac{t^2}{2}\right).$ The particle is at position $x = 2$ at time $t = 4.$

   (a) At time $t = 4,$ is the particle speeding up or slowing down?

   (b) Find all times t in the interval $0 < t < 3$ when the particle changes direction. Justify your answer.

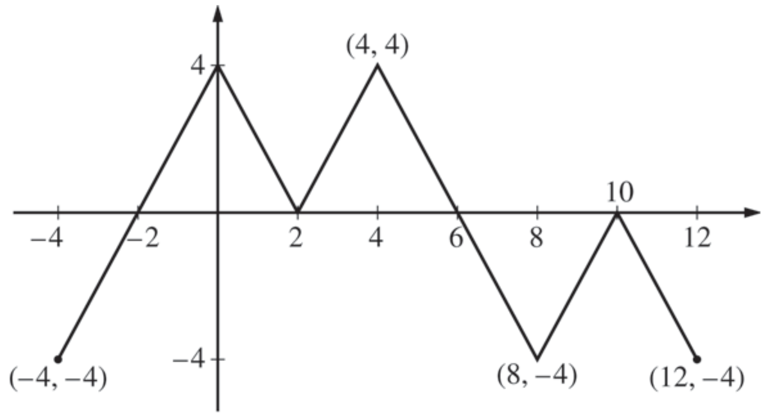
   (c) Find the position of the particle at time $t = 0.$

   (d) Find the total distance the particle travels from time $t = 0$ to time $t = 3.$

END OF PART A OF SECTION II

CALCULUS AB
SECTION II, Part B
Time—60 minutes
Number of problems—4

No calculator is allowed for these problems.



Graph of f

3. The figure above shows the graph of the piecewise-linear function f. For $-4 \leq x \leq 12$, the function g is defined by $g(x) = \int_2^x f(t)\,dt$.

(a) Does g have a relative minimum, a relative maximum, or neither at $x = 10$? Justify your answer.

(b) Does the graph of g have a point of inflection at $x = 4$? Justify your answer.

(c) Find the absolute minimum value and the absolute maximum value of g on the interval $-4 \leq x \leq 12$. Justify your answers.

(d) For $-4 \leq x \leq 12$, find all intervals for which $g(x) \leq 0$.

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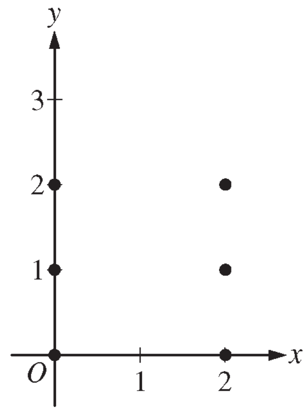
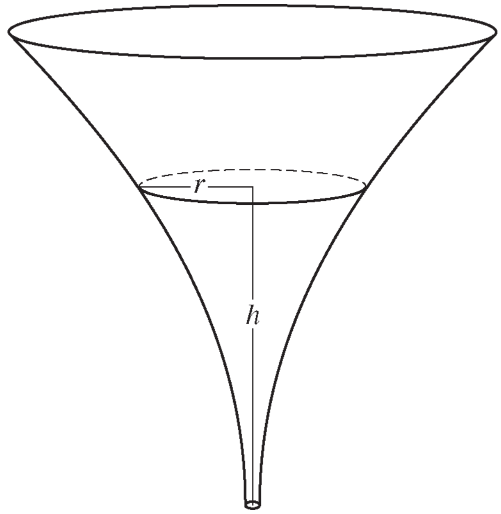
4. Consider the differential equation $\frac{dy}{dx} = \frac{y^2}{x - 1}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.



(b) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(2) = 3$. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 2$. Use your equation to approximate $f(2.1)$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(2) = 3$.



5. The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height h, the radius of the funnel is given by $r = \frac{1}{20}\left(3 + h^2\right)$, where $0 \le h \le 10$. The units of r and h are inches.

   (a) Find the average value of the radius of the funnel.

   (b) Find the volume of the funnel.

   (c) The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is $h = 3$ inches, the radius of the surface of the liquid is decreasing at a rate of $\frac{1}{5}$ inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	–6	3	2	8
2	2	–2	–3	0
3	8	7	6	2
6	4	5	3	–1

6. The functions f and g have continuous second derivatives. The table above gives values of the functions and their derivatives at selected values of x.

   (a) Let $k(x) = f(g(x))$. Write an equation for the line tangent to the graph of k at $x = 3$.

   (b) Let $h(x) = \dfrac{g(x)}{f(x)}$. Find $h'(1)$.

   (c) Evaluate $\int_1^3 f''(2x)\,dx$.

STOP

END OF EXAM

AP® Calculus AB
2015 Free-Response Questions

CALCULUS AB
SECTION II, Part A
Time—30 minutes
Number of problems—2

A graphing calculator is required for these problems.

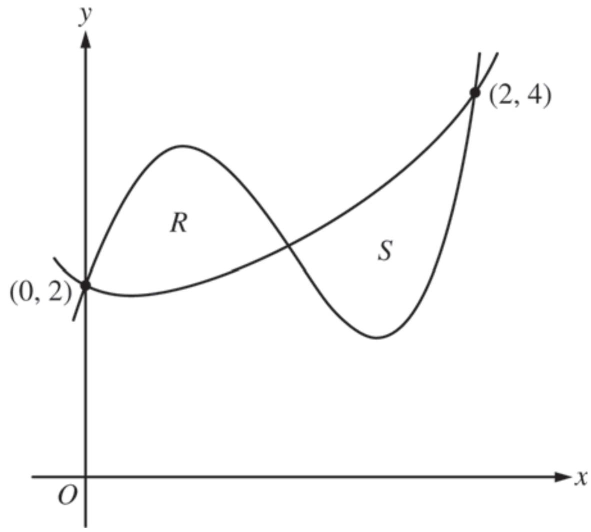
1. The rate at which rainwater flows into a drainpipe is modeled by the function R, where $R(t) = 20\sin\left(\frac{t^2}{35}\right)$ cubic feet per hour, t is measured in hours, and $0 \le t \le 8$. The pipe is partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$ cubic feet per hour, for $0 \le t \le 8$. There are 30 cubic feet of water in the pipe at time $t = 0$.

   (a) How many cubic feet of rainwater flow into the pipe during the 8-hour time interval $0 \le t \le 8$?

   (b) Is the amount of water in the pipe increasing or decreasing at time $t = 3$ hours? Give a reason for your answer.

   (c) At what time t, $0 \le t \le 8$, is the amount of water in the pipe at a minimum? Justify your answer.

   (d) The pipe can hold 50 cubic feet of water before overflowing. For $t > 8$, water continues to flow into and out of the pipe at the given rates until the pipe begins to overflow. Write, but do not solve, an equation involving one or more integrals that gives the time w when the pipe will begin to overflow.



2. Let f and g be the functions defined by $f(x) = 1 + x + e^{x^2 - 2x}$ and $g(x) = x^4 - 6.5x^2 + 6x + 2$. Let R and S be the two regions enclosed by the graphs of f and g shown in the figure above.

(a) Find the sum of the areas of regions R and S.

(b) Region S is the base of a solid whose cross sections perpendicular to the x-axis are squares. Find the volume of the solid.

(c) Let h be the vertical distance between the graphs of f and g in region S. Find the rate at which h changes with respect to x when $x = 1.8$.

END OF PART A OF SECTION II

CALCULUS AB

SECTION II, Part B

Time—60 minutes

Number of problems—4

No calculator is allowed for these problems.

t (minutes)	0	12	20	24	40
$v(t)$ (meters per minute)	0	200	240	–220	150

3. Johanna jogs along a straight path. For $0 \le t \le 40,$ Johanna's velocity is given by a differentiable function v. Selected values of $v(t),$ where t is measured in minutes and $v(t)$ is measured in meters per minute, are given in the table above.

   (a) Use the data in the table to estimate the value of $v'(16)$.

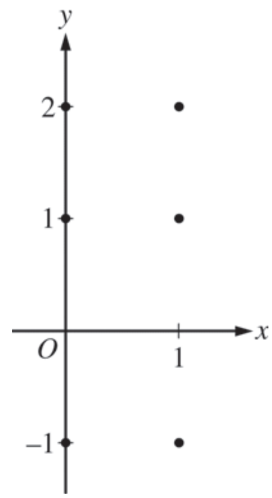
   (b) Using correct units, explain the meaning of the definite integral $\int_0^{40} |v(t)|\, dt$ in the context of the problem. Approximate the value of $\int_0^{40} |v(t)|\, dt$ using a right Riemann sum with the four subintervals indicated in the table.

   (c) Bob is riding his bicycle along the same path. For $0 \le t \le 10,$ Bob's velocity is modeled by $B(t) = t^3 - 6t^2 + 300,$ where t is measured in minutes and $B(t)$ is measured in meters per minute. Find Bob's acceleration at time $t = 5$.

   (d) Based on the model B from part (c), find Bob's average velocity during the interval $0 \le t \le 10$.

4. Consider the differential equation $\frac{dy}{dx} = 2x - y$.

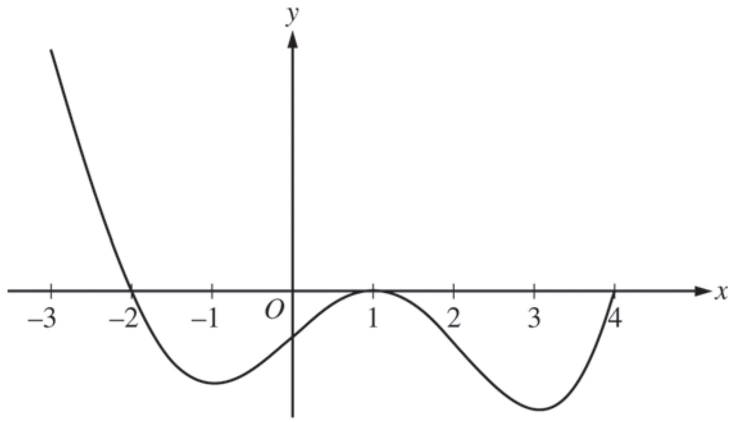
   (a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.



   (b) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Determine the concavity of all solution curves for the given differential equation in Quadrant II. Give a reason for your answer.

   (c) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(2) = 3$. Does f have a relative minimum, a relative maximum, or neither at $x = 2$? Justify your answer.

   (d) Find the values of the constants m and b for which $y = mx + b$ is a solution to the differential equation.



Graph of f'

5. The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the interval $[-3, 4]$. The graph of f' has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$. The areas of the regions bounded by the x-axis and the graph of f' on the intervals $[-2, 1]$ and $[1, 4]$ are 9 and 12, respectively.

(a) Find all x-coordinates at which f has a relative maximum. Give a reason for your answer.

(b) On what open intervals contained in $-3 < x < 4$ is the graph of f both concave down and decreasing? Give a reason for your answer.

(c) Find the x-coordinates of all points of inflection for the graph of f. Give a reason for your answer.

(d) Given that $f(1) = 3$, write an expression for $f(x)$ that involves an integral. Find $f(4)$ and $f(-2)$.

6. Consider the curve given by the equation $y^3 - xy = 2$. It can be shown that $\frac{dy}{dx} = \frac{y}{3y^2 - x}$.

(a) Write an equation for the line tangent to the curve at the point $(-1, 1)$.

(b) Find the coordinates of all points on the curve at which the line tangent to the curve at that point is vertical.

(c) Evaluate $\frac{d^2y}{dx^2}$ at the point on the curve where $x = -1$ and $y = 1$.

STOP

END OF EXAM

AP® Calculus AB 2014 Free-Response Questions

CALCULUS AB
SECTION II, Part A
Time—30 minutes
Number of problems—2

A graphing calculator is required for these problems.

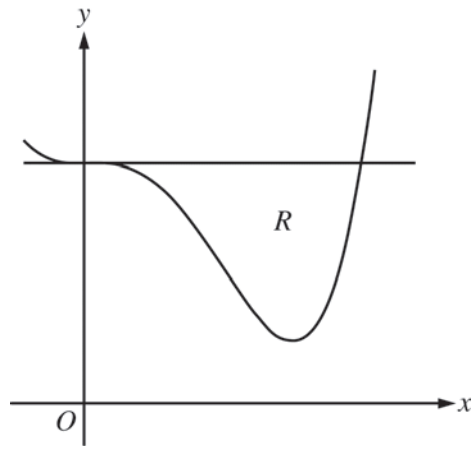
1. Grass clippings are placed in a bin, where they decompose. For $0 \le t \le 30,$ the amount of grass clippings remaining in the bin is modeled by $A(t) = 6.687(0.931)^t,$ where $A(t)$ is measured in pounds and t is measured in days.

   (a) Find the average rate of change of $A(t)$ over the interval $0 \le t \le 30.$ Indicate units of measure.

   (b) Find the value of $A'(15).$ Using correct units, interpret the meaning of the value in the context of the problem.

   (c) Find the time t for which the amount of grass clippings in the bin is equal to the average amount of grass clippings in the bin over the interval $0 \le t \le 30.$

   (d) For $t > 30,$ $L(t),$ the linear approximation to A at $t = 30,$ is a better model for the amount of grass clippings remaining in the bin. Use $L(t)$ to predict the time at which there will be 0.5 pound of grass clippings remaining in the bin. Show the work that leads to your answer.



2. Let R be the region enclosed by the graph of $f(x) = x^4 - 2.3x^3 + 4$ and the horizontal line $y = 4$, as shown in the figure above.

   (a) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

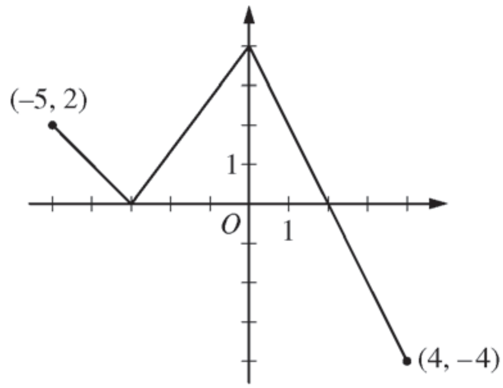
   (b) Region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an isosceles right triangle with a leg in R. Find the volume of the solid.

   (c) The vertical line $x = k$ divides R into two regions with equal areas. Write, but do not solve, an equation involving integral expressions whose solution gives the value k.

END OF PART A OF SECTION II

CALCULUS AB
SECTION II, Part B
Time—60 minutes
Number of problems—4

No calculator is allowed for these problems.



Graph of f

3. The function f is defined on the closed interval $[-5, 4]$. The graph of f consists of three line segments and is shown in the figure above. Let g be the function defined by $g(x) = \int_{-3}^{x} f(t)\,dt$.

(a) Find $g(3)$.

(b) On what open intervals contained in $-5 < x < 4$ is the graph of g both increasing and concave down? Give a reason for your answer.

(c) The function h is defined by $h(x) = \dfrac{g(x)}{5x}$. Find $h'(3)$.

(d) The function p is defined by $p(x) = f\left(x^2 - x\right)$. Find the slope of the line tangent to the graph of p at the point where $x = -1$.

t (minutes)	0	2	5	8	12
$v_A(t)$ (meters/minute)	0	100	40	–120	–150

4. Train A runs back and forth on an east-west section of railroad track. Train A's velocity, measured in meters per minute, is given by a differentiable function $v_A(t)$, where time t is measured in minutes. Selected values for $v_A(t)$ are given in the table above.

   (a) Find the average acceleration of train A over the interval $2 \le t \le 8$.

   (b) Do the data in the table support the conclusion that train A's velocity is -100 meters per minute at some time t with $5 < t < 8$? Give a reason for your answer.

   (c) At time $t = 2$, train A's position is 300 meters east of the Origin Station, and the train is moving to the east. Write an expression involving an integral that gives the position of train A, in meters from the Origin Station, at time $t = 12$. Use a trapezoidal sum with three subintervals indicated by the table to approximate the position of the train at time $t = 12$.

   (d) A second train, train B, travels north from the Origin Station. At time t the velocity of train B is given by $v_B(t) = -5t^2 + 60t + 25$, and at time $t = 2$ the train is 400 meters north of the station. Find the rate, in meters per minute, at which the distance between train A and train B is changing at time $t = 2$.

---

x	-2	$-2 < x < -1$	-1	$-1 < x < 1$	1	$1 < x < 3$	3
$f(x)$	12	Positive	8	Positive	2	Positive	7
$f'(x)$	-5	Negative	0	Negative	0	Positive	$\frac{1}{2}$
$g(x)$	-1	Negative	0	Positive	3	Positive	1
$g'(x)$	2	Positive	$\frac{3}{2}$	Positive	0	Negative	-2

5. The twice-differentiable functions f and g are defined for all real numbers x. Values of f, f', g, and g' for various values of x are given in the table above.

   (a) Find the x-coordinate of each relative minimum of f on the interval $[-2, 3]$. Justify your answers.

   (b) Explain why there must be a value c, for $-1 < c < 1$, such that $f''(c) = 0$.

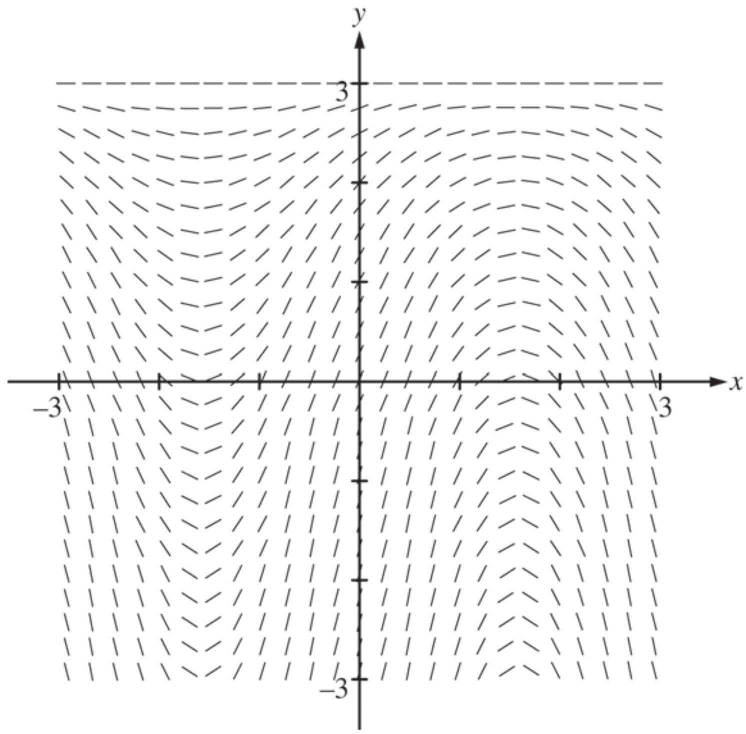
   (c) The function h is defined by $h(x) = \ln(f(x))$. Find $h'(3)$. Show the computations that lead to your answer.

   (d) Evaluate $\int_{-2}^{3} f'(g(x))g'(x)\,dx$.

---

6. Consider the differential equation $\frac{dy}{dx} = (3 - y)\cos x$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(0) = 1$. The function f is defined for all real numbers.

(a) A portion of the slope field of the differential equation is given below. Sketch the solution curve through the point $(0, 1)$.



(b) Write an equation for the line tangent to the solution curve in part (a) at the point $(0, 1)$. Use the equation to approximate $f(0.2)$.

(c) Find $y = f(x)$, the particular solution to the differential equation with the initial condition $f(0) = 1$.

STOP

END OF EXAM

AP® Calculus AB
2013 Free-Response Questions

About the College Board

The College Board is a mission-driven not-for-profit organization that connects students to college success and opportunity. Founded in 1900, the College Board was created to expand access to higher education. Today, the membership association is made up of more than 6,000 of the world's leading educational institutions and is dedicated to promoting excellence and equity in education. Each year, the College Board helps more than seven million students prepare for a successful transition to college through programs and services in college readiness and college success — including the SAT® and the Advanced Placement Program®. The organization also serves the education community through research and advocacy on behalf of students, educators, and schools.

Visit the College Board on the Web: www.collegeboard.org.
AP Central is the official online home for the AP Program: apcentral.collegeboard.org.

CALCULUS AB
SECTION II, Part A
Time—30 minutes
Number of problems—2

A graphing calculator is required for these problems.

1. On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \le t \le 8$. At the beginning of the workday $(t = 0)$, the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \le t \le 8$, the plant processes gravel at a constant rate of 100 tons per hour.

   (a) Find $G'(5)$. Using correct units, interpret your answer in the context of the problem.

   (b) Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.

   (c) Is the amount of unprocessed gravel at the plant increasing or decreasing at time $t = 5$ hours? Show the work that leads to your answer.

   (d) What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.

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2. A particle moves along a straight line. For $0 \le t \le 5$, the velocity of the particle is given by $v(t) = -2 + \left(t^2 + 3t\right)^{6/5} - t^3$, and the position of the particle is given by $s(t)$. It is known that $s(0) = 10$.

   (a) Find all values of t in the interval $2 \le t \le 4$ for which the speed of the particle is 2.

   (b) Write an expression involving an integral that gives the position $s(t)$. Use this expression to find the position of the particle at time $t = 5$.

   (c) Find all times t in the interval $0 \le t \le 5$ at which the particle changes direction. Justify your answer.

   (d) Is the speed of the particle increasing or decreasing at time $t = 4$? Give a reason for your answer.

END OF PART A OF SECTION II

CALCULUS AB
SECTION II, Part B
Time—60 minutes
Number of problems—4

No calculator is allowed for these problems.

t (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

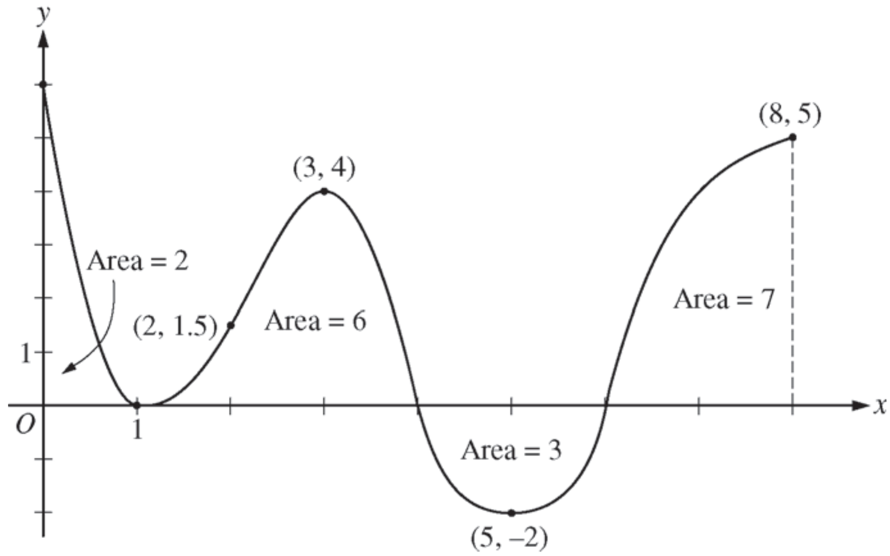
3. Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t, $0 \le t \le 6$, is given by a differentiable function C, where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.

   (a) Use the data in the table to approximate $C'(3.5)$. Show the computations that lead to your answer, and indicate units of measure.

   (b) Is there a time t, $2 \le t \le 4$, at which $C'(t) = 2$? Justify your answer.

   (c) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\frac{1}{6}\int_0^6 C(t)\,dt$. Using correct units, explain the meaning of $\frac{1}{6}\int_0^6 C(t)\,dt$ in the context of the problem.

   (d) The amount of coffee in the cup, in ounces, is modeled by $B(t) = 16 - 16e^{-0.4t}$. Using this model, find the rate at which the amount of coffee in the cup is changing when $t = 5$.



Graph of f'

4. The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the closed interval $0 \le x \le 8$. The graph of f' has horizontal tangent lines at $x = 1$, $x = 3$, and $x = 5$. The areas of the regions between the graph of f' and the x-axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(8) = 4$.

   (a) Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.

   (b) Determine the absolute minimum value of f on the closed interval $0 \le x \le 8$. Justify your answer.

   (c) On what open intervals contained in $0 < x < 8$ is the graph of f both concave down and increasing? Explain your reasoning.

   (d) The function g is defined by $g(x) = (f(x))^3$. If $f(3) = -\frac{5}{2}$, find the slope of the line tangent to the graph of g at $x = 3$.

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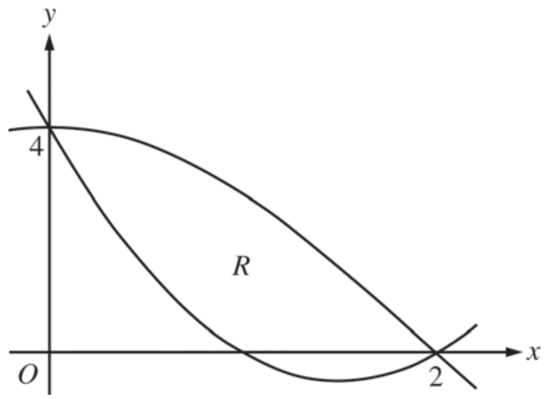

5. Let $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$. Let R be the region bounded by the graphs of f and g, as shown in the figure above.

(a) Find the area of R.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 4$.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.

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6. Consider the differential equation $\frac{dy}{dx} = e^y\left(3x^2 - 6x\right)$. Let $y = f(x)$ be the particular solution to the differential equation that passes through $(1, 0)$.

(a) Write an equation for the line tangent to the graph of f at the point $(1, 0)$. Use the tangent line to approximate $f(1.2)$.

(b) Find $y = f(x)$, the particular solution to the differential equation that passes through $(1, 0)$.

STOP

END OF EXAM

AP® Calculus AB
2012 Free-Response Questions

CALCULUS AB
SECTION II, Part A
Time—30 minutes
Number of problems—2

A graphing calculator is required for these problems.

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

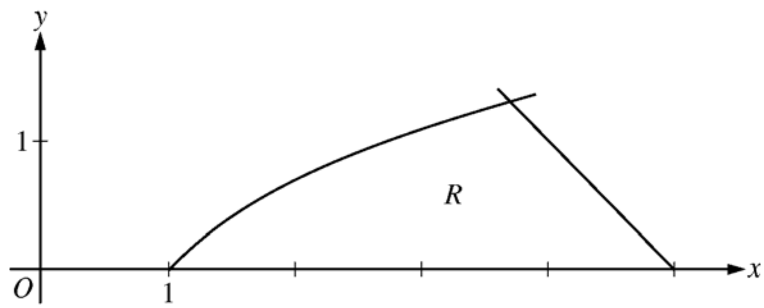
1. The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t = 0$, the temperature of the water is $55°\text{F}$. The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

   (a) Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

   (b) Use the data in the table to evaluate $\int_0^{20} W'(t)\,dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t)\,dt$ in the context of this problem.

   (c) For $0 \le t \le 20$, the average temperature of the water in the tub is $\frac{1}{20}\int_0^{20} W(t)\,dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20}\int_0^{20} W(t)\,dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

   (d) For $20 \le t \le 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t}\cos(0.06t)$. Based on the model, what is the temperature of the water at time $t = 25$?



2. Let R be the region in the first quadrant bounded by the x-axis and the graphs of $y = \ln x$ and $y = 5 - x$, as shown in the figure above.

(a) Find the area of R.

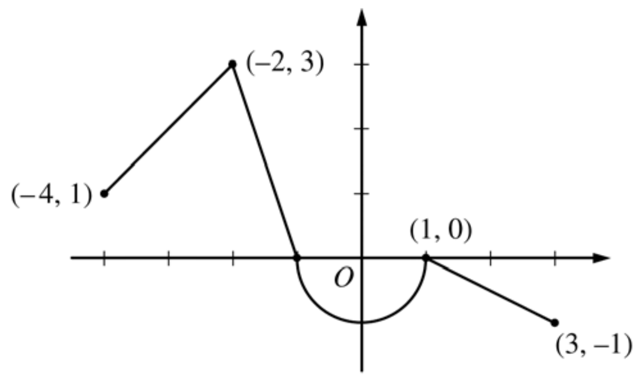
(b) Region R is the base of a solid. For the solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

(c) The horizontal line $y = k$ divides R into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k.

END OF PART A OF SECTION II

CALCULUS AB
SECTION II, Part B
Time—60 minutes
Number of problems—4

No calculator is allowed for these problems.



Graph of f

3. Let f be the continuous function defined on $[-4, 3]$ whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let g be the function given by $g(x) = \int_1^x f(t)\,dt.$

   (a) Find the values of $g(2)$ and $g(-2)$.

   (b) For each of $g'(-3)$ and $g''(-3)$, find the value or state that it does not exist.

   (c) Find the x-coordinate of each point at which the graph of g has a horizontal tangent line. For each of these points, determine whether g has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.

   (d) For $-4 < x < 3$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning.

4. The function f is defined by $f(x) = \sqrt{25 - x^2}$ for $-5 \le x \le 5$.

(a) Find $f'(x)$.

(b) Write an equation for the line tangent to the graph of f at $x = -3$.

(c) Let g be the function defined by $g(x) = \begin{cases} f(x) & \text{for } -5 \le x \le -3 \\ x + 7 & \text{for } -3 < x \le 5. \end{cases}$

Is g continuous at $x = -3$? Use the definition of continuity to explain your answer.

(d) Find the value of $\int_0^5 x\sqrt{25 - x^2}\, dx$.

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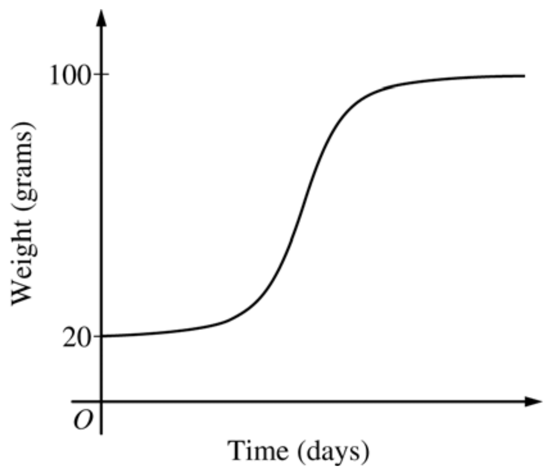
5. The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0,$ when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20.$

(a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.

(b) Find $\frac{d^2B}{dt^2}$ in terms of B. Use $\frac{d^2B}{dt^2}$ to explain why the graph of B cannot resemble the following graph.



(c) Use separation of variables to find $y = B(t),$ the particular solution to the differential equation with initial condition $B(0) = 20.$

6. For $0 \le t \le 12$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = \cos\left(\frac{\pi}{6}t\right)$. The particle is at position $x = -2$ at time $t = 0$.

   (a) For $0 \le t \le 12$, when is the particle moving to the left?

   (b) Write, but do not evaluate, an integral expression that gives the total distance traveled by the particle from time $t = 0$ to time $t = 6$.

   (c) Find the acceleration of the particle at time t. Is the speed of the particle increasing, decreasing, or neither at time $t = 4$? Explain your reasoning.

   (d) Find the position of the particle at time $t = 4$.

STOP

END OF EXAM

AP® Calculus AB
2011 Free-Response Questions

About the College Board

CALCULUS AB
SECTION II, Part A
Time—30 minutes
Number of problems—2

A graphing calculator is required for these problems.

1. For $0 \le t \le 6$, a particle is moving along the x-axis. The particle's position, $x(t)$, is not explicitly given. The velocity of the particle is given by $v(t) = 2\sin\left(e^{t/4}\right) + 1$. The acceleration of the particle is given by $a(t) = \frac{1}{2}e^{t/4}\cos\left(e^{t/4}\right)$ and $x(0) = 2$.

   (a) Is the speed of the particle increasing or decreasing at time $t = 5.5$? Give a reason for your answer.

   (b) Find the average velocity of the particle for the time period $0 \le t \le 6$.

   (c) Find the total distance traveled by the particle from time $t = 0$ to $t = 6$.

   (d) For $0 \le t \le 6$, the particle changes direction exactly once. Find the position of the particle at that time.

WRITE ALL WORK IN THE EXAM BOOKLET.

t (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

2. As a pot of tea cools, the temperature of the tea is modeled by a differentiable function H for $0 \le t \le 10$, where time t is measured in minutes and temperature $H(t)$ is measured in degrees Celsius. Values of $H(t)$ at selected values of time t are shown in the table above.

(a) Use the data in the table to approximate the rate at which the temperature of the tea is changing at time $t = 3.5$. Show the computations that lead to your answer.

(b) Using correct units, explain the meaning of $\frac{1}{10}\int_0^{10} H(t)\,dt$ in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate $\frac{1}{10}\int_0^{10} H(t)\,dt$.

(c) Evaluate $\int_0^{10} H'(t)\,dt$. Using correct units, explain the meaning of the expression in the context of this problem.

(d) At time $t = 0$, biscuits with temperature $100°\text{C}$ were removed from an oven. The temperature of the biscuits at time t is modeled by a differentiable function B for which it is known that $B'(t) = -13.84e^{-0.173t}$. Using the given models, at time $t = 10$, how much cooler are the biscuits than the tea?

WRITE ALL WORK IN THE EXAM BOOKLET.

END OF PART A OF SECTION II

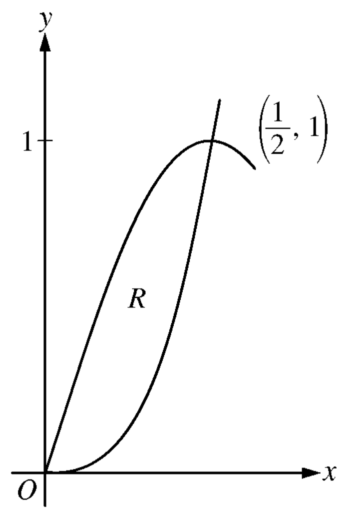
CALCULUS AB

SECTION II, Part B

Time—60 minutes

Number of problems—4

No calculator is allowed for these problems.



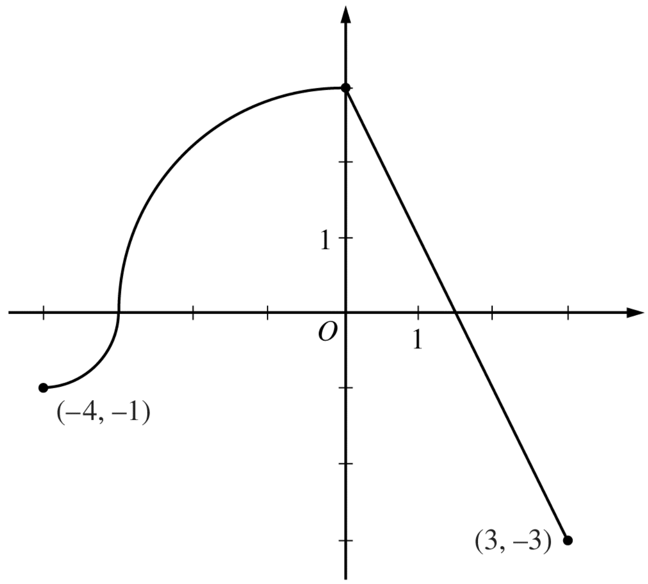
3. Let R be the region in the first quadrant enclosed by the graphs of $f(x) = 8x^3$ and $g(x) = \sin(\pi x)$, as shown in the figure above.

   (a) Write an equation for the line tangent to the graph of f at $x = \frac{1}{2}$.

   (b) Find the area of R.

   (c) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 1$.

WRITE ALL WORK IN THE EXAM BOOKLET.



Graph of f

4. The continuous function f is defined on the interval $-4 \le x \le 3$. The graph of f consists of two quarter circles and one line segment, as shown in the figure above. Let $g(x) = 2x + \int_0^x f(t)\,dt$.

   (a) Find $g(-3)$. Find $g'(x)$ and evaluate $g'(-3)$.

   (b) Determine the x-coordinate of the point at which g has an absolute maximum on the interval $-4 \le x \le 3$. Justify your answer.

   (c) Find all values of x on the interval $-4 < x < 3$ for which the graph of g has a point of inflection. Give a reason for your answer.

   (d) Find the average rate of change of f on the interval $-4 \le x \le 3$. There is no point c, $-4 < c < 3$, for which $f'(c)$ is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem.

WRITE ALL WORK IN THE EXAM BOOKLET.

5. At the beginning of 2010, a landfill contained 1400 tons of solid waste. The increasing function W models the total amount of solid waste stored at the landfill. Planners estimate that W will satisfy the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ for the next 20 years. W is measured in tons, and t is measured in years from the start of 2010.

   (a) Use the line tangent to the graph of W at $t = 0$ to approximate the amount of solid waste that the landfill contains at the end of the first 3 months of 2010 (time $t = \frac{1}{4}$).

   (b) Find $\frac{d^2W}{dt^2}$ in terms of W. Use $\frac{d^2W}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the amount of solid waste that the landfill contains at time $t = \frac{1}{4}$.

   (c) Find the particular solution $W = W(t)$ to the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ with initial condition $W(0) = 1400$.

---

6. Let f be a function defined by $f(x) = \begin{cases} 1 - 2\sin x & \text{for } x \le 0 \\ e^{-4x} & \text{for } x > 0. \end{cases}$

   (a) Show that f is continuous at $x = 0$.

   (b) For $x \ne 0$, express $f'(x)$ as a piecewise-defined function. Find the value of x for which $f'(x) = -3$.

   (c) Find the average value of f on the interval $[-1, 1]$.

WRITE ALL WORK IN THE EXAM BOOKLET.

END OF EXAM

CollegeBoard
Advanced Placement
Program

AP® Calculus AB
2011 Free-Response Questions
Form B

CALCULUS AB
SECTION II, Part A
Time—30 minutes
Number of problems—2

A graphing calculator is required for these problems.

1. A cylindrical can of radius 10 millimeters is used to measure rainfall in Stormville. The can is initially empty, and rain enters the can during a 60-day period. The height of water in the can is modeled by the function S, where $S(t)$ is measured in millimeters and t is measured in days for $0 \le t \le 60$. The rate at which the height of the water is rising in the can is given by $S'(t) = 2\sin(0.03t) + 1.5$.

   (a) According to the model, what is the height of the water in the can at the end of the 60-day period?

   (b) According to the model, what is the average rate of change in the height of water in the can over the 60-day period? Show the computations that lead to your answer. Indicate units of measure.

   (c) Assuming no evaporation occurs, at what rate is the volume of water in the can changing at time $t = 7$? Indicate units of measure.

   (d) During the same 60-day period, rain on Monsoon Mountain accumulates in a can identical to the one in Stormville. The height of the water in the can on Monsoon Mountain is modeled by the function M, where $M(t) = \frac{1}{400}\left(3t^3 - 30t^2 + 330t\right)$. The height $M(t)$ is measured in millimeters, and t is measured in days for $0 \le t \le 60$. Let $D(t) = M'(t) - S'(t)$. Apply the Intermediate Value Theorem to the function D on the interval $0 \le t \le 60$ to justify that there exists a time t, $0 < t < 60$, at which the heights of water in the two cans are changing at the same rate.

---

2. A 12,000-liter tank of water is filled to capacity. At time $t = 0$, water begins to drain out of the tank at a rate modeled by $r(t)$, measured in liters per hour, where r is given by the piecewise-defined function

$$r(t) = \begin{cases} \dfrac{600t}{t+3} & \text{for } 0 \le t \le 5 \\ 1000e^{-0.2t} & \text{for } t > 5 \end{cases}$$

   (a) Is r continuous at $t = 5$? Show the work that leads to your answer.

   (b) Find the average rate at which water is draining from the tank between time $t = 0$ and time $t = 8$ hours.

   (c) Find $r'(3)$. Using correct units, explain the meaning of that value in the context of this problem.

   (d) Write, but do not solve, an equation involving an integral to find the time A when the amount of water in the tank is 9000 liters.

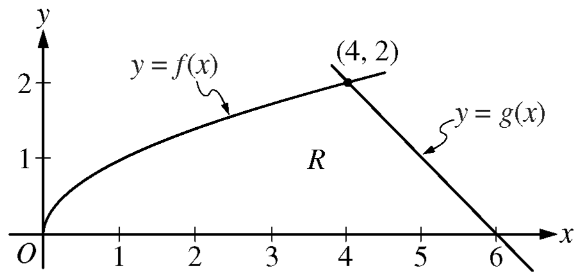
WRITE ALL WORK IN THE EXAM BOOKLET.

END OF PART A OF SECTION II

GO ON TO THE NEXT PAGE.

CALCULUS AB
SECTION II, Part B
Time—60 minutes
Number of problems—4

No calculator is allowed for these problems.



3. The functions f and g are given by $f(x) = \sqrt{x}$ and $g(x) = 6 - x$. Let R be the region bounded by the x-axis and the graphs of f and g, as shown in the figure above.

   (a) Find the area of R.

   (b) The region R is the base of a solid. For each y, where $0 \le y \le 2$, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose base lies in R and whose height is $2y$. Write, but do not evaluate, an integral expression that gives the volume of the solid.

   (c) There is a point P on the graph of f at which the line tangent to the graph of f is perpendicular to the graph of g. Find the coordinates of point P.

---

4. Consider a differentiable function f having domain all positive real numbers, and for which it is known that $f'(x) = (4 - x)x^{-3}$ for $x > 0$.

   (a) Find the x-coordinate of the critical point of f. Determine whether the point is a relative maximum, a relative minimum, or neither for the function f. Justify your answer.

   (b) Find all intervals on which the graph of f is concave down. Justify your answer.

   (c) Given that $f(1) = 2$, determine the function f.

WRITE ALL WORK IN THE EXAM BOOKLET.

t (seconds)	0	10	40	60
$B(t)$ (meters)	100	136	9	49
$v(t)$ (meters per second)	2.0	2.3	2.5	4.6

5. Ben rides a unicycle back and forth along a straight east-west track. The twice-differentiable function B models Ben's position on the track, measured in meters from the western end of the track, at time t, measured in seconds from the start of the ride. The table above gives values for $B(t)$ and Ben's velocity, $v(t)$, measured in meters per second, at selected times t.

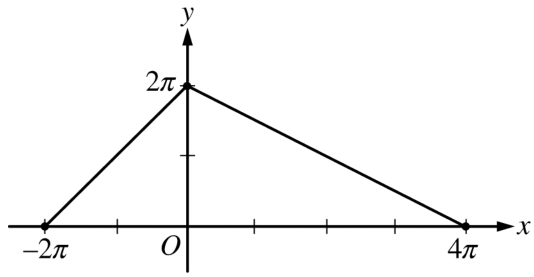
   (a) Use the data in the table to approximate Ben's acceleration at time $t = 5$ seconds. Indicate units of measure.

   (b) Using correct units, interpret the meaning of $\int_0^{60} |v(t)|\, dt$ in the context of this problem. Approximate $\int_0^{60} |v(t)|\, dt$ using a left Riemann sum with the subintervals indicated by the data in the table.

   (c) For $40 \le t \le 60$, must there be a time t when Ben's velocity is 2 meters per second? Justify your answer.

   (d) A light is directly above the western end of the track. Ben rides so that at time t, the distance $L(t)$ between Ben and the light satisfies $(L(t))^2 = 12^2 + (B(t))^2$. At what rate is the distance between Ben and the light changing at time $t = 40$?

WRITE ALL WORK IN THE EXAM BOOKLET.



Graph of g

6. Let g be the piecewise-linear function defined on $[-2\pi, 4\pi]$ whose graph is given above, and let $f(x) = g(x) - \cos\left(\frac{x}{2}\right)$.

   (a) Find $\int_{-2\pi}^{4\pi} f(x)\,dx$. Show the computations that lead to your answer.

   (b) Find all x-values in the open interval $(-2\pi, 4\pi)$ for which f has a critical point.

   (c) Let $h(x) = \int_0^{3x} g(t)\,dt$. Find $h'\left(-\frac{\pi}{3}\right)$.

WRITE ALL WORK IN THE EXAM BOOKLET.

END OF EXAM

Advanced Placement Program

AP® Calculus AB
2010 Free-Response Questions

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

1. There is no snow on Janet's driveway when snow begins to fall at midnight. From midnight to 9 A.M., snow accumulates on the driveway at a rate modeled by $f(t) = 7te^{\cos t}$ cubic feet per hour, where t is measured in hours since midnight. Janet starts removing snow at 6 A.M. $(t = 6)$. The rate $g(t)$, in cubic feet per hour, at which Janet removes snow from the driveway at time t hours after midnight is modeled by

$$g(t) = \begin{cases} 0 & \text{for } 0 \le t < 6 \\ 125 & \text{for } 6 \le t < 7 \\ 108 & \text{for } 7 \le t \le 9. \end{cases}$$

(a) How many cubic feet of snow have accumulated on the driveway by 6 A.M.?

(b) Find the rate of change of the volume of snow on the driveway at 8 A.M.

(c) Let $h(t)$ represent the total amount of snow, in cubic feet, that Janet has removed from the driveway at time t hours after midnight. Express h as a piecewise-defined function with domain $0 \le t \le 9$.

(d) How many cubic feet of snow are on the driveway at 9 A.M.?

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

t (hours)	0	2	5	7	8
$E(t)$ (hundreds of entries)	0	4	13	21	23

2. A zoo sponsored a one-day contest to name a new baby elephant. Zoo visitors deposited entries in a special box between noon $(t = 0)$ and 8 P.M. $(t = 8)$. The number of entries in the box t hours after noon is modeled by a differentiable function E for $0 \le t \le 8$. Values of $E(t)$, in hundreds of entries, at various times t are shown in the table above.

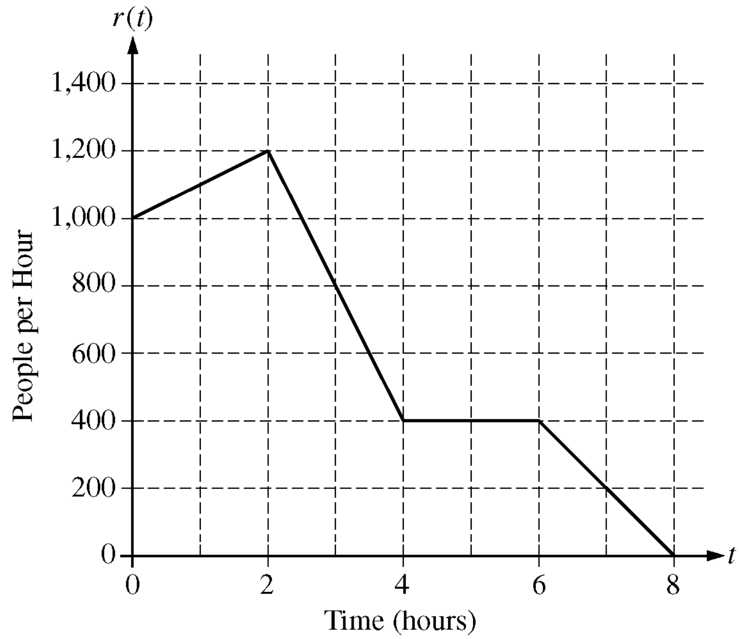
(a) Use the data in the table to approximate the rate, in hundreds of entries per hour, at which entries were being deposited at time $t = 6$. Show the computations that lead to your answer.

(b) Use a trapezoidal sum with the four subintervals given by the table to approximate the value of $\frac{1}{8}\int_0^8 E(t)\,dt$. Using correct units, explain the meaning of $\frac{1}{8}\int_0^8 E(t)\,dt$ in terms of the number of entries.

(c) At 8 P.M., volunteers began to process the entries. They processed the entries at a rate modeled by the function P, where $P(t) = t^3 - 30t^2 + 298t - 976$ hundreds of entries per hour for $8 \le t \le 12$. According to the model, how many entries had not yet been processed by midnight $(t = 12)$?

(d) According to the model from part (c), at what time were the entries being processed most quickly? Justify your answer.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.



3. There are 700 people in line for a popular amusement-park ride when the ride begins operation in the morning. Once it begins operation, the ride accepts passengers until the park closes 8 hours later. While there is a line, people move onto the ride at a rate of 800 people per hour. The graph above shows the rate, $r(t)$, at which people arrive at the ride throughout the day. Time t is measured in hours from the time the ride begins operation.

   (a) How many people arrive at the ride between $t = 0$ and $t = 3$? Show the computations that lead to your answer.

   (b) Is the number of people waiting in line to get on the ride increasing or decreasing between $t = 2$ and $t = 3$? Justify your answer.

   (c) At what time t is the line for the ride the longest? How many people are in line at that time? Justify your answers.

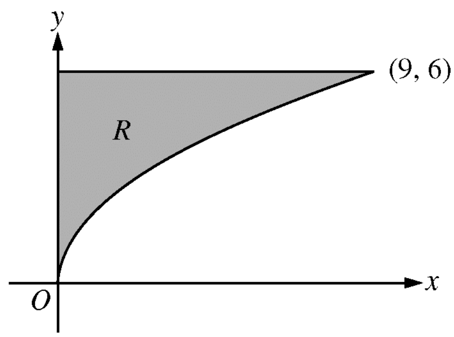
   (d) Write, but do not solve, an equation involving an integral expression of r whose solution gives the earliest time t at which there is no longer a line for the ride.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

END OF PART A OF SECTION II

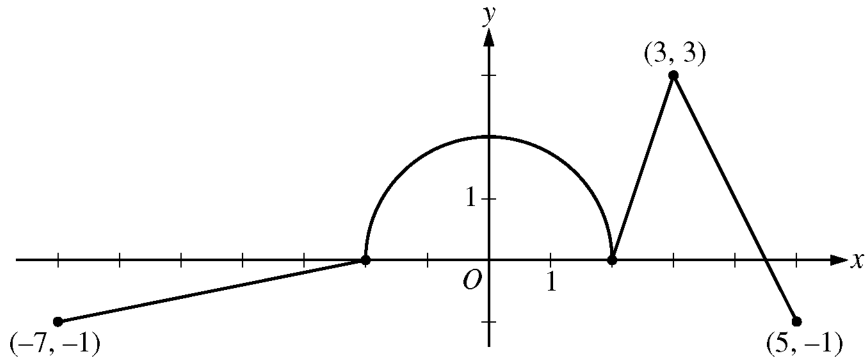
CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



4. Let R be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$, the horizontal line $y = 6$, and the y-axis, as shown in the figure above.
   (a) Find the area of R.
   (b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 7$.
   (c) Region R is the base of a solid. For each y, where $0 \le y \le 6$, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose height is 3 times the length of its base in region R. Write, but do not evaluate, an integral expression that gives the volume of the solid.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.



Graph of g'

5. The function g is defined and differentiable on the closed interval $[-7, 5]$ and satisfies $g(0) = 5$. The graph of $y = g'(x)$, the derivative of g, consists of a semicircle and three line segments, as shown in the figure above.

   (a) Find $g(3)$ and $g(-2)$.

   (b) Find the x-coordinate of each point of inflection of the graph of $y = g(x)$ on the interval $-7 < x < 5$. Explain your reasoning.

   (c) The function h is defined by $h(x) = g(x) - \frac{1}{2}x^2$. Find the x-coordinate of each critical point of h, where $-7 < x < 5$, and classify each critical point as the location of a relative minimum, relative maximum, or neither a minimum nor a maximum. Explain your reasoning.

---

6. Solutions to the differential equation $\frac{dy}{dx} = xy^3$ also satisfy $\frac{d^2y}{dx^2} = y^3\left(1 + 3x^2y^2\right)$. Let $y = f(x)$ be a particular solution to the differential equation $\frac{dy}{dx} = xy^3$ with $f(1) = 2$.

   (a) Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$.

   (b) Use the tangent line equation from part (a) to approximate $f(1.1)$. Given that $f(x) > 0$ for $1 < x < 1.1$, is the approximation for $f(1.1)$ greater than or less than $f(1.1)$? Explain your reasoning.

   (c) Find the particular solution $y = f(x)$ with initial condition $f(1) = 2$.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

END OF EXAM

CollegeBoard
Advanced Placement Program

AP® Calculus AB
2010 Free-Response Questions
Form B

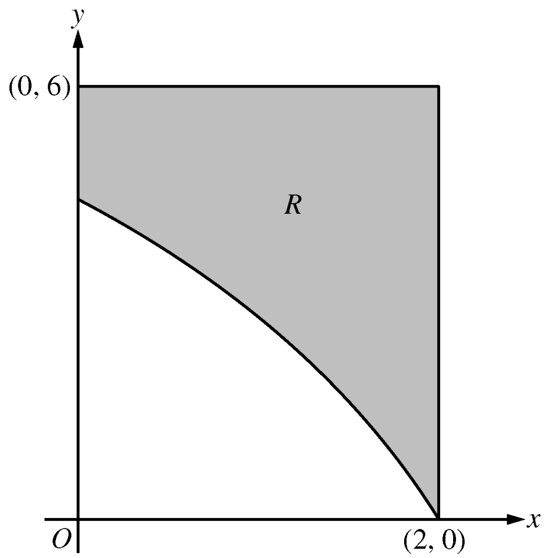
The College Board

The College Board is a not-for-profit membership association whose mission is to connect students to college success and opportunity. Founded in 1900, the College Board is composed of more than 5,700 schools, colleges, universities and other educational organizations. Each year, the College Board serves seven million students and their parents, 23,000 high schools, and 3,800 colleges through major programs and services in college readiness, college admission, guidance, assessment, financial aid and enrollment. Among its widely recognized programs are the SAT®, the PSAT/NMSQT®, the Advanced Placement Program® (AP®), SpringBoard® and ACCUPLACER®. The College Board is committed to the principles of excellence and equity, and that commitment is embodied in all of its programs, services, activities and concerns.

Visit the College Board on the Web: www.collegeboard.com.
AP Central is the official online home for the AP Program: apcentral.collegeboard.com.

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. In the figure above, R is the shaded region in the first quadrant bounded by the graph of $y = 4\ln(3 - x)$, the horizontal line $y = 6$, and the vertical line $x = 2$.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 8$.

   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of the solid.

WRITE ALL WORK IN THE EXAM BOOKLET.

2. The function g is defined for $x > 0$ with $g(1) = 2$, $g'(x) = \sin\left(x + \frac{1}{x}\right)$, and $g''(x) = \left(1 - \frac{1}{x^2}\right)\cos\left(x + \frac{1}{x}\right)$.

   (a) Find all values of x in the interval $0.12 \le x \le 1$ at which the graph of g has a horizontal tangent line.

   (b) On what subintervals of $(0.12, 1)$, if any, is the graph of g concave down? Justify your answer.

   (c) Write an equation for the line tangent to the graph of g at $x = 0.3$.

   (d) Does the line tangent to the graph of g at $x = 0.3$ lie above or below the graph of g for $0.3 < x < 1$? Why?

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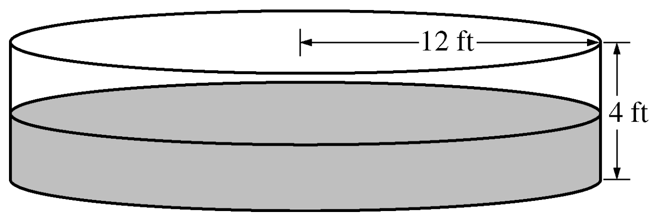
t	0	2	4	6	8	10	12
$P(t)$	0	46	53	57	60	62	63



3. The figure above shows an aboveground swimming pool in the shape of a cylinder with a radius of 12 feet and a height of 4 feet. The pool contains 1000 cubic feet of water at time $t = 0$. During the time interval $0 \le t \le 12$ hours, water is pumped into the pool at the rate $P(t)$ cubic feet per hour. The table above gives values of $P(t)$ for selected values of t. During the same time interval, water is leaking from the pool at the rate $R(t)$ cubic feet per hour, where $R(t) = 25e^{-0.05t}$. (Note: The volume V of a cylinder with radius r and height h is given by $V = \pi r^2 h$.)

   (a) Use a midpoint Riemann sum with three subintervals of equal length to approximate the total amount of water that was pumped into the pool during the time interval $0 \le t \le 12$ hours. Show the computations that lead to your answer.

   (b) Calculate the total amount of water that leaked out of the pool during the time interval $0 \le t \le 12$ hours.

   (c) Use the results from parts (a) and (b) to approximate the volume of water in the pool at time $t = 12$ hours. Round your answer to the nearest cubic foot.

   (d) Find the rate at which the volume of water in the pool is increasing at time $t = 8$ hours. How fast is the water level in the pool rising at $t = 8$ hours? Indicate units of measure in both answers.

WRITE ALL WORK IN THE EXAM BOOKLET.

END OF PART A OF SECTION II

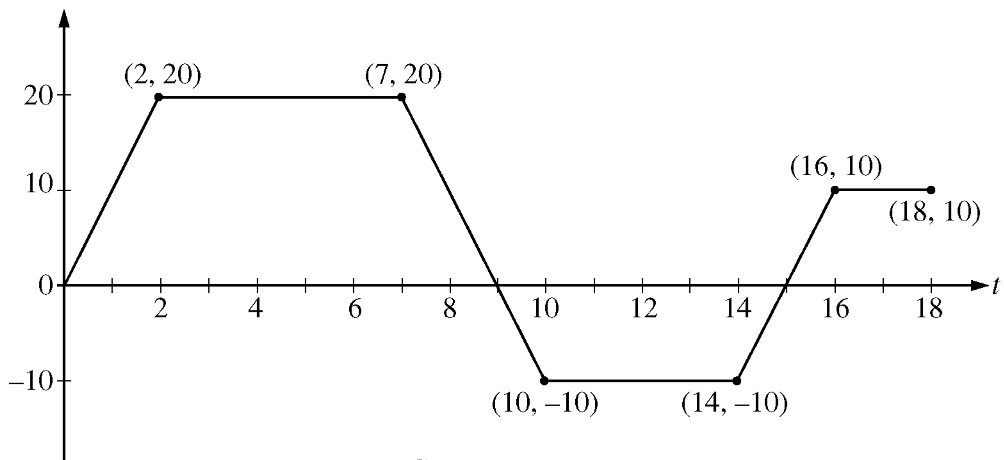
CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Graph of v

4. A squirrel starts at building A at time $t = 0$ and travels along a straight, horizontal wire connected to building B. For $0 \le t \le 18$, the squirrel's velocity is modeled by the piecewise-linear function defined by the graph above.

   (a) At what times in the interval $0 < t < 18$, if any, does the squirrel change direction? Give a reason for your answer.

   (b) At what time in the interval $0 \le t \le 18$ is the squirrel farthest from building A? How far from building A is the squirrel at that time?

   (c) Find the total distance the squirrel travels during the time interval $0 \le t \le 18$.

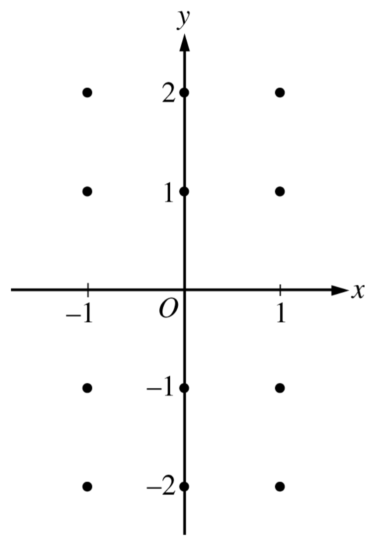
   (d) Write expressions for the squirrel's acceleration $a(t)$, velocity $v(t)$, and distance $x(t)$ from building A that are valid for the time interval $7 < t < 10$.

WRITE ALL WORK IN THE EXAM BOOKLET.

5. Consider the differential equation $\frac{dy}{dx} = \frac{x+1}{y}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated, and for $-1 < x < 1$, sketch the solution curve that passes through the point $(0, -1)$.

(Note: Use the axes provided in the exam booklet.)



(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the *xy*-plane for which $y \neq 0$. Describe all points in the *xy*-plane, $y \neq 0$, for which $\frac{dy}{dx} = -1$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = -2$.

---

6. Two particles move along the x-axis. For $0 \le t \le 6$, the position of particle P at time t is given by $p(t) = 2\cos\left(\frac{\pi}{4}t\right)$, while the position of particle R at time t is given by $r(t) = t^3 - 6t^2 + 9t + 3$.

(a) For $0 \le t \le 6$, find all times t during which particle R is moving to the right.

(b) For $0 \le t \le 6$, find all times t during which the two particles travel in opposite directions.

(c) Find the acceleration of particle P at time $t = 3$. Is particle P speeding up, slowing down, or doing neither at time $t = 3$? Explain your reasoning.

(d) Write, but do not evaluate, an expression for the average distance between the two particles on the interval $1 \le t \le 3$.

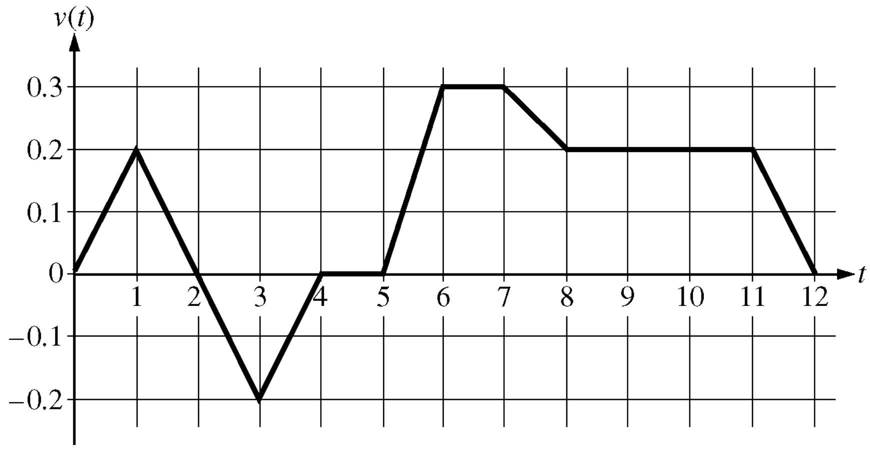
WRITE ALL WORK IN THE EXAM BOOKLET.

END OF EXAM

AP® Calculus AB
2009 Free-Response Questions

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Caren rides her bicycle along a straight road from home to school, starting at home at time $t = 0$ minutes and arriving at school at time $t = 12$ minutes. During the time interval $0 \le t \le 12$ minutes, her velocity $v(t)$, in miles per minute, is modeled by the piecewise-linear function whose graph is shown above.

   (a) Find the acceleration of Caren's bicycle at time $t = 7.5$ minutes. Indicate units of measure.

   (b) Using correct units, explain the meaning of $\int_0^{12} |v(t)|\, dt$ in terms of Caren's trip. Find the value of $\int_0^{12} |v(t)|\, dt$.

   (c) Shortly after leaving home, Caren realizes she left her calculus homework at home, and she returns to get it. At what time does she turn around to go back home? Give a reason for your answer.

   (d) Larry also rides his bicycle along a straight road from home to school in 12 minutes. His velocity is modeled by the function w given by $w(t) = \frac{\pi}{15}\sin\left(\frac{\pi}{12}t\right)$, where $w(t)$ is in miles per minute for $0 \le t \le 12$ minutes. Who lives closer to school: Caren or Larry? Show the work that leads to your answer.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

2. The rate at which people enter an auditorium for a rock concert is modeled by the function R given by $R(t) = 1380t^2 - 675t^3$ for $0 \le t \le 2$ hours; $R(t)$ is measured in people per hour. No one is in the auditorium at time $t = 0$, when the doors open. The doors close and the concert begins at time $t = 2$.

   (a) How many people are in the auditorium when the concert begins?

   (b) Find the time when the rate at which people enter the auditorium is a maximum. Justify your answer.

   (c) The total wait time for all the people in the auditorium is found by adding the time each person waits, starting at the time the person enters the auditorium and ending when the concert begins. The function w models the total wait time for all the people who enter the auditorium before time t. The derivative of w is given by $w'(t) = (2 - t)R(t)$. Find $w(2) - w(1)$, the total wait time for those who enter the auditorium after time $t = 1$.

   (d) On average, how long does a person wait in the auditorium for the concert to begin? Consider all people who enter the auditorium after the doors open, and use the model for total wait time from part (c).

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3. Mighty Cable Company manufactures cable that sells for $120 per meter. For a cable of fixed length, the cost of producing a portion of the cable varies with its distance from the beginning of the cable. Mighty reports that the cost to produce a portion of a cable that is x meters from the beginning of the cable is $6\sqrt{x}$ dollars per meter. (Note: Profit is defined to be the difference between the amount of money received by the company for selling the cable and the company's cost of producing the cable.)

   (a) Find Mighty's profit on the sale of a 25-meter cable.

   (b) Using correct units, explain the meaning of $\int_{25}^{30} 6\sqrt{x}\,dx$ in the context of this problem.

   (c) Write an expression, involving an integral, that represents Mighty's profit on the sale of a cable that is k meters long.

   (d) Find the maximum profit that Mighty could earn on the sale of one cable. Justify your answer.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

END OF PART A OF SECTION II

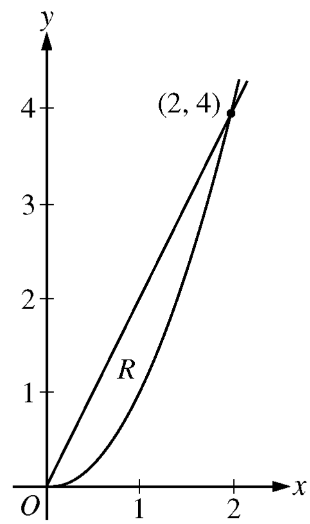
CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



4. Let R be the region in the first quadrant enclosed by the graphs of $y = 2x$ and $y = x^2$, as shown in the figure above.

   (a) Find the area of R.

   (b) The region R is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis has area $A(x) = \sin\left(\frac{\pi}{2}x\right)$. Find the volume of the solid.

   (c) Another solid has the same base R. For this solid, the cross sections perpendicular to the y-axis are squares. Write, but do not evaluate, an integral expression for the volume of the solid.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

x	2	3	5	8	13
$f(x)$	1	4	–2	3	6

5. Let f be a function that is twice differentiable for all real numbers. The table above gives values of f for selected points in the closed interval $2 \le x \le 13$.

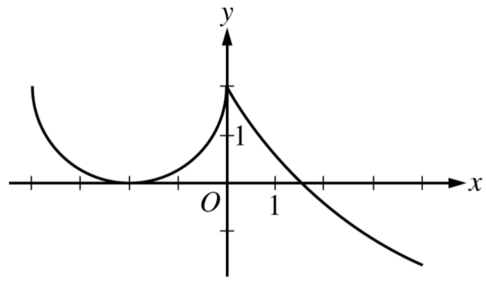
(a) Estimate $f'(4)$. Show the work that leads to your answer.

(b) Evaluate $\int_2^{13} (3 - 5f'(x))\, dx$. Show the work that leads to your answer.

(c) Use a left Riemann sum with subintervals indicated by the data in the table to approximate $\int_2^{13} f(x)\, dx$. Show the work that leads to your answer.

(d) Suppose $f'(5) = 3$ and $f''(x) < 0$ for all x in the closed interval $5 \le x \le 8$. Use the line tangent to the graph of f at $x = 5$ to show that $f(7) \le 4$. Use the secant line for the graph of f on $5 \le x \le 8$ to show that $f(7) \ge \frac{4}{3}$.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.



Graph of f'

6. The derivative of a function f is defined by $f'(x) = \begin{cases} g(x) & \text{for } -4 \le x \le 0 \\ 5e^{-x/3} - 3 & \text{for } \quad 0 < x \le 4 \end{cases}$.

The graph of the continuous function f', shown in the figure above, has x-intercepts at $x = -2$ and $x = 3\ln\left(\frac{5}{3}\right)$. The graph of g on $-4 \le x \le 0$ is a semicircle, and $f(0) = 5$.

(a) For $-4 < x < 4$, find all values of x at which the graph of f has a point of inflection. Justify your answer.

(b) Find $f(-4)$ and $f(4)$.

(c) For $-4 \le x \le 4$, find the value of x at which f has an absolute maximum. Justify your answer.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

END OF EXAM

CollegeBoard
Advanced Placement Program

AP® Calculus AB
2009 Free-Response Questions
Form B

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

1. At a certain height, a tree trunk has a circular cross section. The radius $R(t)$ of that cross section grows at a rate modeled by the function

$$\frac{dR}{dt} = \frac{1}{16}\left(3 + \sin\left(t^2\right)\right) \text{ centimeters per year}$$

for $0 \le t \le 3,$ where time t is measured in years. At time $t = 0,$ the radius is 6 centimeters. The area of the cross section at time t is denoted by $A(t).$

(a) Write an expression, involving an integral, for the radius $R(t)$ for $0 \le t \le 3.$ Use your expression to find $R(3).$

(b) Find the rate at which the cross-sectional area $A(t)$ is increasing at time $t = 3$ years. Indicate units of measure.

(c) Evaluate $\int_0^3 A'(t)\,dt.$ Using appropriate units, interpret the meaning of that integral in terms of cross-sectional area.

WRITE ALL WORK IN THE EXAM BOOKLET.

2. A storm washed away sand from a beach, causing the edge of the water to get closer to a nearby road. The rate at which the distance between the road and the edge of the water was changing during the storm is modeled by $f(t) = \sqrt{t} + \cos t - 3$ meters per hour, t hours after the storm began. The edge of the water was 35 meters from the road when the storm began, and the storm lasted 5 hours. The derivative of $f(t)$ is $f'(t) = \dfrac{1}{2\sqrt{t}} - \sin t$.

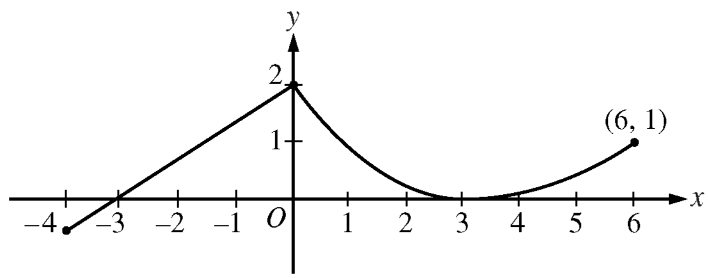
   (a) What was the distance between the road and the edge of the water at the end of the storm?

   (b) Using correct units, interpret the value $f'(4) = 1.007$ in terms of the distance between the road and the edge of the water.

   (c) At what time during the 5 hours of the storm was the distance between the road and the edge of the water decreasing most rapidly? Justify your answer.

   (d) After the storm, a machine pumped sand back onto the beach so that the distance between the road and the edge of the water was growing at a rate of $g(p)$ meters per day, where p is the number of days since pumping began. Write an equation involving an integral expression whose solution would give the number of days that sand must be pumped to restore the original distance between the road and the edge of the water.

WRITE ALL WORK IN THE EXAM BOOKLET.



Graph of f

3. A continuous function f is defined on the closed interval $-4 \le x \le 6$. The graph of f consists of a line segment and a curve that is tangent to the x-axis at $x = 3$, as shown in the figure above. On the interval $0 < x < 6$, the function f is twice differentiable, with $f''(x) > 0$.

   (a) Is f differentiable at $x = 0$? Use the definition of the derivative with one-sided limits to justify your answer.

   (b) For how many values of a, $-4 \le a < 6$, is the average rate of change of f on the interval $[a, 6]$ equal to 0 ? Give a reason for your answer.

   (c) Is there a value of a, $-4 \le a < 6$, for which the Mean Value Theorem, applied to the interval $[a, 6]$, guarantees a value c, $a < c < 6$, at which $f'(c) = \frac{1}{3}$? Justify your answer.

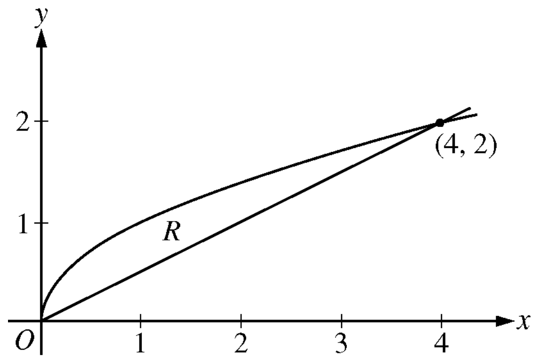
   (d) The function g is defined by $g(x) = \int_0^x f(t)\,dt$ for $-4 \le x \le 6$. On what intervals contained in $[-4, 6]$ is the graph of g concave up? Explain your reasoning.

WRITE ALL WORK IN THE EXAM BOOKLET.

END OF PART A OF SECTION II

CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



4. Let R be the region bounded by the graphs of $y = \sqrt{x}$ and $y = \dfrac{x}{2}$, as shown in the figure above.

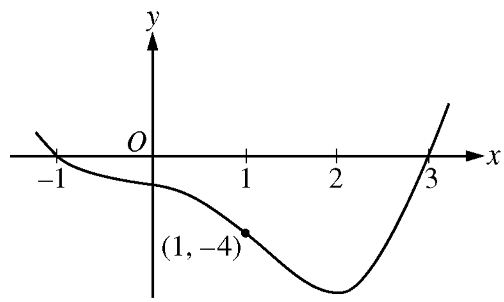
   (a) Find the area of R.

   (b) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are squares. Find the volume of this solid.

   (c) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 2$.

WRITE ALL WORK IN THE EXAM BOOKLET.

GO ON TO THE NEXT PAGE.



Graph of f'

5. Let f be a twice-differentiable function defined on the interval $-1.2 < x < 3.2$ with $f(1) = 2$. The graph of f', the derivative of f, is shown above. The graph of f' crosses the x-axis at $x = -1$ and $x = 3$ and has a horizontal tangent at $x = 2$. Let g be the function given by $g(x) = e^{f(x)}$.

   (a) Write an equation for the line tangent to the graph of g at $x = 1$.

   (b) For $-1.2 < x < 3.2$, find all values of x at which g has a local maximum. Justify your answer.

   (c) The second derivative of g is $g''(x) = e^{f(x)}\left[\left(f'(x)\right)^2 + f''(x)\right]$. Is $g''(-1)$ positive, negative, or zero? Justify your answer.

   (d) Find the average rate of change of g', the derivative of g, over the interval $[1, 3]$.

WRITE ALL WORK IN THE EXAM BOOKLET.

t (seconds)	0	8	20	25	32	40
$v(t)$ (meters per second)	3	5	–10	–8	–4	7

6. The velocity of a particle moving along the x-axis is modeled by a differentiable function v, where the position x is measured in meters, and time t is measured in seconds. Selected values of $v(t)$ are given in the table above. The particle is at position $x = 7$ meters when $t = 0$ seconds.

   (a) Estimate the acceleration of the particle at $t = 36$ seconds. Show the computations that lead to your answer. Indicate units of measure.

   (b) Using correct units, explain the meaning of $\int_{20}^{40} v(t)\,dt$ in the context of this problem. Use a trapezoidal sum with the three subintervals indicated by the data in the table to approximate $\int_{20}^{40} v(t)\,dt$.

   (c) For $0 \le t \le 40,$ must the particle change direction in any of the subintervals indicated by the data in the table? If so, identify the subintervals and explain your reasoning. If not, explain why not.

   (d) Suppose that the acceleration of the particle is positive for $0 < t < 8$ seconds. Explain why the position of the particle at $t = 8$ seconds must be greater than $x = 30$ meters.

WRITE ALL WORK IN THE EXAM BOOKLET.

END OF EXAM

CollegeBoard
Advanced Placement Program

AP® Calculus AB
2008 Free-Response Questions

The College Board: Connecting Students to College Success

The College Board is a not-for-profit membership association whose mission is to connect students to college success and opportunity. Founded in 1900, the association is composed of more than 5,000 schools, colleges, universities, and other educational organizations. Each year, the College Board serves seven million students and their parents, 23,000 high schools, and 3,500 colleges through major programs and services in college admissions, guidance, assessment, financial aid, enrollment, and teaching and learning. Among its best-known programs are the SAT®, the PSAT/NMSQT®, and the Advanced Placement Program® (AP®). The College Board is committed to the principles of excellence and equity, and that commitment is embodied in all of its programs, services, activities, and concerns.

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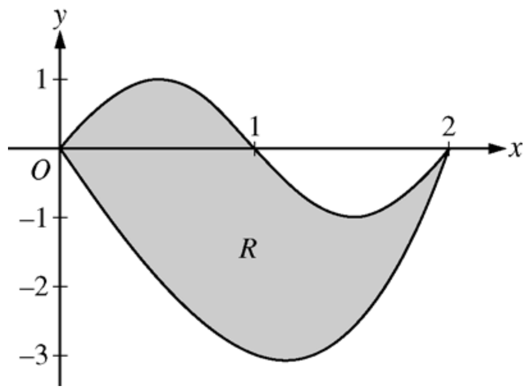
AP Central is the official online home for the AP Program: apcentral.collegeboard.com.

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let R be the region bounded by the graphs of $y = \sin(\pi x)$ and $y = x^3 - 4x$, as shown in the figure above.

   (a) Find the area of R.

   (b) The horizontal line $y = -2$ splits the region R into two parts. Write, but do not evaluate, an integral expression for the area of the part of R that is below this horizontal line.

   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

   (d) The region R models the surface of a small pond. At all points in R at a distance x from the y-axis, the depth of the water is given by $h(x) = 3 - x$. Find the volume of water in the pond.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

2. Concert tickets went on sale at noon $(t = 0)$ and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time t is modeled by a twice-differentiable function L for $0 \le t \le 9.$ Values of $L(t)$ at various times t are shown in the table above.

   (a) Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. $(t = 5.5).$ Show the computations that lead to your answer. Indicate units of measure.

   (b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.

   (c) For $0 \le t \le 9,$ what is the fewest number of times at which $L'(t)$ must equal 0 ? Give a reason for your answer.

   (d) The rate at which tickets were sold for $0 \le t \le 9$ is modeled by $r(t) = 550te^{-t/2}$ tickets per hour. Based on the model, how many tickets were sold by 3 P.M. $(t = 3),$ to the nearest whole number?

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

3. Oil is leaking from a pipeline on the surface of a lake and forms an oil slick whose volume increases at a constant rate of 2000 cubic centimeters per minute. The oil slick takes the form of a right circular cylinder with both its radius and height changing with time. (Note: The volume V of a right circular cylinder with radius r and height h is given by $V = \pi r^2 h.$)

   (a) At the instant when the radius of the oil slick is 100 centimeters and the height is 0.5 centimeter, the radius is increasing at the rate of 2.5 centimeters per minute. At this instant, what is the rate of change of the height of the oil slick with respect to time, in centimeters per minute?

   (b) A recovery device arrives on the scene and begins removing oil. The rate at which oil is removed is $R(t) = 400\sqrt{t}$ cubic centimeters per minute, where t is the time in minutes since the device began working. Oil continues to leak at the rate of 2000 cubic centimeters per minute. Find the time t when the oil slick reaches its maximum volume. Justify your answer.

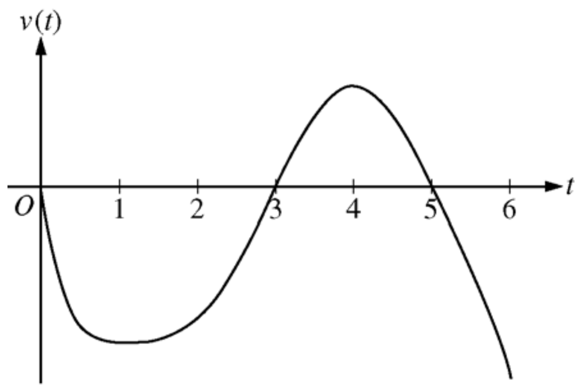
   (c) By the time the recovery device began removing oil, 60,000 cubic centimeters of oil had already leaked. Write, but do not evaluate, an expression involving an integral that gives the volume of oil at the time found in part (b).

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

END OF PART A OF SECTION II

CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



Graph of v

4. A particle moves along the x-axis so that its velocity at time t, for $0 \le t \le 6$, is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 0$, $t = 3$, and $t = 5$, and the graph has horizontal tangents at $t = 1$ and $t = 4$. The areas of the regions bounded by the t-axis and the graph of v on the intervals $[0, 3]$, $[3, 5]$, and $[5, 6]$ are 8, 3, and 2, respectively. At time $t = 0$, the particle is at $x = -2$.

   (a) For $0 \le t \le 6$, find both the time and the position of the particle when the particle is farthest to the left. Justify your answer.

   (b) For how many values of t, where $0 \le t \le 6$, is the particle at $x = -8$? Explain your reasoning.

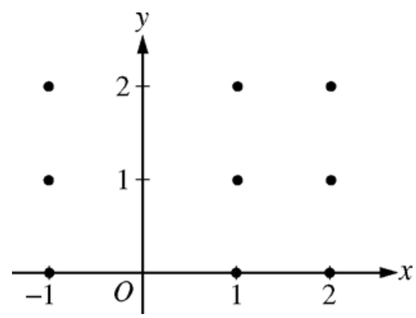
   (c) On the interval $2 < t < 3$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

   (d) During what time intervals, if any, is the acceleration of the particle negative? Justify your answer.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

5. Consider the differential equation $\dfrac{dy}{dx} = \dfrac{y-1}{x^2}$, where $x \neq 0$.

   (a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.
   (Note: Use the axes provided in the exam booklet.)



   (b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(2) = 0$.

   (c) For the particular solution $y = f(x)$ described in part (b), find $\lim\limits_{x \to \infty} f(x)$.

---

6. Let f be the function given by $f(x) = \dfrac{\ln x}{x}$ for all $x > 0$. The derivative of f is given by $f'(x) = \dfrac{1 - \ln x}{x^2}$.

   (a) Write an equation for the line tangent to the graph of f at $x = e^2$.

   (b) Find the x-coordinate of the critical point of f. Determine whether this point is a relative minimum, a relative maximum, or neither for the function f. Justify your answer.

   (c) The graph of the function f has exactly one point of inflection. Find the x-coordinate of this point.

   (d) Find $\lim\limits_{x \to 0^+} f(x)$.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

END OF EXAM

AP® Calculus AB
2008 Free-Response Questions
Form B

The College Board: Connecting Students to College Success

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AP Central is the official online home for the AP Program: apcentral.collegeboard.com.

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

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1. Let R be the region in the first quadrant bounded by the graphs of $y = \sqrt{x}$ and $y = \dfrac{x}{3}$.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is rotated about the vertical line $x = -1$.

   (c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the y-axis are squares. Find the volume of this solid.

WRITE ALL WORK IN THE EXAM BOOKLET.

GO ON TO THE NEXT PAGE.

2. For time $t \geq 0$ hours, let $r(t) = 120\left(1 - e^{-10t^2}\right)$ represent the speed, in kilometers per hour, at which a car travels along a straight road. The number of liters of gasoline used by the car to travel x kilometers is modeled by $g(x) = 0.05x\left(1 - e^{-x/2}\right)$.

   (a) How many kilometers does the car travel during the first 2 hours?

   (b) Find the rate of change with respect to time of the number of liters of gasoline used by the car when $t = 2$ hours. Indicate units of measure.

   (c) How many liters of gasoline have been used by the car when it reaches a speed of 80 kilometers per hour?

WRITE ALL WORK IN THE EXAM BOOKLET.

Distance from the river's edge (feet)	0	8	14	22	24
Depth of the water (feet)	0	7	8	2	0

3. A scientist measures the depth of the Doe River at Picnic Point. The river is 24 feet wide at this location. The measurements are taken in a straight line perpendicular to the edge of the river. The data are shown in the table above. The velocity of the water at Picnic Point, in feet per minute, is modeled by $v(t) = 16 + 2\sin\left(\sqrt{t+10}\right)$ for $0 \le t \le 120$ minutes.

   (a) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the area of the cross section of the river at Picnic Point, in square feet. Show the computations that lead to your answer.

   (b) The volumetric flow at a location along the river is the product of the cross-sectional area and the velocity of the water at that location. Use your approximation from part (a) to estimate the average value of the volumetric flow at Picnic Point, in cubic feet per minute, from $t = 0$ to $t = 120$ minutes.

   (c) The scientist proposes the function f, given by $f(x) = 8\sin\left(\frac{\pi x}{24}\right)$, as a model for the depth of the water, in feet, at Picnic Point x feet from the river's edge. Find the area of the cross section of the river at Picnic Point based on this model.

   (d) Recall that the volumetric flow is the product of the cross-sectional area and the velocity of the water at a location. To prevent flooding, water must be diverted if the average value of the volumetric flow at Picnic Point exceeds 2100 cubic feet per minute for a 20-minute period. Using your answer from part (c), find the average value of the volumetric flow during the time interval $40 \le t \le 60$ minutes. Does this value indicate that the water must be diverted?

WRITE ALL WORK IN THE EXAM BOOKLET.

END OF PART A OF SECTION II

CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

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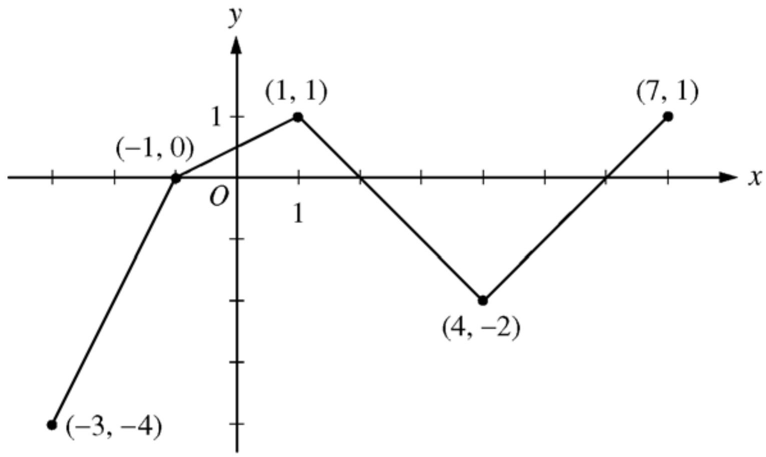
4. The functions f and g are given by $f(x) = \int_0^{3x} \sqrt{4 + t^2}\, dt$ and $g(x) = f(\sin x)$.

   (a) Find $f'(x)$ and $g'(x)$.

   (b) Write an equation for the line tangent to the graph of $y = g(x)$ at $x = \pi$.

   (c) Write, but do not evaluate, an integral expression that represents the maximum value of g on the interval $0 \le x \le \pi$. Justify your answer.

WRITE ALL WORK IN THE EXAM BOOKLET.



Graph of g'

5. Let g be a continuous function with $g(2) = 5$. The graph of the piecewise-linear function g', the derivative of g, is shown above for $-3 \le x \le 7$.

(a) Find the x-coordinate of all points of inflection of the graph of $y = g(x)$ for $-3 < x < 7$. Justify your answer.

(b) Find the absolute maximum value of g on the interval $-3 \le x \le 7$. Justify your answer.

(c) Find the average rate of change of $g(x)$ on the interval $-3 \le x \le 7$.

(d) Find the average rate of change of $g'(x)$ on the interval $-3 \le x \le 7$. Does the Mean Value Theorem applied on the interval $-3 \le x \le 7$ guarantee a value of c, for $-3 < c < 7$, such that $g''(c)$ is equal to this average rate of change? Why or why not?

WRITE ALL WORK IN THE EXAM BOOKLET.

6. Consider the closed curve in the xy-plane given by

$$x^2 + 2x + y^4 + 4y = 5.$$

(a) Show that $\frac{dy}{dx} = \frac{-(x+1)}{2(y^3+1)}$.

(b) Write an equation for the line tangent to the curve at the point $(-2, 1)$.

(c) Find the coordinates of the two points on the curve where the line tangent to the curve is vertical.

(d) Is it possible for this curve to have a horizontal tangent at points where it intersects the x-axis? Explain your reasoning.

WRITE ALL WORK IN THE EXAM BOOKLET.

END OF EXAM

AP® Calculus AB
2007 Free-Response Questions

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

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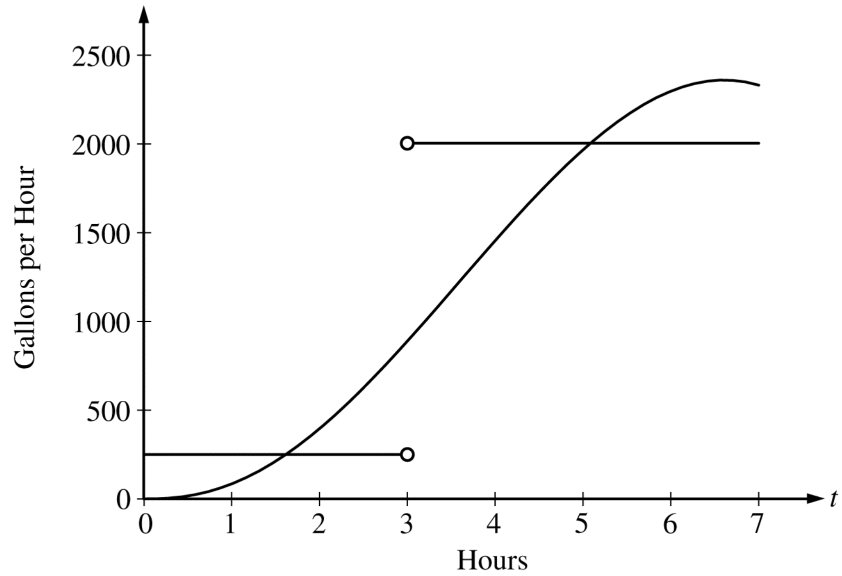
1. Let R be the region in the first and second quadrants bounded above by the graph of $y = \dfrac{20}{1 + x^2}$ and below by the horizontal line $y = 2$.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is rotated about the x-axis.

   (c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles. Find the volume of this solid.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.



2. The amount of water in a storage tank, in gallons, is modeled by a continuous function on the time interval $0 \le t \le 7$, where t is measured in hours. In this model, rates are given as follows:

   (i) The rate at which water enters the tank is $f(t) = 100t^2 \sin\left(\sqrt{t}\right)$ gallons per hour for $0 \le t \le 7$.

   (ii) The rate at which water leaves the tank is

$$g(t) = \begin{cases} 250 & \text{for } 0 \le t < 3 \\ 2000 & \text{for } 3 < t \le 7 \end{cases} \text{ gallons per hour.}$$

   The graphs of f and g, which intersect at $t = 1.617$ and $t = 5.076$, are shown in the figure above. At time $t = 0$, the amount of water in the tank is 5000 gallons.

   (a) How many gallons of water enter the tank during the time interval $0 \le t \le 7$? Round your answer to the nearest gallon.

   (b) For $0 \le t \le 7$, find the time intervals during which the amount of water in the tank is decreasing. Give a reason for each answer.

   (c) For $0 \le t \le 7$, at what time t is the amount of water in the tank greatest? To the nearest gallon, compute the amount of water at this time. Justify your answer.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	–4	4	2
4	–1	3	6	7

3. The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of x. The function h is given by $h(x) = f(g(x)) - 6$.

   (a) Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.

   (b) Explain why there must be a value c for $1 < c < 3$ such that $h'(c) = -5$.

   (c) Let w be the function given by $w(x) = \int_1^{g(x)} f(t)\,dt$. Find the value of $w'(3)$.

   (d) If g^{-1} is the inverse function of g, write an equation for the line tangent to the graph of $y = g^{-1}(x)$ at $x = 2$.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

END OF PART A OF SECTION II

CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

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4. A particle moves along the x-axis with position at time t given by $x(t) = e^{-t}\sin t$ for $0 \le t \le 2\pi$.

   (a) Find the time t at which the particle is farthest to the left. Justify your answer.

   (b) Find the value of the constant A for which $x(t)$ satisfies the equation $Ax''(t) + x'(t) + x(t) = 0$ for $0 < t < 2\pi$.

---

t (minutes)	0	2	5	7	11	12
$r'(t)$ (feet per minute)	5.7	4.0	2.0	1.2	0.6	0.5

5. The volume of a spherical hot air balloon expands as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by a twice-differentiable function r of time t, where t is measured in minutes. For $0 < t < 12$, the graph of r is concave down. The table above gives selected values of the rate of change, $r'(t)$, of the radius of the balloon over the time interval $0 \le t \le 12$. The radius of the balloon is 30 feet when $t = 5$. (Note: The volume of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$.)

   (a) Estimate the radius of the balloon when $t = 5.4$ using the tangent line approximation at $t = 5$. Is your estimate greater than or less than the true value? Give a reason for your answer.

   (b) Find the rate of change of the volume of the balloon with respect to time when $t = 5$. Indicate units of measure.

   (c) Use a right Riemann sum with the five subintervals indicated by the data in the table to approximate $\int_0^{12} r'(t)\,dt$. Using correct units, explain the meaning of $\int_0^{12} r'(t)\,dt$ in terms of the radius of the balloon.

   (d) Is your approximation in part (c) greater than or less than $\int_0^{12} r'(t)\,dt$? Give a reason for your answer.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

GO ON TO THE NEXT PAGE.

6. Let f be the function defined by $f(x) = k\sqrt{x} - \ln x$ for $x > 0$, where k is a positive constant.

   (a) Find $f'(x)$ and $f''(x)$.

   (b) For what value of the constant k does f have a critical point at $x = 1$? For this value of k, determine whether f has a relative minimum, relative maximum, or neither at $x = 1$. Justify your answer.

   (c) For a certain value of the constant k, the graph of f has a point of inflection on the x-axis. Find this value of k.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

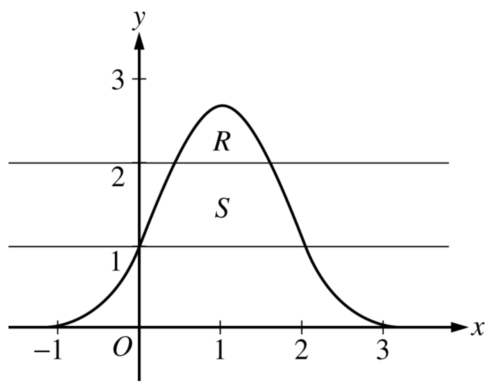
END OF EXAM

AP® Calculus AB
2007 Free-Response Questions
Form B

The College Board: Connecting Students to College Success

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



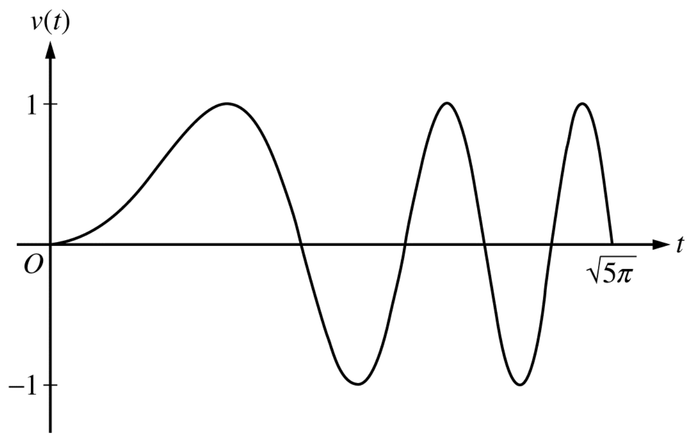
1. Let R be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal line $y = 2,$ and let S be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal lines $y = 1$ and $y = 2,$ as shown above.

   (a) Find the area of R.

   (b) Find the area of S.

   (c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 1$.

WRITE ALL WORK IN THE EXAM BOOKLET.



2. A particle moves along the x-axis so that its velocity v at time $t \geq 0$ is given by $v(t) = \sin\left(t^2\right)$. The graph of v is shown above for $0 \leq t \leq \sqrt{5\pi}$. The position of the particle at time t is $x(t)$ and its position at time $t = 0$ is $x(0) = 5$.

(a) Find the acceleration of the particle at time $t = 3$.

(b) Find the total distance traveled by the particle from time $t = 0$ to $t = 3$.

(c) Find the position of the particle at time $t = 3$.

(d) For $0 \leq t \leq \sqrt{5\pi}$, find the time t at which the particle is farthest to the right. Explain your answer.

WRITE ALL WORK IN THE EXAM BOOKLET.

GO ON TO THE NEXT PAGE.

3. The wind chill is the temperature, in degrees Fahrenheit $(°\text{F})$, a human feels based on the air temperature, in degrees Fahrenheit, and the wind velocity v, in miles per hour (mph). If the air temperature is $32°\text{F}$, then the wind chill is given by $W(v) = 55.6 - 22.1v^{0.16}$ and is valid for $5 \le v \le 60$.

   (a) Find $W'(20)$. Using correct units, explain the meaning of $W'(20)$ in terms of the wind chill.

   (b) Find the average rate of change of W over the interval $5 \le v \le 60$. Find the value of v at which the instantaneous rate of change of W is equal to the average rate of change of W over the interval $5 \le v \le 60$.

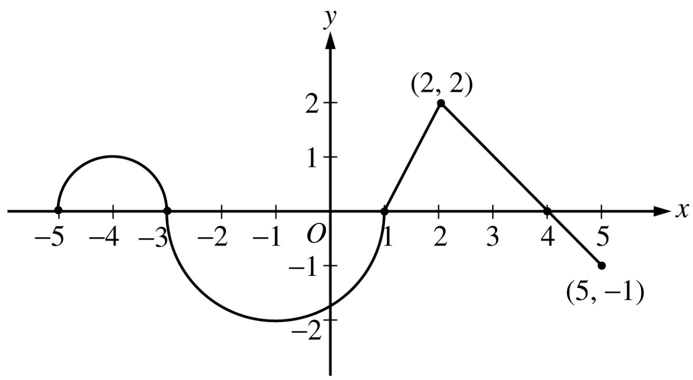
   (c) Over the time interval $0 \le t \le 4$ hours, the air temperature is a constant $32°\text{F}$. At time $t = 0$, the wind velocity is $v = 20$ mph. If the wind velocity increases at a constant rate of 5 mph per hour, what is the rate of change of the wind chill with respect to time at $t = 3$ hours? Indicate units of measure.

WRITE ALL WORK IN THE EXAM BOOKLET.

END OF PART A OF SECTION II

CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



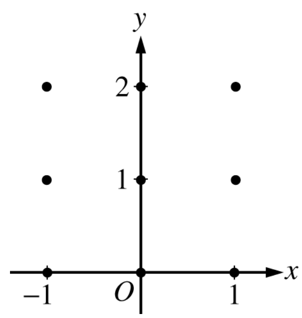
Graph of f'

4. Let f be a function defined on the closed interval $-5 \le x \le 5$ with $f(1) = 3$. The graph of f', the derivative of f, consists of two semicircles and two line segments, as shown above.
   (a) For $-5 < x < 5$, find all values x at which f has a relative maximum. Justify your answer.
   (b) For $-5 < x < 5$, find all values x at which the graph of f has a point of inflection. Justify your answer.
   (c) Find all intervals on which the graph of f is concave up and also has positive slope. Explain your reasoning.
   (d) Find the absolute minimum value of $f(x)$ over the closed interval $-5 \le x \le 5$. Explain your reasoning.

WRITE ALL WORK IN THE EXAM BOOKLET.

5. Consider the differential equation $\frac{dy}{dx} = \frac{1}{2}x + y - 1$.

   (a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.
   (Note: Use the axes provided in the exam booklet.)



   (b) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Describe the region in the xy-plane in which all solution curves to the differential equation are concave up.

   (c) Let $y = f(x)$ be a particular solution to the differential equation with the initial condition $f(0) = 1$. Does f have a relative minimum, a relative maximum, or neither at $x = 0$? Justify your answer.

   (d) Find the values of the constants m and b, for which $y = mx + b$ is a solution to the differential equation.

---

6. Let f be a twice-differentiable function such that $f(2) = 5$ and $f(5) = 2$. Let g be the function given by $g(x) = f(f(x))$.

   (a) Explain why there must be a value c for $2 < c < 5$ such that $f'(c) = -1$.

   (b) Show that $g'(2) = g'(5)$. Use this result to explain why there must be a value k for $2 < k < 5$ such that $g''(k) = 0$.

   (c) Show that if $f''(x) = 0$ for all x, then the graph of g does not have a point of inflection.

   (d) Let $h(x) = f(x) - x$. Explain why there must be a value r for $2 < r < 5$ such that $h(r) = 0$.

WRITE ALL WORK IN THE EXAM BOOKLET.

END OF EXAM

AP® Calculus AB
2006 Free-Response Questions

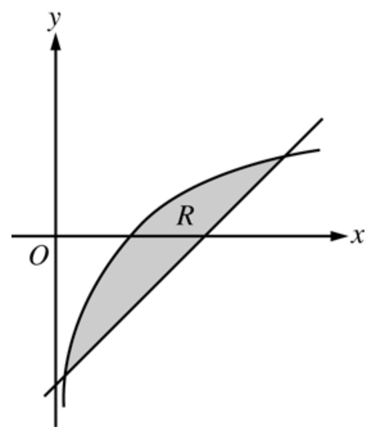
The College Board: Connecting Students to College Success

The College Board is a not-for-profit membership association whose mission is to connect students to college success and opportunity. Founded in 1900, the association is composed of more than 5,000 schools, colleges, universities, and other educational organizations. Each year, the College Board serves seven million students and their parents, 23,000 high schools, and 3,500 colleges through major programs and services in college admissions, guidance, assessment, financial aid, enrollment, and teaching and learning. Among its best-known programs are the SAT®, the PSAT/NMSQT®, and the Advanced Placement Program® (AP®). The College Board is committed to the principles of excellence and equity, and that commitment is embodied in all of its programs, services, activities, and concerns.

Visit the College Board on the Web: www.collegeboard.com.
AP Central is the official online home for the AP Program: apcentral.collegeboard.com.

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



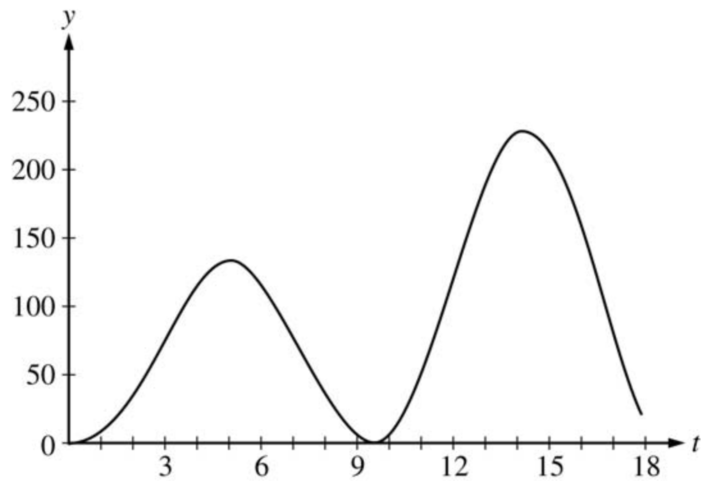
1. Let R be the shaded region bounded by the graph of $y = \ln x$ and the line $y = x - 2$, as shown above.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -3$.

   (c) Write, but do not evaluate, an integral expression that can be used to find the volume of the solid generated when R is rotated about the y-axis.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.



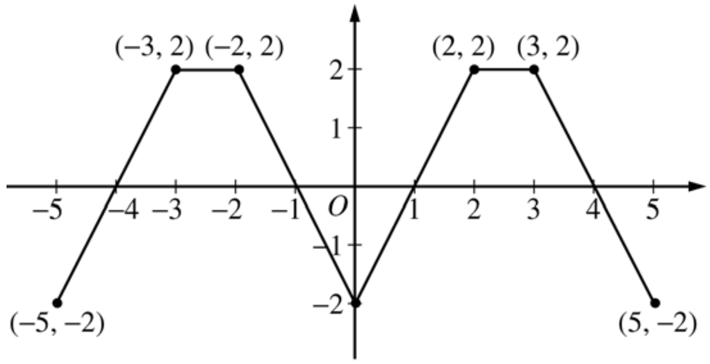
2. At an intersection in Thomasville, Oregon, cars turn left at the rate $L(t) = 60\sqrt{t}\sin^2\left(\frac{t}{3}\right)$ cars per hour over the time interval $0 \le t \le 18$ hours. The graph of $y = L(t)$ is shown above.

(a) To the nearest whole number, find the total number of cars turning left at the intersection over the time interval $0 \le t \le 18$ hours.

(b) Traffic engineers will consider turn restrictions when $L(t) \ge 150$ cars per hour. Find all values of t for which $L(t) \ge 150$ and compute the average value of L over this time interval. Indicate units of measure.

(c) Traffic engineers will install a signal if there is any two-hour time interval during which the product of the total number of cars turning left and the total number of oncoming cars traveling straight through the intersection is greater than 200,000. In every two-hour time interval, 500 oncoming cars travel straight through the intersection. Does this intersection require a traffic signal? Explain the reasoning that leads to your conclusion.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.



Graph of f

3. The graph of the function f shown above consists of six line segments. Let g be the function given by $g(x) = \int_0^x f(t)\,dt$.

   (a) Find $g(4)$, $g'(4)$, and $g''(4)$.

   (b) Does g have a relative minimum, a relative maximum, or neither at $x = 1$? Justify your answer.

   (c) Suppose that f is defined for all real numbers x and is periodic with a period of length 5. The graph above shows two periods of f. Given that $g(5) = 2$, find $g(10)$ and write an equation for the line tangent to the graph of g at $x = 108$.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

END OF PART A OF SECTION II

CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

t (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

4. Rocket A has positive velocity $v(t)$ after being launched upward from an initial height of 0 feet at time $t = 0$ seconds. The velocity of the rocket is recorded for selected values of t over the interval $0 \le t \le 80$ seconds, as shown in the table above.

   (a) Find the average acceleration of rocket A over the time interval $0 \le t \le 80$ seconds. Indicate units of measure.

   (b) Using correct units, explain the meaning of $\int_{10}^{70} v(t)\,dt$ in terms of the rocket's flight. Use a midpoint Riemann sum with 3 subintervals of equal length to approximate $\int_{10}^{70} v(t)\,dt$.

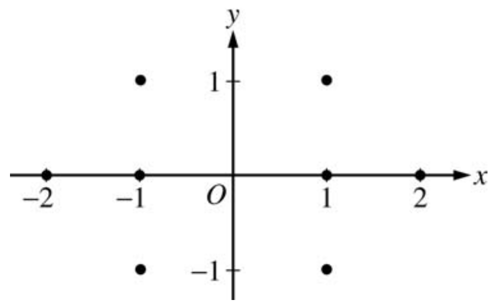
   (c) Rocket B is launched upward with an acceleration of $a(t) = \dfrac{3}{\sqrt{t+1}}$ feet per second per second. At time $t = 0$ seconds, the initial height of the rocket is 0 feet, and the initial velocity is 2 feet per second. Which of the two rockets is traveling faster at time $t = 80$ seconds? Explain your answer.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

GO ON TO THE NEXT PAGE.

5. Consider the differential equation $\frac{dy}{dx} = \frac{1 + y}{x}$, where $x \neq 0$.

   (a) On the axes provided, sketch a slope field for the given differential equation at the eight points indicated.
   (Note: Use the axes provided in the pink exam booklet.)



   (b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(-1) = 1$ and state its domain.

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6. The twice-differentiable function f is defined for all real numbers and satisfies the following conditions:

$$f(0) = 2,\ \ f'(0) = -4,\ \text{and}\ \ f''(0) = 3.$$

   (a) The function g is given by $g(x) = e^{ax} + f(x)$ for all real numbers, where a is a constant. Find $g'(0)$ and $g''(0)$ in terms of a. Show the work that leads to your answers.

   (b) The function h is given by $h(x) = \cos(kx) f(x)$ for all real numbers, where k is a constant. Find $h'(x)$ and write an equation for the line tangent to the graph of h at $x = 0$.

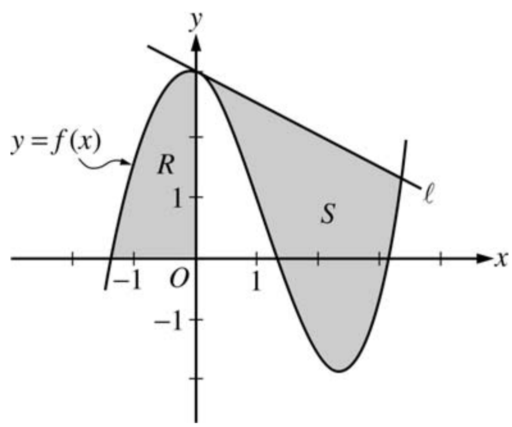
WRITE ALL WORK IN THE PINK EXAM BOOKLET.

END OF EXAM

AP® Calculus AB
2006 Free-Response Questions
Form B

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



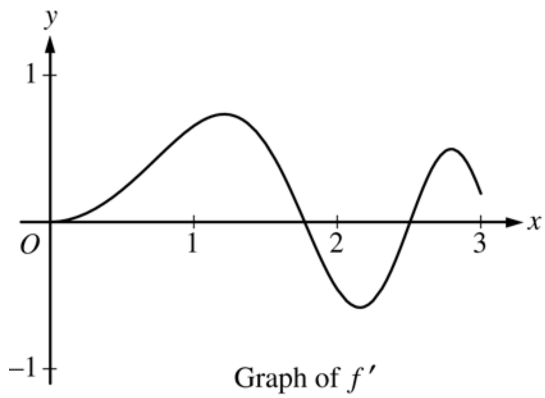
1. Let f be the function given by $f(x) = \frac{x^3}{4} - \frac{x^2}{3} - \frac{x}{2} + 3\cos x$. Let R be the shaded region in the second quadrant bounded by the graph of f, and let S be the shaded region bounded by the graph of f and line ℓ, the line tangent to the graph of f at $x = 0$, as shown above.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

   (c) Write, but do not evaluate, an integral expression that can be used to find the area of S.

WRITE ALL WORK IN THE EXAM BOOKLET.



Graph of f'

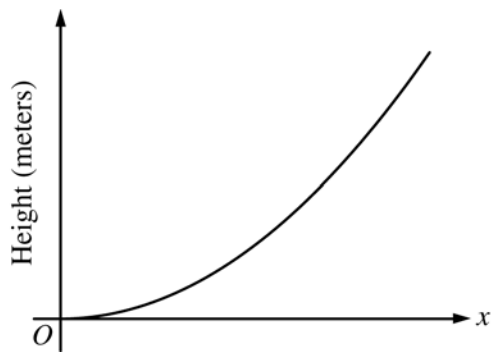
2. Let f be the function defined for $x \geq 0$ with $f(0) = 5$ and f', the first derivative of f, given by $f'(x) = e^{(-x/4)} \sin(x^2)$. The graph of $y = f'(x)$ is shown above.

(a) Use the graph of f' to determine whether the graph of f is concave up, concave down, or neither on the interval $1.7 < x < 1.9$. Explain your reasoning.

(b) On the interval $0 \leq x \leq 3$, find the value of x at which f has an absolute maximum. Justify your answer.

(c) Write an equation for the line tangent to the graph of f at $x = 2$.

WRITE ALL WORK IN THE EXAM BOOKLET.



3. The figure above is the graph of a function of x, which models the height of a skateboard ramp. The function meets the following requirements.

   (i) At $x = 0$, the value of the function is 0, and the slope of the graph of the function is 0.

   (ii) At $x = 4$, the value of the function is 1, and the slope of the graph of the function is 1.

   (iii) Between $x = 0$ and $x = 4$, the function is increasing.

   (a) Let $f(x) = ax^2$, where a is a nonzero constant. Show that it is not possible to find a value for a so that f meets requirement (ii) above.

   (b) Let $g(x) = cx^3 - \frac{x^2}{16}$, where c is a nonzero constant. Find the value of c so that g meets requirement (ii) above. Show the work that leads to your answer.

   (c) Using the function g and your value of c from part (b), show that g does not meet requirement (iii) above.

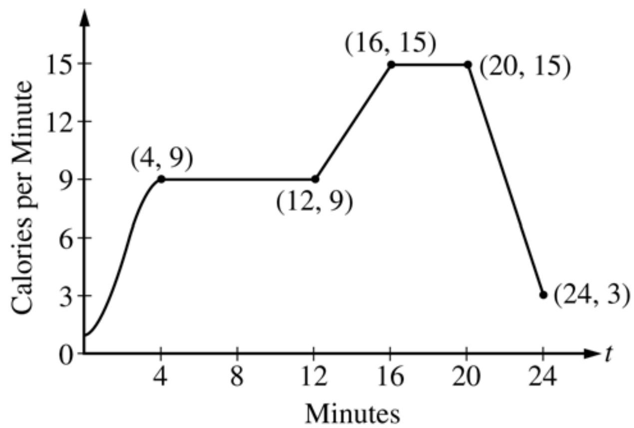
   (d) Let $h(x) = \frac{x^n}{k}$, where k is a nonzero constant and n is a positive integer. Find the values of k and n so that h meets requirement (ii) above. Show that h also meets requirements (i) and (iii) above.

WRITE ALL WORK IN THE EXAM BOOKLET.

END OF PART A OF SECTION II

CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



4. The rate, in calories per minute, at which a person using an exercise machine burns calories is modeled by the function f. In the figure above, $f(t) = -\frac{1}{4}t^3 + \frac{3}{2}t^2 + 1$ for $0 \le t \le 4$ and f is piecewise linear for $4 \le t \le 24$.

(a) Find $f'(22)$. Indicate units of measure.

(b) For the time interval $0 \le t \le 24$, at what time t is f increasing at its greatest rate? Show the reasoning that supports your answer.

(c) Find the total number of calories burned over the time interval $6 \le t \le 18$ minutes.

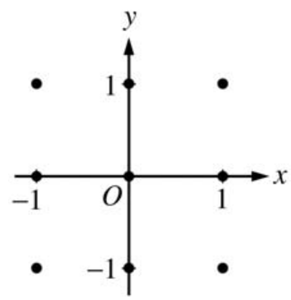
(d) The setting on the machine is now changed so that the person burns $f(t) + c$ calories per minute. For this setting, find c so that an average of 15 calories per minute is burned during the time interval $6 \le t \le 18$.

WRITE ALL WORK IN THE EXAM BOOKLET.

GO ON TO THE NEXT PAGE.

5. Consider the differential equation $\frac{dy}{dx} = (y-1)^2 \cos(\pi x)$.

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.
(Note: Use the axes provided in the exam booklet.)



(b) There is a horizontal line with equation $y = c$ that satisfies this differential equation. Find the value of c.

(c) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = 0$.

t (sec)	0	15	25	30	35	50	60
$v(t)$ (ft/sec)	–20	–30	–20	–14	–10	0	10
$a(t)$ $\left(\text{ft}/\text{sec}^2\right)$	1	5	2	1	2	4	2

6. A car travels on a straight track. During the time interval $0 \le t \le 60$ seconds, the car's velocity v, measured in feet per second, and acceleration a, measured in feet per second per second, are continuous functions. The table above shows selected values of these functions.

(a) Using appropriate units, explain the meaning of $\int_{30}^{60} |v(t)|\, dt$ in terms of the car's motion. Approximate $\int_{30}^{60} |v(t)|\, dt$ using a trapezoidal approximation with the three subintervals determined by the table.

(b) Using appropriate units, explain the meaning of $\int_{0}^{30} a(t)\, dt$ in terms of the car's motion. Find the exact value of $\int_{0}^{30} a(t)\, dt$.

(c) For $0 < t < 60$, must there be a time t when $v(t) = -5$? Justify your answer.

(d) For $0 < t < 60$, must there be a time t when $a(t) = 0$? Justify your answer.

WRITE ALL WORK IN THE EXAM BOOKLET.

END OF EXAM

AP® Calculus AB
2005 Free-Response Questions

The College Board: Connecting Students to College Success

The College Board is a not-for-profit membership association whose mission is to connect students to college success and opportunity. Founded in 1900, the association is composed of more than 4,700 schools, colleges, universities, and other educational organizations. Each year, the College Board serves over three and a half million students and their parents, 23,000 high schools, and 3,500 colleges through major programs and services in college admissions, guidance, assessment, financial aid, enrollment, and teaching and learning. Among its best-known programs are the SAT®, the PSAT/NMSQT®, and the Advanced Placement Program® (AP®). The College Board is committed to the principles of excellence and equity, and that commitment is embodied in all of its programs, services, activities, and concerns.

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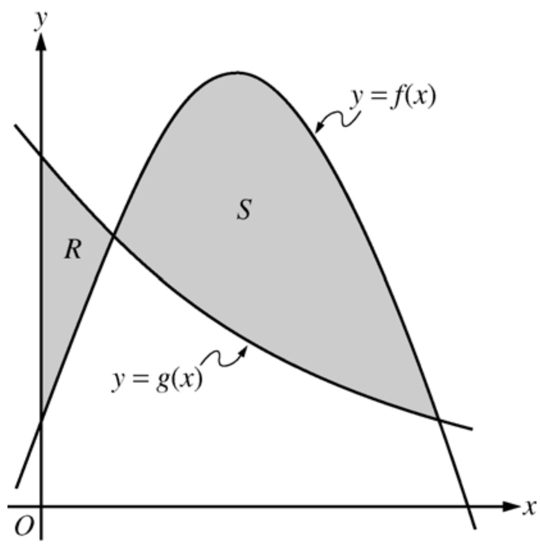
AP Central is the official online home for the AP Program and Pre-AP: apcentral.collegeboard.com.

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let f and g be the functions given by $f(x) = \frac{1}{4} + \sin(\pi x)$ and $g(x) = 4^{-x}$. Let R be the shaded region in the first quadrant enclosed by the y-axis and the graphs of f and g, and let S be the shaded region in the first quadrant enclosed by the graphs of f and g, as shown in the figure above.

   (a) Find the area of R.

   (b) Find the area of S.

   (c) Find the volume of the solid generated when S is revolved about the horizontal line $y = -1$.

WRITE ALL WORK IN THE TEST BOOKLET.

2. The tide removes sand from Sandy Point Beach at a rate modeled by the function $R,$ given by

$$R(t) = 2 + 5\sin\left(\frac{4\pi t}{25}\right).$$

A pumping station adds sand to the beach at a rate modeled by the function $S,$ given by

$$S(t) = \frac{15t}{1 + 3t}.$$

Both $R(t)$ and $S(t)$ have units of cubic yards per hour and t is measured in hours for $0 \le t \le 6.$ At time $t = 0,$ the beach contains 2500 cubic yards of sand.

(a) How much sand will the tide remove from the beach during this 6-hour period? Indicate units of measure.

(b) Write an expression for $Y(t),$ the total number of cubic yards of sand on the beach at time $t.$

(c) Find the rate at which the total amount of sand on the beach is changing at time $t = 4.$

(d) For $0 \le t \le 6,$ at what time t is the amount of sand on the beach a minimum? What is the minimum value? Justify your answers.

WRITE ALL WORK IN THE TEST BOOKLET.

Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ (°C)	100	93	70	62	55

3. A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature $T(x)$, in degrees Celsius $(°\text{C})$, of the wire x cm from the heated end. The function T is decreasing and twice differentiable.

   (a) Estimate $T'(7)$. Show the work that leads to your answer. Indicate units of measure.

   (b) Write an integral expression in terms of $T(x)$ for the average temperature of the wire. Estimate the average temperature of the wire using a trapezoidal sum with the four subintervals indicated by the data in the table. Indicate units of measure.

   (c) Find $\int_0^8 T'(x)\,dx$, and indicate units of measure. Explain the meaning of $\int_0^8 T'(x)\,dx$ in terms of the temperature of the wire.

   (d) Are the data in the table consistent with the assertion that $T''(x) > 0$ for every x in the interval $0 < x < 8$? Explain your answer.

WRITE ALL WORK IN THE TEST BOOKLET.

END OF PART A OF SECTION II

CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

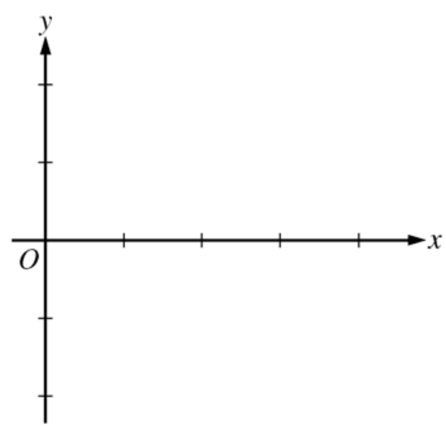
x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3	$3 < x < 4$
$f(x)$	–1	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	4	Positive	0	Positive	DNE	Negative	–3	Negative
$f''(x)$	–2	Negative	0	Positive	DNE	Negative	0	Positive

4. Let f be a function that is continuous on the interval $[0, 4)$. The function f is twice differentiable except at $x = 2$. The function f and its derivatives have the properties indicated in the table above, where DNE indicates that the derivatives of f do not exist at $x = 2$.

   (a) For $0 < x < 4$, find all values of x at which f has a relative extremum. Determine whether f has a relative maximum or a relative minimum at each of these values. Justify your answer.

   (b) On the axes provided, sketch the graph of a function that has all the characteristics of f.

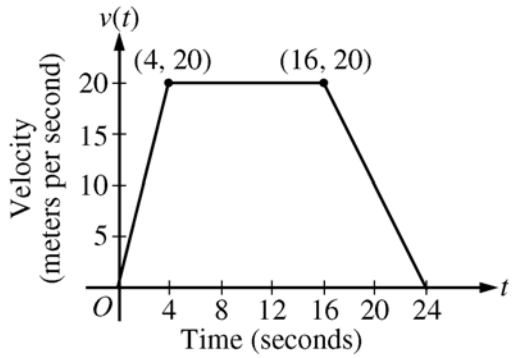
   (Note: Use the axes provided in the pink test booklet.)



   (c) Let g be the function defined by $g(x) = \int_1^x f(t)\,dt$ on the open interval $(0, 4)$. For $0 < x < 4$, find all values of x at which g has a relative extremum. Determine whether g has a relative maximum or a relative minimum at each of these values. Justify your answer.

   (d) For the function g defined in part (c), find all values of x, for $0 < x < 4$, at which the graph of g has a point of inflection. Justify your answer.

WRITE ALL WORK IN THE TEST BOOKLET.



5. A car is traveling on a straight road. For $0 \le t \le 24$ seconds, the car's velocity $v(t)$, in meters per second, is modeled by the piecewise-linear function defined by the graph above.

(a) Find $\int_0^{24} v(t)\,dt$. Using correct units, explain the meaning of $\int_0^{24} v(t)\,dt$.

(b) For each of $v'(4)$ and $v'(20)$, find the value or explain why it does not exist. Indicate units of measure.

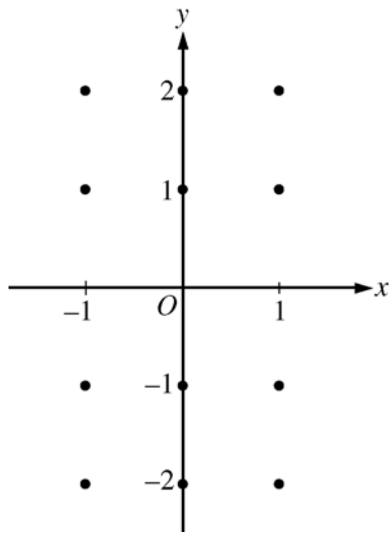
(c) Let $a(t)$ be the car's acceleration at time t, in meters per second per second. For $0 < t < 24$, write a piecewise-defined function for $a(t)$.

(d) Find the average rate of change of v over the interval $8 \le t \le 20$. Does the Mean Value Theorem guarantee a value of c, for $8 < c < 20$, such that $v'(c)$ is equal to this average rate of change? Why or why not?

WRITE ALL WORK IN THE TEST BOOKLET.

6. Consider the differential equation $\dfrac{dy}{dx} = -\dfrac{2x}{y}$.

   (a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.
   (**Note: Use the axes provided in the pink test booklet.**)



   (b) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = -1$. Write an equation for the line tangent to the graph of f at $(1, -1)$ and use it to approximate $f(1.1)$.

   (c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(1) = -1$.

WRITE ALL WORK IN THE TEST BOOKLET.

END OF EXAM

AP® Calculus AB
2005 Free-Response Questions
Form B

The College Board: Connecting Students to College Success

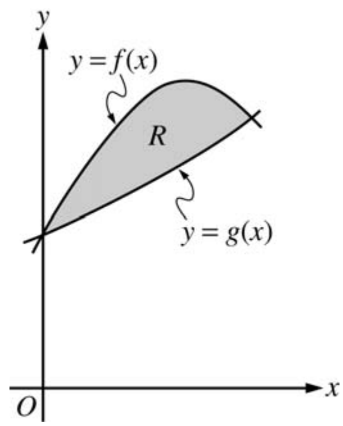
CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let f and g be the functions given by $f(x) = 1 + \sin(2x)$ and $g(x) = e^{x/2}$. Let R be the shaded region in the first quadrant enclosed by the graphs of f and g as shown in the figure above.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is revolved about the x-axis.

   (c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles with diameters extending from $y = f(x)$ to $y = g(x)$. Find the volume of this solid.

WRITE ALL WORK IN THE TEST BOOKLET.

2. A water tank at Camp Newton holds 1200 gallons of water at time $t = 0$. During the time interval $0 \le t \le 18$ hours, water is pumped into the tank at the rate

$$W(t) = 95\sqrt{t}\sin^2\left(\frac{t}{6}\right) \text{ gallons per hour.}$$

During the same time interval, water is removed from the tank at the rate

$$R(t) = 275\sin^2\left(\frac{t}{3}\right) \text{ gallons per hour.}$$

(a) Is the amount of water in the tank increasing at time $t = 15$? Why or why not?

(b) To the nearest whole number, how many gallons of water are in the tank at time $t = 18$?

(c) At what time t, for $0 \le t \le 18$, is the amount of water in the tank at an absolute minimum? Show the work that leads to your conclusion.

(d) For $t > 18$, no water is pumped into the tank, but water continues to be removed at the rate $R(t)$ until the tank becomes empty. Let k be the time at which the tank becomes empty. Write, but do not solve, an equation involving an integral expression that can be used to find the value of k.

WRITE ALL WORK IN THE TEST BOOKLET.

3. A particle moves along the x-axis so that its velocity v at time t, for $0 \le t \le 5$, is given by $v(t) = \ln\left(t^2 - 3t + 3\right)$. The particle is at position $x = 8$ at time $t = 0$.

   (a) Find the acceleration of the particle at time $t = 4$.

   (b) Find all times t in the open interval $0 < t < 5$ at which the particle changes direction. During which time intervals, for $0 \le t \le 5$, does the particle travel to the left?

   (c) Find the position of the particle at time $t = 2$.

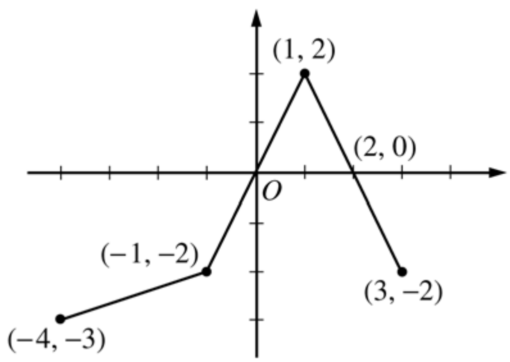
   (d) Find the average speed of the particle over the interval $0 \le t \le 2$.

WRITE ALL WORK IN THE TEST BOOKLET.

END OF PART A OF SECTION II

CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



Graph of f

4. The graph of the function f above consists of three line segments.

(a) Let g be the function given by $g(x) = \int_{-4}^{x} f(t)\,dt$. For each of $g(-1)$, $g'(-1)$, and $g''(-1)$, find the value or state that it does not exist.

(b) For the function g defined in part (a), find the x-coordinate of each point of inflection of the graph of g on the open interval $-4 < x < 3$. Explain your reasoning.

(c) Let h be the function given by $h(x) = \int_{x}^{3} f(t)\,dt$. Find all values of x in the closed interval $-4 \le x \le 3$ for which $h(x) = 0$.

(d) For the function h defined in part (c), find all intervals on which h is decreasing. Explain your reasoning.

WRITE ALL WORK IN THE TEST BOOKLET.

5. Consider the curve given by $y^2 = 2 + xy$.

   (a) Show that $\dfrac{dy}{dx} = \dfrac{y}{2y - x}$.

   (b) Find all points (x, y) on the curve where the line tangent to the curve has slope $\dfrac{1}{2}$.

   (c) Show that there are no points (x, y) on the curve where the line tangent to the curve is horizontal.

   (d) Let x and y be functions of time t that are related by the equation $y^2 = 2 + xy$. At time $t = 5$, the value of y is 3 and $\dfrac{dy}{dt} = 6$. Find the value of $\dfrac{dx}{dt}$ at time $t = 5$.

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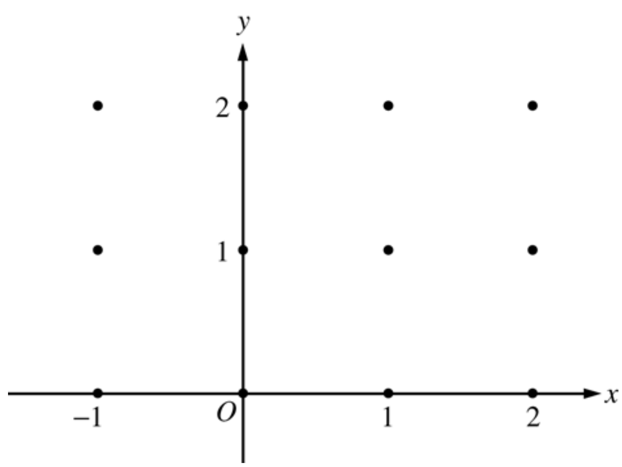
6. Consider the differential equation $\dfrac{dy}{dx} = \dfrac{-xy^2}{2}$. Let $y = f(x)$ be the particular solution to this differential equation with the initial condition $f(-1) = 2$.

   (a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated. **(Note: Use the axes provided in the test booklet.)**



   (b) Write an equation for the line tangent to the graph of f at $x = -1$.

   (c) Find the solution $y = f(x)$ to the given differential equation with the initial condition $f(-1) = 2$.

WRITE ALL WORK IN THE TEST BOOKLET.

END OF EXAM

CollegeBoard
Advanced Placement Program

AP® Calculus AB
2004 Free-Response Questions

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

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1. Traffic flow is defined as the rate at which cars pass through an intersection, measured in cars per minute. The traffic flow at a particular intersection is modeled by the function F defined by

$$F(t) = 82 + 4\sin\left(\frac{t}{2}\right) \text{ for } 0 \le t \le 30,$$

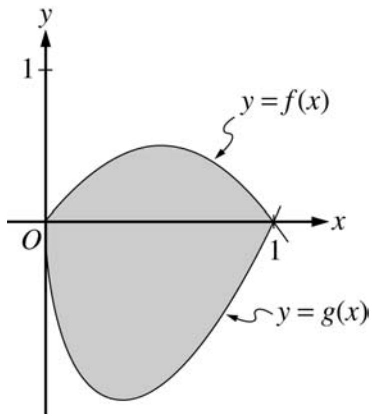
where $F(t)$ is measured in cars per minute and t is measured in minutes.

(a) To the nearest whole number, how many cars pass through the intersection over the 30-minute period?

(b) Is the traffic flow increasing or decreasing at $t = 7$? Give a reason for your answer.

(c) What is the average value of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

(d) What is the average rate of change of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.



2. Let f and g be the functions given by $f(x) = 2x(1 - x)$ and $g(x) = 3(x - 1)\sqrt{x}$ for $0 \le x \le 1$. The graphs of f and g are shown in the figure above.

(a) Find the area of the shaded region enclosed by the graphs of f and g.

(b) Find the volume of the solid generated when the shaded region enclosed by the graphs of f and g is revolved about the horizontal line $y = 2$.

(c) Let h be the function given by $h(x) = kx(1 - x)$ for $0 \le x \le 1$. For each $k > 0$, the region (not shown) enclosed by the graphs of h and g is the base of a solid with square cross sections perpendicular to the x-axis. There is a value of k for which the volume of this solid is equal to 15. Write, but do not solve, an equation involving an integral expression that could be used to find the value of k.

GO ON TO THE NEXT PAGE.

3. A particle moves along the y-axis so that its velocity v at time $t \geq 0$ is given by $v(t) = 1 - \tan^{-1}\left(e^t\right)$. At time $t = 0$, the particle is at $y = -1$. (Note: $\tan^{-1} x = \arctan x$)

   (a) Find the acceleration of the particle at time $t = 2$.

   (b) Is the speed of the particle increasing or decreasing at time $t = 2$? Give a reason for your answer.

   (c) Find the time $t \geq 0$ at which the particle reaches its highest point. Justify your answer.

   (d) Find the position of the particle at time $t = 2$. Is the particle moving toward the origin or away from the origin at time $t = 2$? Justify your answer.

END OF PART A OF SECTION II

CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

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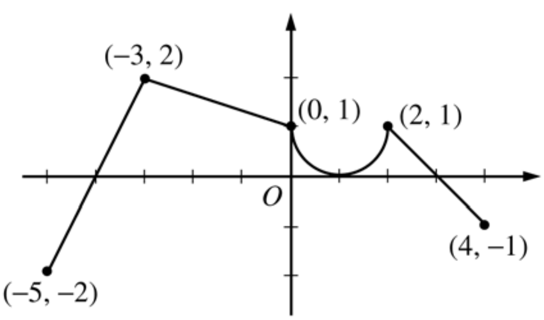
4. Consider the curve given by $x^2 + 4y^2 = 7 + 3xy$.

   (a) Show that $\dfrac{dy}{dx} = \dfrac{3y - 2x}{8y - 3x}$.

   (b) Show that there is a point P with x-coordinate 3 at which the line tangent to the curve at P is horizontal. Find the y-coordinate of P.

   (c) Find the value of $\dfrac{d^2y}{dx^2}$ at the point P found in part (b). Does the curve have a local maximum, a local minimum, or neither at the point P? Justify your answer.



Graph of f

5. The graph of the function f shown above consists of a semicircle and three line segments. Let g be the function given by $g(x) = \int_{-3}^{x} f(t)\,dt$.

   (a) Find $g(0)$ and $g'(0)$.

   (b) Find all values of x in the open interval $(-5, 4)$ at which g attains a relative maximum. Justify your answer.

   (c) Find the absolute minimum value of g on the closed interval $[-5, 4]$. Justify your answer.

   (d) Find all values of x in the open interval $(-5, 4)$ at which the graph of g has a point of inflection.

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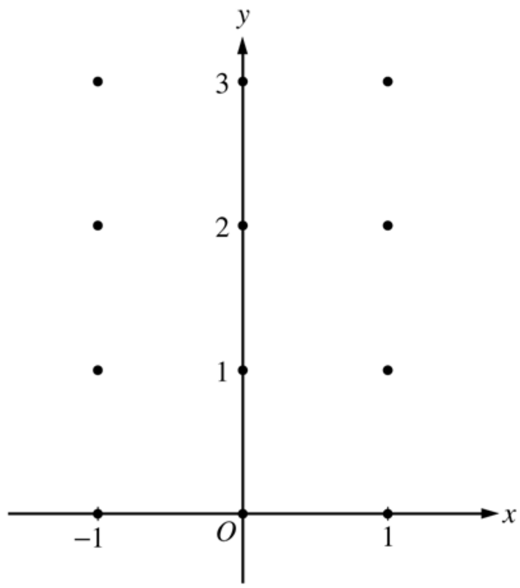
6. Consider the differential equation $\frac{dy}{dx} = x^2(y - 1)$.

   (a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated. (**Note: Use the axes provided in the pink test booklet.**)



   (b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are positive.

   (c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$.

END OF EXAMINATION

CollegeBoard
Advanced Placement Program

AP® Calculus AB
2004 Free-Response Questions
Form B

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CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

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1. Let R be the region enclosed by the graph of $y = \sqrt{x - 1}$, the vertical line $x = 10$, and the x-axis.
   (a) Find the area of R.
   (b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 3$.
   (c) Find the volume of the solid generated when R is revolved about the vertical line $x = 10$.

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2. For $0 \le t \le 31$, the rate of change of the number of mosquitoes on Tropical Island at time t days is modeled by $R(t) = 5\sqrt{t}\cos\left(\frac{t}{5}\right)$ mosquitoes per day. There are 1000 mosquitoes on Tropical Island at time $t = 0$.
   (a) Show that the number of mosquitoes is increasing at time $t = 6$.
   (b) At time $t = 6$, is the number of mosquitoes increasing at an increasing rate, or is the number of mosquitoes increasing at a decreasing rate? Give a reason for your answer.
   (c) According to the model, how many mosquitoes will be on the island at time $t = 31$? Round your answer to the nearest whole number.
   (d) To the nearest whole number, what is the maximum number of mosquitoes for $0 \le t \le 31$? Show the analysis that leads to your conclusion.

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t (minutes)	0	5	10	15	20	25	30	35	40
$v(t)$ (miles per minute)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

3. A test plane flies in a straight line with positive velocity $v(t)$, in miles per minute at time t minutes, where v is a differentiable function of t. Selected values of $v(t)$ for $0 \le t \le 40$ are shown in the table above.

(a) Use a midpoint Riemann sum with four subintervals of equal length and values from the table to approximate $\int_0^{40} v(t)\,dt$. Show the computations that lead to your answer. Using correct units, explain the meaning of $\int_0^{40} v(t)\,dt$ in terms of the plane's flight.

(b) Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval $0 < t < 40$? Justify your answer.

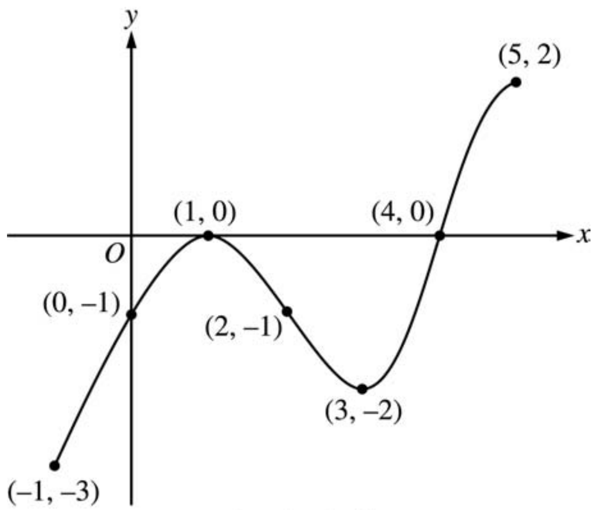
(c) The function f, defined by $f(t) = 6 + \cos\left(\frac{t}{10}\right) + 3\sin\left(\frac{7t}{40}\right)$, is used to model the velocity of the plane, in miles per minute, for $0 \le t \le 40$. According to this model, what is the acceleration of the plane at $t = 23$? Indicate units of measure.

(d) According to the model f, given in part (c), what is the average velocity of the plane, in miles per minute, over the time interval $0 \le t \le 40$?

END OF PART A OF SECTION II

CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



Graph of f'

4. The figure above shows the graph of f', the derivative of the function f, on the closed interval $-1 \le x \le 5$. The graph of f' has horizontal tangent lines at $x = 1$ and $x = 3$. The function f is twice differentiable with $f(2) = 6$.

   (a) Find the x-coordinate of each of the points of inflection of the graph of f. Give a reason for your answer.

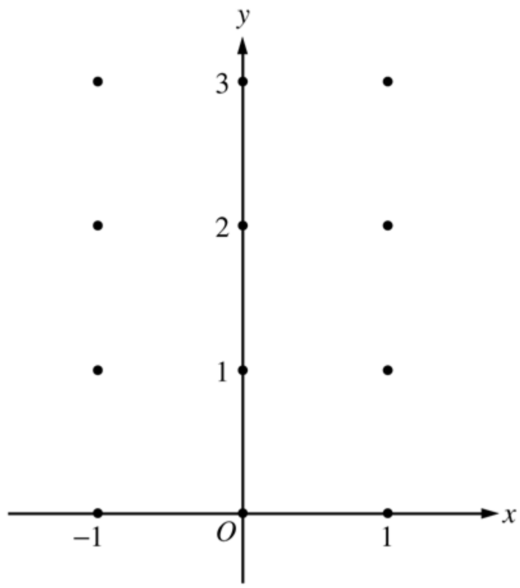
   (b) At what value of x does f attain its absolute minimum value on the closed interval $-1 \le x \le 5$? At what value of x does f attain its absolute maximum value on the closed interval $-1 \le x \le 5$? Show the analysis that leads to your answers.

   (c) Let g be the function defined by $g(x) = x f(x)$. Find an equation for the line tangent to the graph of g at $x = 2$.

GO ON TO THE NEXT PAGE.

5. Consider the differential equation $\dfrac{dy}{dx} = x^4(y - 2)$.

   (a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated. **(Note: Use the axes provided in the test booklet.)**



   (b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are negative.

   (c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 0$.

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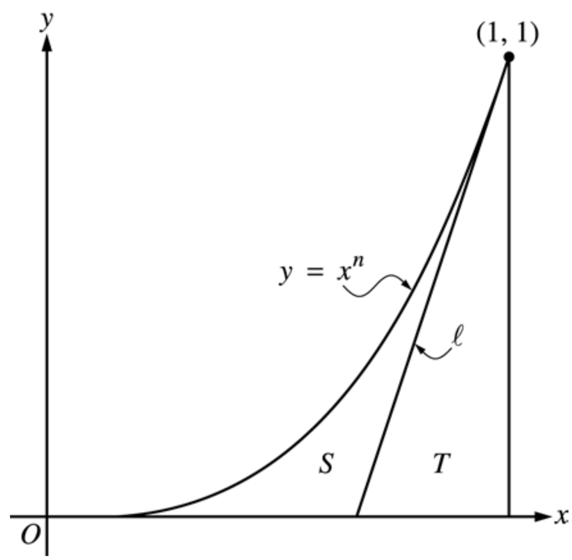


6. Let ℓ be the line tangent to the graph of $y = x^n$ at the point $(1, 1)$, where $n > 1$, as shown above.

   (a) Find $\int_0^1 x^n\,dx$ in terms of n.

   (b) Let T be the triangular region bounded by ℓ, the x-axis, and the line $x = 1$. Show that the area of T is $\frac{1}{2n}$.

   (c) Let S be the region bounded by the graph of $y = x^n$, the line ℓ, and the x-axis. Express the area of S in terms of n and determine the value of n that maximizes the area of S.

END OF EXAMINATION

AP® Calculus AB
2003 Free-Response Questions

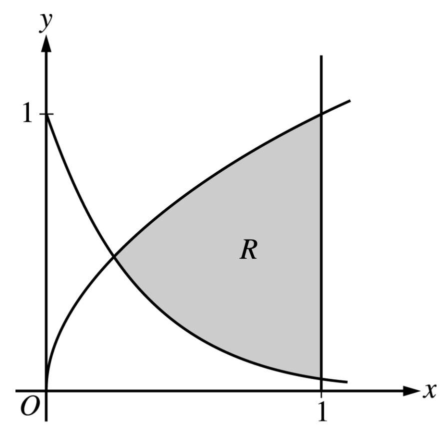
CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let R be the shaded region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-3x}$ and the vertical line $x = 1$, as shown in the figure above.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 1$.

   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a rectangle whose height is 5 times the length of its base in region R. Find the volume of this solid.

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2. A particle moves along the x-axis so that its velocity at time t is given by

$$v(t) = -(t + 1)\sin\left(\frac{t^2}{2}\right).$$

At time $t = 0$, the particle is at position $x = 1$.

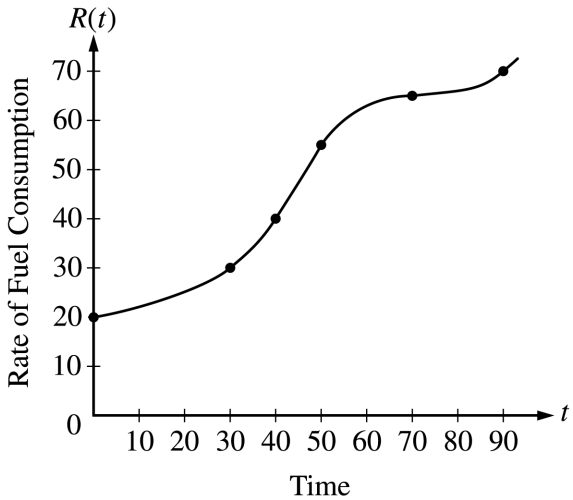
(a) Find the acceleration of the particle at time $t = 2$. Is the speed of the particle increasing at $t = 2$? Why or why not?

(b) Find all times t in the open interval $0 < t < 3$ when the particle changes direction. Justify your answer.

(c) Find the total distance traveled by the particle from time $t = 0$ until time $t = 3$.

(d) During the time interval $0 \le t \le 3$, what is the greatest distance between the particle and the origin? Show the work that leads to your answer.

GO ON TO THE NEXT PAGE.



t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

3. The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function R of time t. The graph of R and a table of selected values of $R(t)$, for the time interval $0 \le t \le 90$ minutes, are shown above.

   (a) Use data from the table to find an approximation for $R'(45)$. Show the computations that lead to your answer. Indicate units of measure.

   (b) The rate of fuel consumption is increasing fastest at time $t = 45$ minutes. What is the value of $R''(45)$? Explain your reasoning.

   (c) Approximate the value of $\int_0^{90} R(t)\,dt$ using a left Riemann sum with the five subintervals indicated by the data in the table. Is this numerical approximation less than the value of $\int_0^{90} R(t)\,dt$? Explain your reasoning.

   (d) For $0 < b \le 90$ minutes, explain the meaning of $\int_0^{b} R(t)\,dt$ in terms of fuel consumption for the plane. Explain the meaning of $\frac{1}{b}\int_0^{b} R(t)\,dt$ in terms of fuel consumption for the plane. Indicate units of measure in both answers.

END OF PART A OF SECTION II

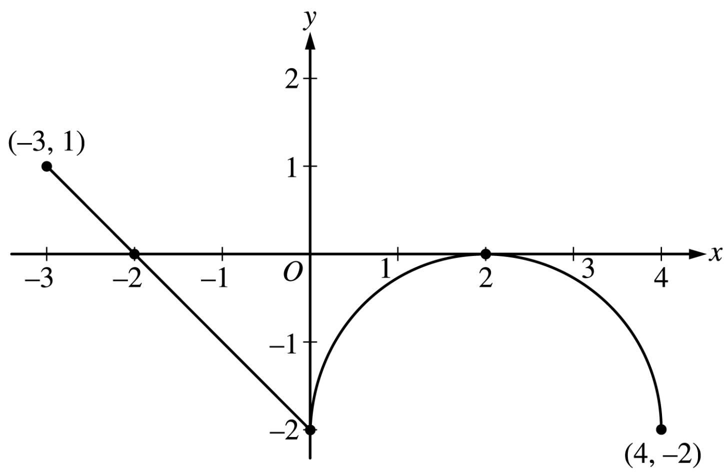
CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Graph of f'

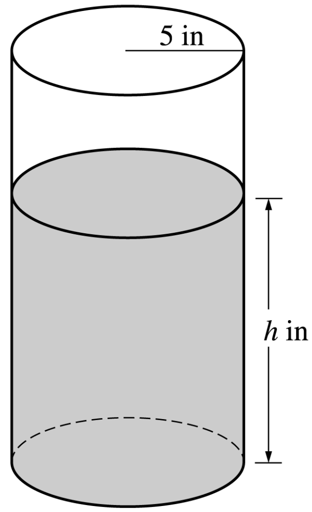
4. Let f be a function defined on the closed interval $-3 \le x \le 4$ with $f(0) = 3$. The graph of f', the derivative of f, consists of one line segment and a semicircle, as shown above.

   (a) On what intervals, if any, is f increasing? Justify your answer.

   (b) Find the x-coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$. Justify your answer.

   (c) Find an equation for the line tangent to the graph of f at the point $(0, 3)$.

   (d) Find $f(-3)$ and $f(4)$. Show the work that leads to your answers.



5. A coffeepot has the shape of a cylinder with radius 5 inches, as shown in the figure above. Let h be the depth of the coffee in the pot, measured in inches, where h is a function of time t, measured in seconds. The volume V of coffee in the pot is changing at the rate of $-5\pi\sqrt{h}$ cubic inches per second. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)

   (a) Show that $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$.

   (b) Given that $h = 17$ at time $t = 0$, solve the differential equation $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$ for h as a function of t.

   (c) At what time t is the coffeepot empty?

---

6. Let f be the function defined by

$$f(x) = \begin{cases} \sqrt{x+1} & \text{for } 0 \le x \le 3 \\ 5 - x & \text{for } 3 < x \le 5. \end{cases}$$

   (a) Is f continuous at $x = 3$? Explain why or why not.

   (b) Find the average value of $f(x)$ on the closed interval $0 \le x \le 5$.

   (c) Suppose the function g is defined by

$$g(x) = \begin{cases} k\sqrt{x+1} & \text{for } 0 \le x \le 3 \\ mx + 2 & \text{for } 3 < x \le 5, \end{cases}$$

   where k and m are constants. If g is differentiable at $x = 3$, what are the values of k and m?

END OF EXAMINATION

AP® Calculus AB
2003 Free-Response Questions
Form B

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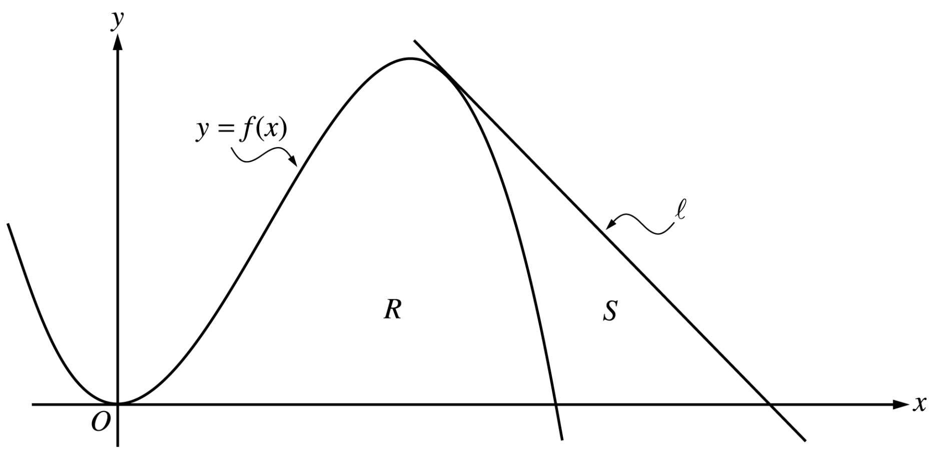
CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let f be the function given by $f(x) = 4x^2 - x^3$, and let ℓ be the line $y = 18 - 3x$, where ℓ is tangent to the graph of f. Let R be the region bounded by the graph of f and the x-axis, and let S be the region bounded by the graph of f, the line ℓ, and the x-axis, as shown above.

   (a) Show that ℓ is tangent to the graph of $y = f(x)$ at the point $x = 3$.

   (b) Find the area of S.

   (c) Find the volume of the solid generated when R is revolved about the x-axis.

GO ON TO THE NEXT PAGE.

2. A tank contains 125 gallons of heating oil at time $t = 0$. During the time interval $0 \le t \le 12$ hours, heating oil is pumped into the tank at the rate

$$H(t) = 2 + \frac{10}{(1 + \ln(t + 1))} \text{ gallons per hour.}$$

During the same time interval, heating oil is removed from the tank at the rate

$$R(t) = 12 \sin\left(\frac{t^2}{47}\right) \text{ gallons per hour.}$$

(a) How many gallons of heating oil are pumped into the tank during the time interval $0 \le t \le 12$ hours?

(b) Is the level of heating oil in the tank rising or falling at time $t = 6$ hours? Give a reason for your answer.

(c) How many gallons of heating oil are in the tank at time $t = 12$ hours?

(d) At what time t, for $0 \le t \le 12$, is the volume of heating oil in the tank the least? Show the analysis that leads to your conclusion.

GO ON TO THE NEXT PAGE.

Distance x (mm)	0	60	120	180	240	300	360
Diameter $B(x)$ (mm)	24	30	28	30	26	24	26

3. A blood vessel is 360 millimeters (mm) long with circular cross sections of varying diameter. The table above gives the measurements of the diameter of the blood vessel at selected points along the length of the blood vessel, where x represents the distance from one end of the blood vessel and $B(x)$ is a twice-differentiable function that represents the diameter at that point.

   (a) Write an integral expression in terms of $B(x)$ that represents the average radius, in mm, of the blood vessel between $x = 0$ and $x = 360$.

   (b) Approximate the value of your answer from part (a) using the data from the table and a midpoint Riemann sum with three subintervals of equal length. Show the computations that lead to your answer.

   (c) Using correct units, explain the meaning of $\pi \int_{125}^{275} \left(\frac{B(x)}{2} \right)^2 dx$ in terms of the blood vessel.

   (d) Explain why there must be at least one value x, for $0 < x < 360$, such that $B''(x) = 0$.

END OF PART A OF SECTION II

CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

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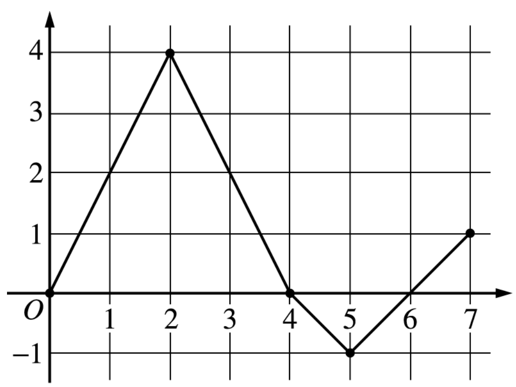
4. A particle moves along the x-axis with velocity at time $t \geq 0$ given by $v(t) = -1 + e^{1-t}$.
   (a) Find the acceleration of the particle at time $t = 3$.
   (b) Is the speed of the particle increasing at time $t = 3$? Give a reason for your answer.
   (c) Find all values of t at which the particle changes direction. Justify your answer.
   (d) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 3$.



Graph of f

5. Let f be a function defined on the closed interval $[0, 7]$. The graph of f, consisting of four line segments, is shown above. Let g be the function given by $g(x) = \int_2^x f(t)\,dt$.
   (a) Find $g(3)$, $g'(3)$, and $g''(3)$.
   (b) Find the average rate of change of g on the interval $0 \leq x \leq 3$.
   (c) For how many values c, where $0 < c < 3$, is $g'(c)$ equal to the average rate found in part (b) ? Explain your reasoning.
   (d) Find the x-coordinate of each point of inflection of the graph of g on the interval $0 < x < 7$. Justify your answer.

---

6. Let f be the function satisfying $f'(x) = x\sqrt{f(x)}$ for all real numbers x, where $f(3) = 25$.

   (a) Find $f''(3)$.

   (b) Write an expression for $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = x\sqrt{y}$ with the initial condition $f(3) = 25$.

END OF EXAMINATION

AP® Calculus AB
2002 Free-Response Questions

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

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1. Let f and g be the functions given by $f(x) = e^x$ and $g(x) = \ln x$.

   (a) Find the area of the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and $x = 1$.

   (b) Find the volume of the solid generated when the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and $x = 1$ is revolved about the line $y = 4$.

   (c) Let h be the function given by $h(x) = f(x) - g(x)$. Find the absolute minimum value of $h(x)$ on the closed interval $\frac{1}{2} \le x \le 1$, and find the absolute maximum value of $h(x)$ on the closed interval $\frac{1}{2} \le x \le 1$. Show the analysis that leads to your answers.

---

2. The rate at which people enter an amusement park on a given day is modeled by the function E defined by

$$E(t) = \frac{15600}{\left(t^2 - 24t + 160\right)}.$$

The rate at which people leave the same amusement park on the same day is modeled by the function L defined by

$$L(t) = \frac{9890}{\left(t^2 - 38t + 370\right)}.$$

Both $E(t)$ and $L(t)$ are measured in people per hour and time t is measured in hours after midnight. These functions are valid for $9 \le t \le 23,$ the hours during which the park is open. At time $t = 9,$ there are no people in the park.

(a) How many people have entered the park by 5:00 P.M. ($t = 17$)? Round your answer to the nearest whole number.

(b) The price of admission to the park is \$15 until 5:00 P.M. ($t = 17$). After 5:00 P.M., the price of admission to the park is \$11. How many dollars are collected from admissions to the park on the given day? Round your answer to the nearest whole number.

(c) Let $H(t) = \int_9^t \left(E(x) - L(x)\right) dx$ for $9 \le t \le 23.$ The value of $H(17)$ to the nearest whole number is 3725. Find the value of $H'(17)$, and explain the meaning of $H(17)$ and $H'(17)$ in the context of the amusement park.

(d) At what time t, for $9 \le t \le 23,$ does the model predict that the number of people in the park is a maximum?

GO ON TO THE NEXT PAGE.

3. An object moves along the x-axis with initial position $x(0) = 2$. The velocity of the object at time $t \geq 0$ is given by $v(t) = \sin\left(\frac{\pi}{3}t\right)$.

   (a) What is the acceleration of the object at time $t = 4$?

   (b) Consider the following two statements.

   Statement I: For $3 < t < 4.5$, the velocity of the object is decreasing.
   Statement II: For $3 < t < 4.5$, the speed of the object is increasing.

   Are either or both of these statements correct? For each statement provide a reason why it is correct or not correct.

   (c) What is the total distance traveled by the object over the time interval $0 \leq t \leq 4$?

   (d) What is the position of the object at time $t = 4$?

END OF PART A OF SECTION II

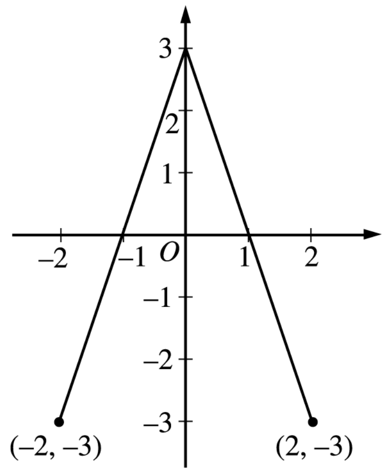
CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Graph of f

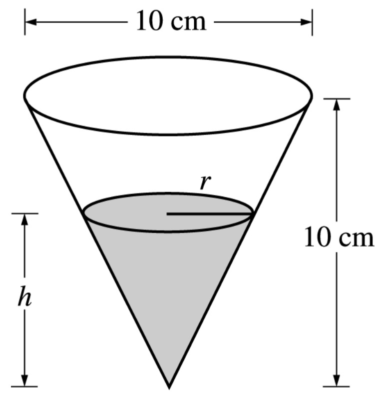
4. The graph of the function f shown above consists of two line segments. Let g be the function given by $g(x) = \int_0^x f(t)\, dt$.

   (a) Find $g(-1)$, $g'(-1)$, and $g''(-1)$.

   (b) For what values of x in the open interval $(-2, 2)$ is g increasing? Explain your reasoning.

   (c) For what values of x in the open interval $(-2, 2)$ is the graph of g concave down? Explain your reasoning.

   (d) On the axes provided, sketch the graph of g on the closed interval $[-2, 2]$.
   (Note: The axes are provided in the pink test booklet only.)



5. A container has the shape of an open right circular cone, as shown in the figure above. The height of the container is 10 cm and the diameter of the opening is 10 cm. Water in the container is evaporating so that its depth h is changing at the constant rate of $\frac{-3}{10}$ cm/hr.

   (Note: The volume of a cone of height h and radius r is given by $V = \frac{1}{3}\pi r^2 h$.)

   (a) Find the volume V of water in the container when $h = 5$ cm. Indicate units of measure.

   (b) Find the rate of change of the volume of water in the container, with respect to time, when $h = 5$ cm. Indicate units of measure.

   (c) Show that the rate of change of the volume of water in the container due to evaporation is directly proportional to the exposed surface area of the water. What is the constant of proportionality?

GO ON TO THE NEXT PAGE.

x	-1.5	-1.0	-0.5	0	0.5	1.0	1.5
$f(x)$	-1	-4	-6	-7	-6	-4	-1
$f'(x)$	-7	-5	-3	0	3	5	7

6. Let f be a function that is differentiable for all real numbers. The table above gives the values of f and its derivative f' for selected points x in the closed interval $-1.5 \le x \le 1.5$. The second derivative of f has the property that $f''(x) > 0$ for $-1.5 \le x \le 1.5$.

   (a) Evaluate $\int_0^{1.5} (3f'(x) + 4)\,dx$. Show the work that leads to your answer.

   (b) Write an equation of the line tangent to the graph of f at the point where $x = 1$. Use this line to approximate the value of $f(1.2)$. Is this approximation greater than or less than the actual value of $f(1.2)$? Give a reason for your answer.

   (c) Find a positive real number r having the property that there must exist a value c with $0 < c < 0.5$ and $f''(c) = r$. Give a reason for your answer.

   (d) Let g be the function given by $g(x) = \begin{cases} 2x^2 - x - 7 & \text{for } x < 0 \\ 2x^2 + x - 7 & \text{for } x \ge 0. \end{cases}$

   The graph of g passes through each of the points $(x, f(x))$ given in the table above. Is it possible that f and g are the same function? Give a reason for your answer.

END OF EXAMINATION

AP® Calculus AB
2002 Free-Response Questions
Form B

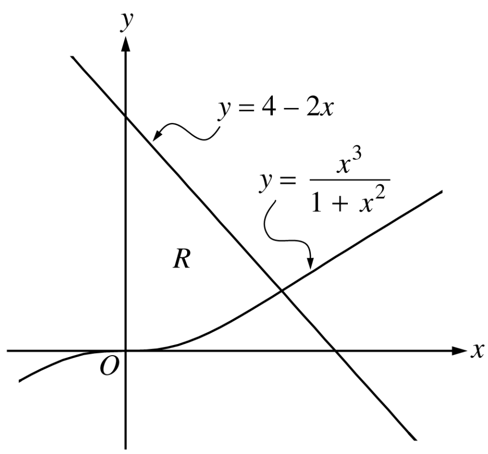
CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let R be the region bounded by the y-axis and the graphs of $y = \dfrac{x^3}{1 + x^2}$ and $y = 4 - 2x$, as shown in the figure above.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is revolved about the x-axis.

   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

2. The number of gallons, $P(t)$, of a pollutant in a lake changes at the rate $P'(t) = 1 - 3e^{-0.2\sqrt{t}}$ gallons per day, where t is measured in days. There are 50 gallons of the pollutant in the lake at time $t = 0$. The lake is considered to be safe when it contains 40 gallons or less of pollutant.

   (a) Is the amount of pollutant increasing at time $t = 9$? Why or why not?

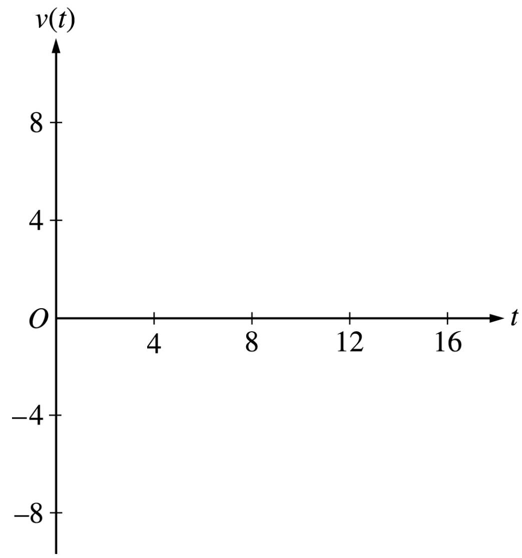
   (b) For what value of t will the number of gallons of pollutant be at its minimum? Justify your answer.

   (c) Is the lake safe when the number of gallons of pollutant is at its minimum? Justify your answer.

   (d) An investigator uses the tangent line approximation to $P(t)$ at $t = 0$ as a model for the amount of pollutant in the lake. At what time t does this model predict that the lake becomes safe?

3. A particle moves along the x-axis so that its velocity v at any time t, for $0 \le t \le 16$, is given by $v(t) = e^{2\sin t} - 1$. At time $t = 0$, the particle is at the origin.

   (a) On the axes provided, sketch the graph of $v(t)$ for $0 \le t \le 16$.
   (Note: Use the axes provided in the test booklet.)



   (b) During what intervals of time is the particle moving to the left? Give a reason for your answer.

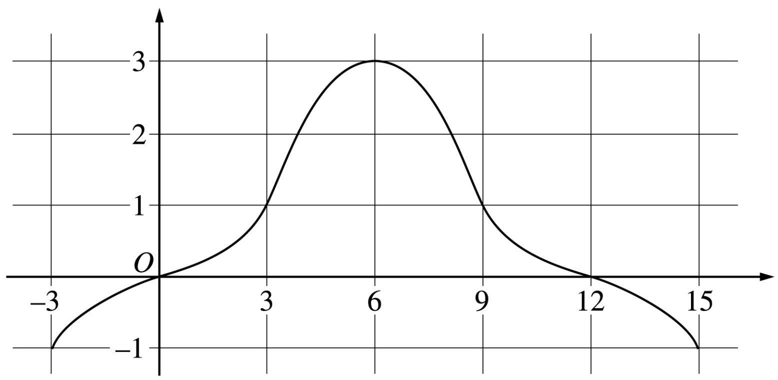
   (c) Find the total distance traveled by the particle from $t = 0$ to $t = 4$.

   (d) Is there any time t, $0 < t \le 16$, at which the particle returns to the origin? Justify your answer.

END OF PART A OF SECTION II

CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



Graph of f

4. The graph of a differentiable function f on the closed interval $[-3, 15]$ is shown in the figure above. The graph of f has a horizontal tangent line at $x = 6$. Let $g(x) = 5 + \int_6^x f(t)\,dt$ for $-3 \le x \le 15$.

   (a) Find $g(6)$, $g'(6)$, and $g''(6)$.

   (b) On what intervals is g decreasing? Justify your answer.

   (c) On what intervals is the graph of g concave down? Justify your answer.

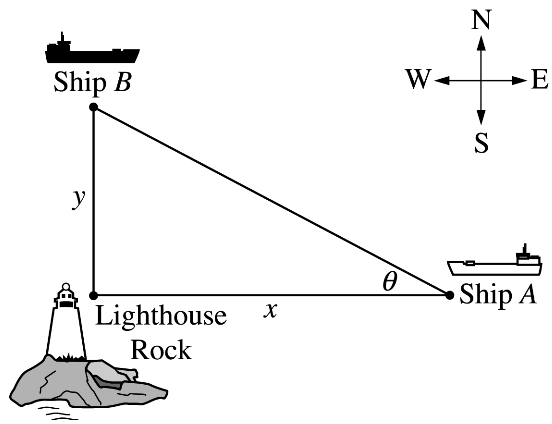
   (d) Find a trapezoidal approximation of $\int_{-3}^{15} f(t)\,dt$ using six subintervals of length $\Delta t = 3$.

5. Consider the differential equation $\dfrac{dy}{dx} = \dfrac{3 - x}{y}$.

   (a) Let $y = f(x)$ be the particular solution to the given differential equation for $1 < x < 5$ such that the line $y = -2$ is tangent to the graph of f. Find the x-coordinate of the point of tangency, and determine whether f has a local maximum, local minimum, or neither at this point. Justify your answer.

   (b) Let $y = g(x)$ be the particular solution to the given differential equation for $-2 < x < 8$, with the initial condition $g(6) = -4$. Find $y = g(x)$.

GO ON TO THE NEXT PAGE.



6. Ship A is traveling due west toward Lighthouse Rock at a speed of 15 kilometers per hour (km/hr). Ship B is traveling due north away from Lighthouse Rock at a speed of 10 km/hr. Let x be the distance between Ship A and Lighthouse Rock at time t, and let y be the distance between Ship B and Lighthouse Rock at time t, as shown in the figure above.

   (a) Find the distance, in kilometers, between Ship A and Ship B when $x = 4$ km and $y = 3$ km.

   (b) Find the rate of change, in km/hr, of the distance between the two ships when $x = 4$ km and $y = 3$ km.

   (c) Let θ be the angle shown in the figure. Find the rate of change of θ, in radians per hour, when $x = 4$ km and $y = 3$ km.

END OF EXAMINATION

AP Calculus AB
2001 Free-Response Questions

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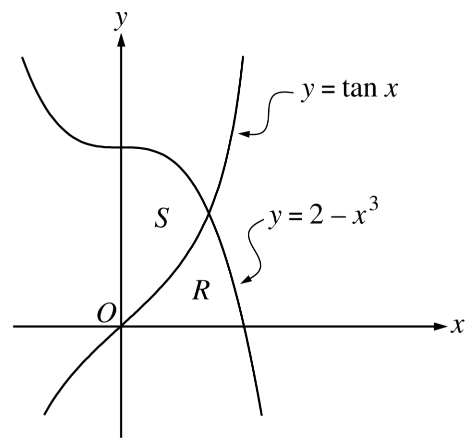
CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let R and S be the regions in the first quadrant shown in the figure above. The region R is bounded by the x-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$. The region S is bounded by the y-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$.

   (a) Find the area of R.

   (b) Find the area of S.

   (c) Find the volume of the solid generated when S is revolved about the x-axis.

t (days)	$W(t)$ (°C)
0	20
3	31
6	28
9	24
12	22
15	21

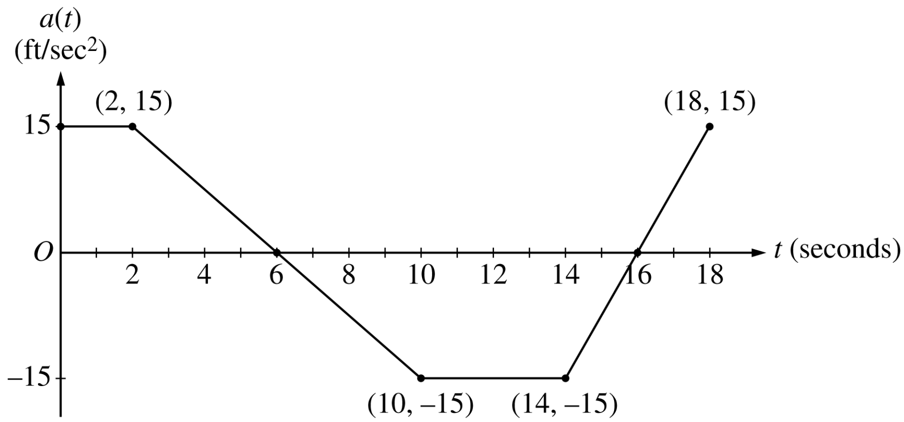
2. The temperature, in degrees Celsius (°C), of the water in a pond is a differentiable function W of time t. The table above shows the water temperature as recorded every 3 days over a 15-day period.

(a) Use data from the table to find an approximation for $W'(12)$. Show the computations that lead to your answer. Indicate units of measure.

(b) Approximate the average temperature, in degrees Celsius, of the water over the time interval $0 \le t \le 15$ days by using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days.

(c) A student proposes the function P, given by $P(t) = 20 + 10te^{(-t/3)}$, as a model for the temperature of the water in the pond at time t, where t is measured in days and $P(t)$ is measured in degrees Celsius. Find $P'(12)$. Using appropriate units, explain the meaning of your answer in terms of water temperature.

(d) Use the function P defined in part (c) to find the average value, in degrees Celsius, of $P(t)$ over the time interval $0 \le t \le 15$ days.



3. A car is traveling on a straight road with velocity 55 ft/sec at time $t = 0$. For $0 \le t \le 18$ seconds, the car's acceleration $a(t)$, in ft/sec^2, is the piecewise linear function defined by the graph above.

(a) Is the velocity of the car increasing at $t = 2$ seconds? Why or why not?

(b) At what time in the interval $0 \le t \le 18$, other than $t = 0$, is the velocity of the car 55 ft/sec? Why?

(c) On the time interval $0 \le t \le 18$, what is the car's absolute maximum velocity, in ft/sec, and at what time does it occur? Justify your answer.

(d) At what times in the interval $0 \le t \le 18$, if any, is the car's velocity equal to zero? Justify your answer.

END OF PART A OF SECTION II

CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

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4. Let h be a function defined for all $x \neq 0$ such that $h(4) = -3$ and the derivative of h is given by $h'(x) = \dfrac{x^2 - 2}{x}$ for all $x \neq 0$.

   (a) Find all values of x for which the graph of h has a horizontal tangent, and determine whether h has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

   (b) On what intervals, if any, is the graph of h concave up? Justify your answer.

   (c) Write an equation for the line tangent to the graph of h at $x = 4$.

   (d) Does the line tangent to the graph of h at $x = 4$ lie above or below the graph of h for $x > 4$? Why?

---

5. A cubic polynomial function f is defined by

$$f(x) = 4x^3 + ax^2 + bx + k$$

   where a, b, and k are constants. The function f has a local minimum at $x = -1$, and the graph of f has a point of inflection at $x = -2$.

   (a) Find the values of a and b.

   (b) If $\int_0^1 f(x)\,dx = 32$, what is the value of k ?

---

6. The function f is differentiable for all real numbers. The point $\left(3, \frac{1}{4}\right)$ is on the graph of $y = f(x)$, and the slope at each point (x, y) on the graph is given by $\dfrac{dy}{dx} = y^2(6 - 2x)$.

   (a) Find $\dfrac{d^2y}{dx^2}$ and evaluate it at the point $\left(3, \frac{1}{4}\right)$.

   (b) Find $y = f(x)$ by solving the differential equation $\dfrac{dy}{dx} = y^2(6 - 2x)$ with the initial condition $f(3) = \dfrac{1}{4}$.

END OF EXAMINATION

2000 Advanced Placement Program®
Free-Response Questions

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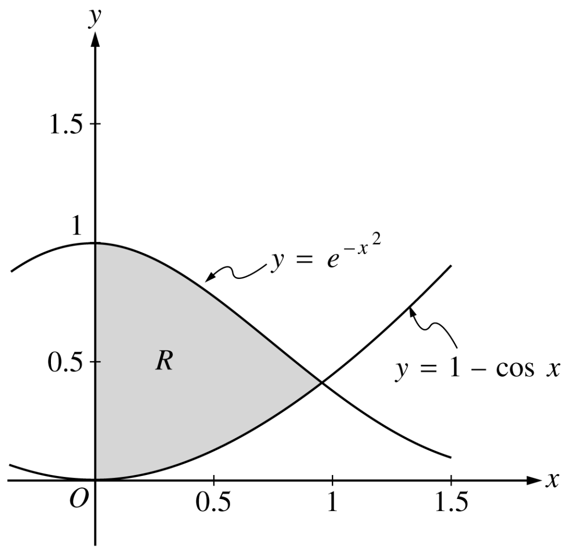
CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$, and the y-axis, as shown in the figure above.

   (a) Find the area of the region R.

   (b) Find the volume of the solid generated when the region R is revolved about the x-axis.

   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

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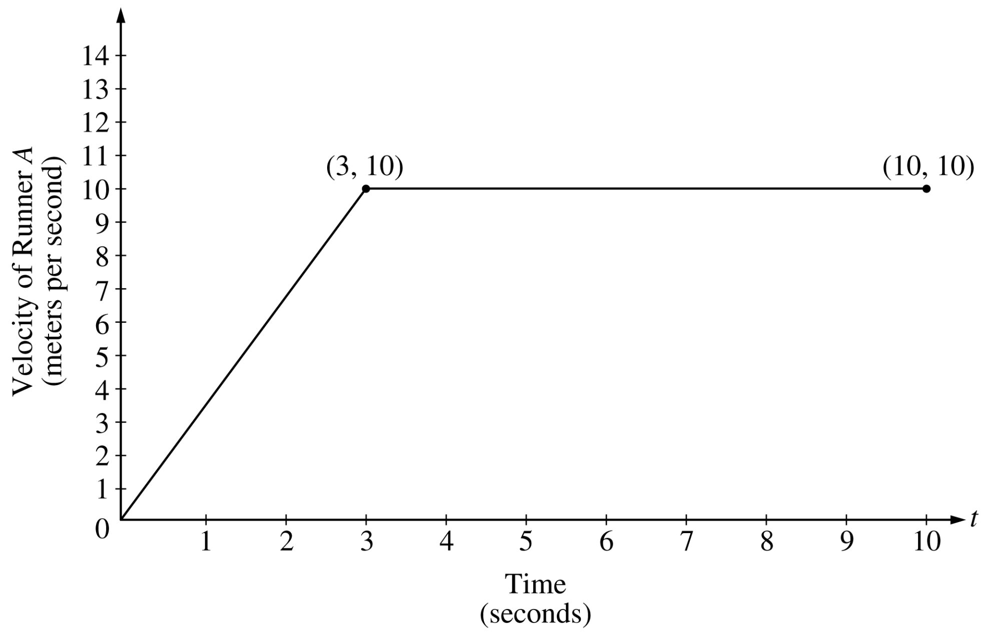

2. Two runners, A and B, run on a straight racetrack for $0 \le t \le 10$ seconds. The graph above, which consists of two line segments, shows the velocity, in meters per second, of Runner A. The velocity, in meters per second, of Runner B is given by the function v defined by $v(t) = \dfrac{24t}{2t+3}$.

   (a) Find the velocity of Runner A and the velocity of Runner B at time $t = 2$ seconds. Indicate units of measure.

   (b) Find the acceleration of Runner A and the acceleration of Runner B at time $t = 2$ seconds. Indicate units of measure.

   (c) Find the total distance run by Runner A and the total distance run by Runner B over the time interval $0 \le t \le 10$ seconds. Indicate units of measure.

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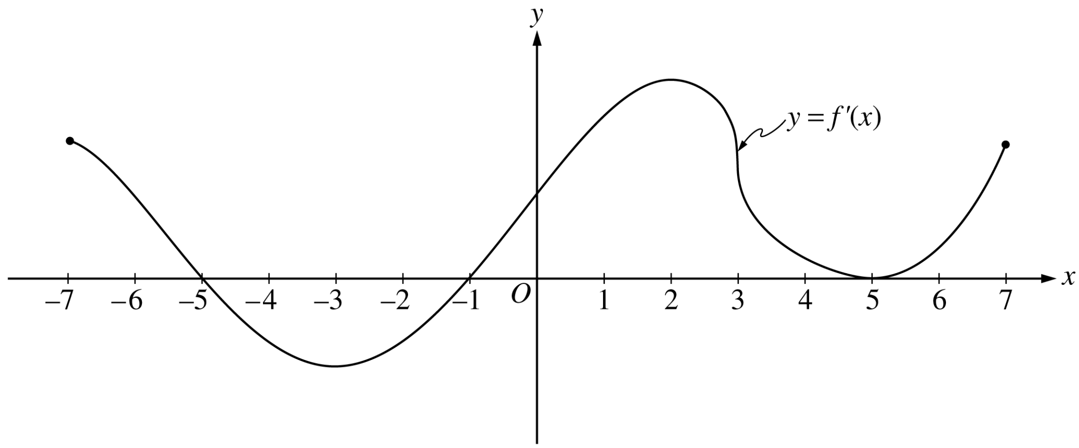

3. The figure above shows the graph of f', the derivative of the function f, for $-7 \le x \le 7$. The graph of f' has horizontal tangent lines at $x = -3$, $x = 2$, and $x = 5$, and a vertical tangent line at $x = 3$.

   (a) Find all values of x, for $-7 < x < 7$, at which f attains a relative minimum. Justify your answer.

   (b) Find all values of x, for $-7 < x < 7$, at which f attains a relative maximum. Justify your answer.

   (c) Find all values of x, for $-7 < x < 7$, at which $f''(x) < 0$.

   (d) At what value of x, for $-7 \le x \le 7$, does f attain its absolute maximum? Justify your answer.

END OF PART A OF SECTION II

CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

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4. Water is pumped into an underground tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of $\sqrt{t+1}$ gallons per minute, for $0 \le t \le 120$ minutes. At time $t = 0$, the tank contains 30 gallons of water.

   (a) How many gallons of water leak out of the tank from time $t = 0$ to $t = 3$ minutes?

   (b) How many gallons of water are in the tank at time $t = 3$ minutes?

   (c) Write an expression for $A(t)$, the total number of gallons of water in the tank at time t.

   (d) At what time t, for $0 \le t \le 120$, is the amount of water in the tank a maximum? Justify your answer.

---

5. Consider the curve given by $xy^2 - x^3y = 6$.

   (a) Show that $\dfrac{dy}{dx} = \dfrac{3x^2y - y^2}{2xy - x^3}$.

   (b) Find all points on the curve whose x-coordinate is 1, and write an equation for the tangent line at each of these points.

   (c) Find the x-coordinate of each point on the curve where the tangent line is vertical.

---

6. Consider the differential equation $\dfrac{dy}{dx} = \dfrac{3x^2}{e^{2y}}$.

   (a) Find a solution $y = f(x)$ to the differential equation satisfying $f(0) = \dfrac{1}{2}$.

   (b) Find the domain and range of the function f found in part (a).

END OF EXAMINATION

AP Calculus AB
1999 Free-Response Questions

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1999

The College Board

Advanced Placement Examination

CALCULUS AB

SECTION II

Time — 1 hour and 30 minutes

Number of problems — 6

Percent of total grade — 50

REMEMBER TO SHOW YOUR SETUPS AS DESCRIBED IN THE GENERAL INSTRUCTIONS.

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1. A particle moves along the y-axis with velocity given by $v(t) = t\sin(t^2)$ for $t \geq 0$.

   (a) In which direction (up or down) is the particle moving at time $t = 1.5$? Why?

   (b) Find the acceleration of the particle at time $t = 1.5$. Is the velocity of the particle increasing at $t = 1.5$? Why or why not?

   (c) Given that $y(t)$ is the position of the particle at time t and that $y(0) = 3$, find $y(2)$.

   (d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

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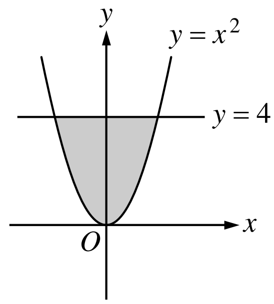

2. The shaded region, R, is bounded by the graph of $y = x^2$ and the line $y = 4$, as shown in the figure above.

   (a) Find the area of R.

   (b) Find the volume of the solid generated by revolving R about the x-axis.

   (c) There exists a number k, $k > 4$, such that when R is revolved about the line $y = k$, the resulting solid has the same volume as the solid in part (b). Write, but do not solve, an equation involving an integral expression that can be used to find the value of k.

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t (hours)	$R(t)$ (gallons per hour)
0	9.6
3	10.4
6	10.8
9	11.2
12	11.4
15	11.3
18	10.7
21	10.2
24	9.6

3. The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t. The table above shows the rate as measured every 3 hours for a 24-hour period.

   (a) Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate $\int_0^{24} R(t)dt$. Using correct units, explain the meaning of your answer in terms of water flow.

   (b) Is there some time t, $0 < t < 24$, such that $R'(t) = 0$? Justify your answer.

   (c) The rate of water flow $R(t)$ can be approximated by $Q(t) = \frac{1}{79}\left(768 + 23t - t^2\right)$. Use $Q(t)$ to approximate the average rate of water flow during the 24-hour time period. Indicate units of measure.

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4. Suppose that the function f has a continuous second derivative for all x, and that $f(0) = 2$, $f'(0) = -3$, and $f''(0) = 0$. Let g be a function whose derivative is given by $g'(x) = e^{-2x}(3f(x) + 2f'(x))$ for all x.

(a) Write an equation of the line tangent to the graph of f at the point where $x = 0$.

(b) Is there sufficient information to determine whether or not the graph of f has a point of inflection when $x = 0$? Explain your answer.

(c) Given that $g(0) = 4$, write an equation of the line tangent to the graph of g at the point where $x = 0$.

(d) Show that $g''(x) = e^{-2x}(-6f(x) - f'(x) + 2f''(x))$. Does g have a local maximum at $x = 0$? Justify your answer.

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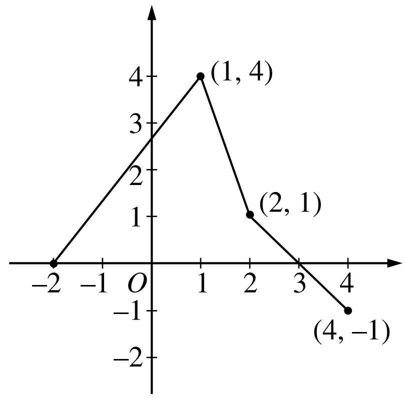

5. The graph of the function f, consisting of three line segments, is given above. Let $g(x) = \int_1^x f(t)\,dt$.

(a) Compute $g(4)$ and $g(-2)$.

(b) Find the instantaneous rate of change of g, with respect to x, at $x = 1$.

(c) Find the absolute minimum value of g on the closed interval $[-2, 4]$. Justify your answer.

(d) The second derivative of g is not defined at $x = 1$ and $x = 2$. How many of these values are x-coordinates of points of inflection of the graph of g? Justify your answer.

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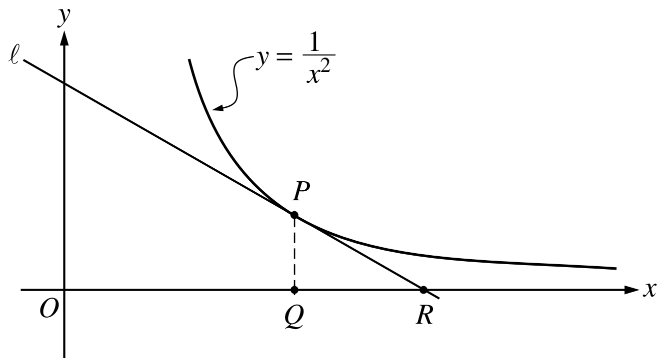

6. In the figure above, line ℓ is tangent to the graph of $y = \frac{1}{x^2}$ at point P, with coordinates $\left(w, \frac{1}{w^2}\right)$, where $w > 0$. Point Q has coordinates $(w, 0)$. Line ℓ crosses the x-axis at point R, with coordinates $(k, 0)$.

   (a) Find the value of k when $w = 3$.

   (b) For all $w > 0$, find k in terms of w.

   (c) Suppose that w is increasing at the constant rate of 7 units per second. When $w = 5$, what is the rate of change of k with respect to time?

   (d) Suppose that w is increasing at the constant rate of 7 units per second. When $w = 5$, what is the rate of change of the area of $\triangle PQR$ with respect to time? Determine whether the area is increasing or decreasing at this instant.

END OF EXAMINATION

AP® Calculus AB
1998 Free-Response Questions

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1998 AP Calculus AB Free-Response Questions

CALCULUS AB

Section II

Time—1 hour and 30 minutes

Number of problems—6

Percent of total grade—50

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS ON THIS SECTION OF THE EXAMINATION.

REMEMBER TO SHOW YOUR SETUPS AS DESCRIBED IN THE GENERAL INSTRUCTIONS.

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1. Let R be the region bounded by the x–axis, the graph of $y = \sqrt{x}$, and the line $x = 4$.

   (a) Find the area of the region R.

   (b) Find the value of h such that the vertical line $x = h$ divides the region R into two regions of equal area.

   (c) Find the volume of the solid generated when R is revolved about the x–axis.

   (d) The vertical line $x = k$ divides the region R into two regions such that when these two regions are revolved about the x–axis, they generate solids with equal volumes. Find the value of k.

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2. Let f be the function given by $f(x) = 2xe^{2x}$.
   (a) Find $\lim_{x\to-\infty} f(x)$ and $\lim_{x\to\infty} f(x)$.
   (b) Find the absolute minimum value of f. Justify that your answer is an absolute minimum.
   (c) What is the range of f?
   (d) Consider the family of functions defined by $y = bxe^{bx}$, where b is a nonzero constant. Show that the absolute minimum value of bxe^{bx} is the same for all nonzero values of b.

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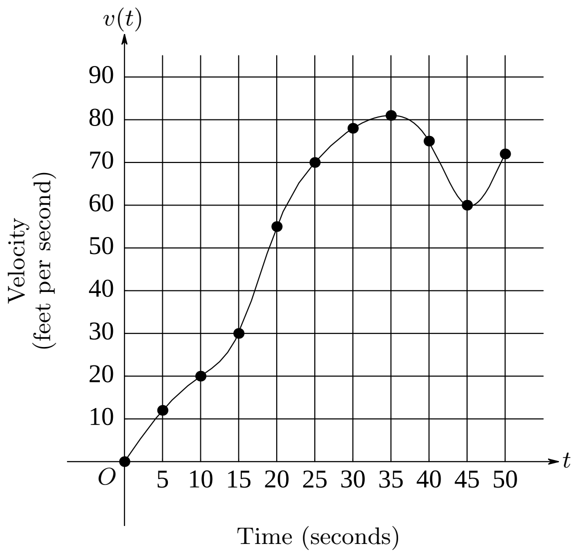

t (seconds)	$v(t)$ (feet per second)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72

3. The graph of the velocity $v(t)$, in ft/sec, of a car traveling on a straight road, for $0 \leq t \leq 50$, is shown above. A table of values for $v(t)$, at 5 second intervals of time t, is shown to the right of the graph.

   (a) During what intervals of time is the acceleration of the car positive? Give a reason for your answer.

   (b) Find the average acceleration of the car, in ft/sec^2, over the interval $0 \leq t \leq 50$.

   (c) Find one approximation for the acceleration of the car, in ft/sec^2, at $t = 40$. Show the computations you used to arrive at your answer.

   (d) Approximate $\int_0^{50} v(t)\,dt$ with a Riemann sum, using the midpoints of five subintervals of equal length. Using correct units, explain the meaning of this integral.

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4. Let f be a function with $f(1) = 4$ such that for all points (x, y) on the graph of f the slope is given by $\dfrac{3x^2+1}{2y}$.
   (a) Find the slope of the graph of f at the point where $x = 1$.
   (b) Write an equation for the line tangent to the graph of f at $x = 1$ and use it to approximate $f(1.2)$.
   (c) Find $f(x)$ by solving the separable differential equation $\dfrac{dy}{dx} = \dfrac{3x^2+1}{2y}$ with the initial condition $f(1) = 4$.
   (d) Use your solution from part (c) to find $f(1.2)$.

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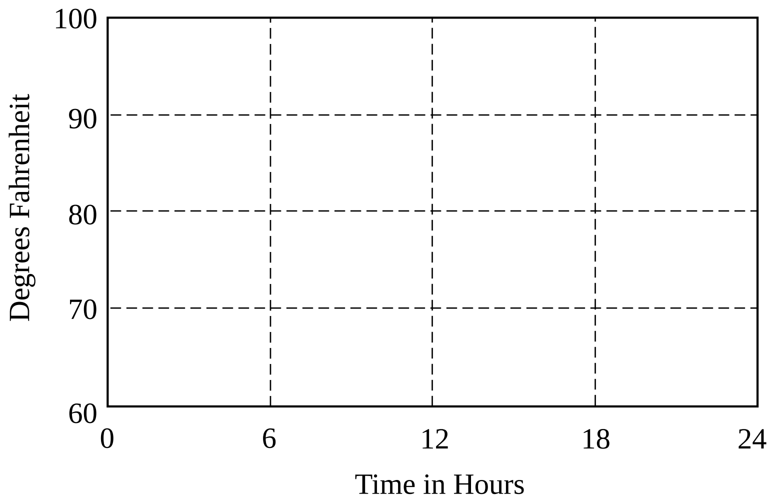
5. The temperature outside a house during a 24-hour period is given by

$$F(t) = 80 - 10\cos\left(\frac{\pi t}{12}\right),\ 0 \le t \le 24,$$

where $F(t)$ is measured in degrees Fahrenheit and t is measured in hours.

(a) Sketch the graph of F on the grid below.



(b) Find the average temperature, to the nearest degree Fahrenheit, between $t = 6$ and $t = 14$.

(c) An air conditioner cooled the house whenever the outside temperature was at or above 78 degrees Fahrenheit. For what values of t was the air conditioner cooling the house?

(d) The cost of cooling the house accumulates at the rate of \$0.05 per hour for each degree the outside temperature exceeds 78 degrees Fahrenheit. What was the total cost, to the nearest cent, to cool the house for this 24–hour period?

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6. Consider the curve defined by $2y^3 + 6x^2y - 12x^2 + 6y = 1$.

   (a) Show that $\dfrac{dy}{dx} = \dfrac{4x - 2xy}{x^2 + y^2 + 1}$.

   (b) Write an equation of each horizontal tangent line to the curve.

   (c) The line through the origin with slope -1 is tangent to the curve at point P. Find the x– and y–coordinates of point P.

END OF EXAMINATION