



## AP Calculus AB 2000 Scoring Guidelines

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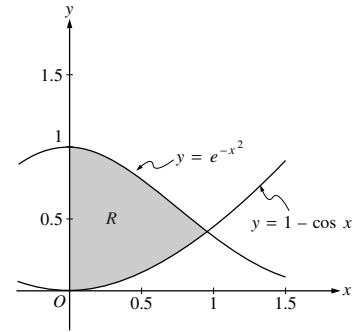
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## AP Calculus AB-1 / BC-1

Let  $R$  be the shaded region in the first quadrant enclosed by the graphs of  $y = e^{-x^2}$ ,  $y = 1 - \cos x$ , and the  $y$ -axis, as shown in the figure above.

- Find the area of the region  $R$ .
- Find the volume of the solid generated when the region  $R$  is revolved about the  $x$ -axis.
- The region  $R$  is the base of a solid. For this solid, each cross section perpendicular to the  $x$ -axis is a square. Find the volume of this solid.



Region  $R$

$$e^{-x^2} = 1 - \cos x \text{ at } x = 0.941944 = A$$

$$\begin{aligned} \text{(a) Area} &= \int_0^A (e^{-x^2} - (1 - \cos x)) dx \\ &= 0.590 \text{ or } 0.591 \end{aligned}$$

$$\begin{aligned} \text{(b) Volume} &= \pi \int_0^A \left( (e^{-x^2})^2 - (1 - \cos x)^2 \right) dx \\ &= 0.55596\pi = 1.746 \text{ or } 1.747 \end{aligned}$$

$$\begin{aligned} \text{(c) Volume} &= \int_0^A \left( e^{-x^2} - (1 - \cos x) \right)^2 dx \\ &= 0.461 \end{aligned}$$

1 : Correct limits in an integral in (a), (b), or (c).

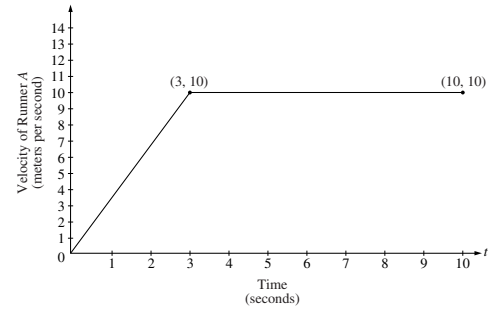
2  $\left\{ \begin{array}{l} 1 : \text{integrand} \\ 1 : \text{answer} \end{array} \right.$

3  $\left\{ \begin{array}{l} 2 : \text{integrand and constant} \\ < -1 > \text{ each error} \\ 1 : \text{answer} \end{array} \right.$

3  $\left\{ \begin{array}{l} 2 : \text{integrand} \\ < -1 > \text{ each error} \\ \text{Note: } 0/2 \text{ if not of the form} \\ \quad k \int_c^d (f(x) - g(x))^2 dx \\ 1 : \text{answer} \end{array} \right.$

## AP Calculus AB–2 / BC–2

Two runners,  $A$  and  $B$ , run on a straight racetrack for  $0 \leq t \leq 10$  seconds. The graph above, which consists of two line segments, shows the velocity, in meters per second, of Runner  $A$ . The velocity, in meters per second, of Runner  $B$  is given by the function  $v$  defined by  $v(t) = \frac{24t}{2t + 3}$ .



- Find the velocity of Runner  $A$  and the velocity of Runner  $B$  at time  $t = 2$  seconds. Indicate units of measure.
- Find the acceleration of Runner  $A$  and the acceleration of Runner  $B$  at time  $t = 2$  seconds. Indicate units of measure.
- Find the total distance run by Runner  $A$  and the total distance run by Runner  $B$  over the time interval  $0 \leq t \leq 10$  seconds. Indicate units of measure.

$$\begin{aligned} \text{(a) Runner } A: \text{ velocity} &= \frac{10}{3} \cdot 2 = \frac{20}{3} \\ &= 6.666 \text{ or } 6.667 \text{ meters/sec} \end{aligned}$$

$$\text{Runner } B: v(2) = \frac{48}{7} = 6.857 \text{ meters/sec}$$

$$\text{(b) Runner } A: \text{ acceleration} = \frac{10}{3} = 3.333 \text{ meters/sec}^2$$

$$\begin{aligned} \text{Runner } B: a(2) = v'(2) &= \frac{72}{(2t + 3)^2} \Big|_{t=2} \\ &= \frac{72}{49} = 1.469 \text{ meters/sec}^2 \end{aligned}$$

$$\text{(c) Runner } A: \text{ distance} = \frac{1}{2}(3)(10) + 7(10) = 85 \text{ meters}$$

$$\text{Runner } B: \text{ distance} = \int_0^{10} \frac{24t}{2t + 3} dt = 83.336 \text{ meters}$$

(units) meters/sec in part (a), meters/sec<sup>2</sup> in part (b), and meters in part (c), or equivalent.

$$2 \left\{ \begin{array}{l} 1: \text{ velocity for Runner } A \\ 1: \text{ velocity for Runner } B \end{array} \right.$$

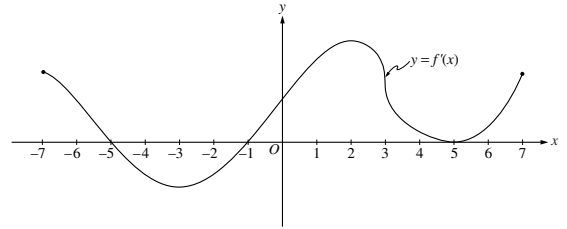
$$2 \left\{ \begin{array}{l} 1: \text{ acceleration for Runner } A \\ 1: \text{ acceleration for Runner } B \end{array} \right.$$

$$4 \left\{ \begin{array}{l} 2: \text{ distance for Runner } A \\ 1: \text{ method} \\ 1: \text{ answer} \\ 2: \text{ distance for Runner } B \\ 1: \text{ integral} \\ 1: \text{ answer} \end{array} \right.$$

1: units

## AP Calculus AB–3

The figure above shows the graph of  $f'$ , the derivative of the function  $f$ , for  $-7 \leq x \leq 7$ . The graph of  $f'$  has horizontal tangent lines at  $x = -3$ ,  $x = 2$ , and  $x = 5$ , and a vertical tangent line at  $x = 3$ .



- (a) Find all values of  $x$ , for  $-7 < x < 7$ , at which  $f$  attains a relative minimum. Justify your answer.
- (b) Find all values of  $x$ , for  $-7 < x < 7$ , at which  $f$  attains a relative maximum. Justify your answer.
- (c) Find all values of  $x$ , for  $-7 < x < 7$ , at which  $f''(x) < 0$ .
- (d) At what value of  $x$ , for  $-7 \leq x \leq 7$ , does  $f$  attain its absolute maximum? Justify your answer.

(a)  $x = -1$

$f'(x)$  changes from negative to positive at  $x = -1$

(b)  $x = -5$

$f'(x)$  changes from positive to negative at  $x = -5$

(c)  $f''(x)$  exists and  $f'$  is decreasing on the intervals  $(-7, -3)$ ,  $(2, 3)$ , and  $(3, 5)$

(d)  $x = 7$

The absolute maximum must occur at  $x = -5$  or at an endpoint.

$f(-5) > f(-7)$  because  $f$  is increasing on  $(-7, -5)$

The graph of  $f'$  shows that the magnitude of the negative change in  $f$  from  $x = -5$  to  $x = -1$  is smaller than the positive change in  $f$  from  $x = -1$  to  $x = 7$ .

Therefore the net change in  $f$  is positive from  $x = -5$  to  $x = 7$ , and  $f(7) > f(-5)$ . So  $f(7)$  is the absolute maximum.

$$2 \begin{cases} 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$$

$$2 \begin{cases} 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$$

$$2 \begin{cases} 1 : (-7, -3) \\ 1 : (2, 3) \cup (3, 5) \end{cases}$$

$$3 \begin{cases} 1 : \text{answer} \\ 1 : \text{identifies } x = -5 \text{ and } x = 7 \\ \quad \text{as candidates} \\ \quad \text{— or —} \\ \quad \text{indicates that the graph of } f \\ \quad \text{increases, decreases, then increases} \\ 1 : \text{justifies } f(7) > f(-5) \end{cases}$$

## AP Calculus AB-4

Water is pumped into an underground tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of  $\sqrt{t+1}$  gallons per minute, for  $0 \leq t \leq 120$  minutes. At time  $t = 0$ , the tank contains 30 gallons of water.

- (a) How many gallons of water leak out of the tank from time  $t = 0$  to  $t = 3$  minutes?  
 (b) How many gallons of water are in the tank at time  $t = 3$  minutes?  
 (c) Write an expression for  $A(t)$ , the total number of gallons of water in the tank at time  $t$ .  
 (d) At what time  $t$ , for  $0 \leq t \leq 120$ , is the amount of water in the tank a maximum? Justify your answer.

(a) Method 1:  $\int_0^3 \sqrt{t+1} dt = \frac{2}{3}(t+1)^{3/2} \Big|_0^3 = \frac{14}{3}$

– or –

Method 2:  $L(t)$  = gallons leaked in first  $t$  minutes

$$\frac{dL}{dt} = \sqrt{t+1}; \quad L(t) = \frac{2}{3}(t+1)^{3/2} + C$$

$$L(0) = 0; \quad C = -\frac{2}{3}$$

$$L(t) = \frac{2}{3}(t+1)^{3/2} - \frac{2}{3}; \quad L(3) = \frac{14}{3}$$

(b)  $30 + 8 \cdot 3 - \frac{14}{3} = \frac{148}{3}$

(c) Method 1:

$$\begin{aligned} ( ) &= 30 + \int_0^t (8 - \sqrt{x+1}) \\ &= 30 + 8t - \int_0^t \sqrt{x+1} \end{aligned}$$

– or –

Method 2:

$$\frac{dA}{dt} = 8 - \sqrt{t+1}$$

$$A(t) = 8t - \frac{2}{3}(t+1)^{3/2} + C$$

$$30 = 8(0) - \frac{2}{3}(0+1)^{3/2} + C; \quad C = \frac{92}{3}$$

$$A(t) = 8t - \frac{2}{3}(t+1)^{3/2} + \frac{92}{3}$$

- (d)  $A'(t) = 8 - \sqrt{t+1} = 0$  when  $t = 63$   
 $A'(t)$  is positive for  $0 < t < 63$  and negative for  $63 < t < 120$ . Therefore there is a maximum at  $t = 63$ .

Method 1:

$$3 \begin{cases} 2 : \text{definite integral} \\ 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$$

– or –

Method 2:

$$3 \begin{cases} 1 : \text{antiderivative with } C \\ 1 : \text{solves for } C \text{ using } L(0) = 0 \\ 1 : \text{answer} \end{cases}$$

1 : answer

Method 1:

$$2 \begin{cases} 1 : 30 + 8t \\ 1 : -\int_0^t \sqrt{x+1} dx \end{cases}$$

– or –

Method 2:

$$2 \begin{cases} 1 : \text{antiderivative with } C \\ 1 : \text{answer} \end{cases}$$

$$3 \begin{cases} 1 : \text{sets } A'(t) = 0 \\ 1 : \text{solves for } t \\ 1 : \text{justification} \end{cases}$$

## AP Calculus AB–5 / BC–5

Consider the curve given by  $xy^2 - x^3y = 6$ .

- (a) Show that  $\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$ .
- (b) Find all points on the curve whose  $x$ -coordinate is 1, and write an equation for the tangent line at each of these points.
- (c) Find the  $x$ -coordinate of each point on the curve where the tangent line is vertical.

(a)  $y^2 + 2xy \frac{dy}{dx} - 3x^2y - x^3 \frac{dy}{dx} = 0$

$$\frac{dy}{dx}(2xy - x^3) = 3x^2y - y^2$$

$$\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$$

(b) When  $x = 1$ ,  $y^2 - y = 6$   
 $y^2 - y - 6 = 0$   
 $(y - 3)(y + 2) = 0$   
 $y = 3, y = -2$

At  $(1, 3)$ ,  $\frac{dy}{dx} = \frac{9 - 9}{6 - 1} = 0$

Tangent line equation is  $y = 3$

At  $(1, -2)$ ,  $\frac{dy}{dx} = \frac{-6 - 4}{-4 - 1} = \frac{-10}{-5} = 2$

Tangent line equation is  $y + 2 = 2(x - 1)$

(c) Tangent line is vertical when  $2xy - x^3 = 0$

$$x(2y - x^2) = 0 \text{ gives } x = 0 \text{ or } y = \frac{1}{2}x^2$$

There is no point on the curve with  $x$ -coordinate 0.

When  $y = \frac{1}{2}x^2$ ,  $\frac{1}{4}x^5 - \frac{1}{2}x^5 = 6$

$$-\frac{1}{4}x^5 = 6$$

$$x = \sqrt[5]{-24}$$

$$2 \left\{ \begin{array}{l} 1 : \text{implicit differentiation} \\ 1 : \text{verifies expression for } \frac{dy}{dx} \end{array} \right.$$

$$4 \left\{ \begin{array}{l} 1 : y^2 - y = 6 \\ 1 : \text{solves for } y \\ 2 : \text{tangent lines} \end{array} \right.$$

Note: 0/4 if not solving an equation of the form  $y^2 - y = k$

$$3 \left\{ \begin{array}{l} 1 : \text{sets denominator of } \frac{dy}{dx} \text{ equal to 0} \\ 1 : \text{substitutes } y = \frac{1}{2}x^2 \text{ or } x = \pm\sqrt{2y} \\ \text{into the equation for the curve} \\ 1 : \text{solves for } x\text{-coordinate} \end{array} \right.$$

## AP Calculus AB-6

Consider the differential equation  $\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$ .

- (a) Find a solution  $y = f(x)$  to the differential equation satisfying  $f(0) = \frac{1}{2}$ .  
 (b) Find the domain and range of the function  $f$  found in part (a).

(a)  $e^{2y} dy = 3x^2 dx$

$$\frac{1}{2}e^{2y} = x^3 + C_1$$

$$e^{2y} = 2x^3 + C$$

$$y = \frac{1}{2} \ln(2x^3 + C)$$

$$\frac{1}{2} = \frac{1}{2} \ln(0 + C); \quad C = e$$

$$y = \frac{1}{2} \ln(2x^3 + e)$$

$$6 \left\{ \begin{array}{l} 1 : \text{separates variables} \\ 1 : \text{antiderivative of } dy \text{ term} \\ 1 : \text{antiderivative of } dx \text{ term} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition } f(0) = \frac{1}{2} \\ 1 : \text{solves for } y \\ \text{Note: } 0/1 \text{ if } y \text{ is not a logarithmic function of } x \end{array} \right.$$

Note: max 3/6 [1-1-1-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables

(b) Domain:  $2x^3 + e > 0$

$$x^3 > -\frac{1}{2}e$$

$$x > \left(-\frac{1}{2}e\right)^{1/3} = -\left(\frac{1}{2}e\right)^{1/3}$$

Range:  $-\infty < y < \infty$

$$3 \left\{ \begin{array}{l} 1 : 2x^3 + e > 0 \\ 1 : \text{domain} \\ \text{Note: } 0/1 \text{ if } 0 \text{ is not in the domain} \\ 1 : \text{range} \end{array} \right.$$

Note: 0/3 if  $y$  is not a logarithmic function of  $x$