



**AP[®] Calculus AB
2002 Free-Response Questions
Form B**

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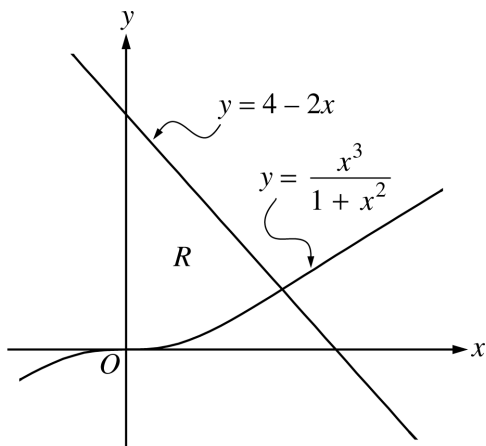
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CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

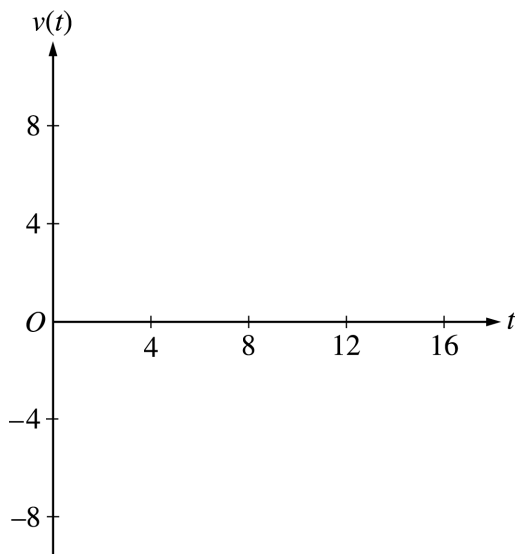


1. Let R be the region bounded by the y -axis and the graphs of $y = \frac{x^3}{1+x^2}$ and $y = 4 - 2x$, as shown in the figure above.
- Find the area of R .
 - Find the volume of the solid generated when R is revolved about the x -axis.
 - The region R is the base of a solid. For this solid, each cross section perpendicular to the x -axis is a square. Find the volume of this solid.
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2. The number of gallons, $P(t)$, of a pollutant in a lake changes at the rate $P'(t) = 1 - 3e^{-0.2\sqrt{t}}$ gallons per day, where t is measured in days. There are 50 gallons of the pollutant in the lake at time $t = 0$. The lake is considered to be safe when it contains 40 gallons or less of pollutant.
- Is the amount of pollutant increasing at time $t = 9$? Why or why not?
 - For what value of t will the number of gallons of pollutant be at its minimum? Justify your answer.
 - Is the lake safe when the number of gallons of pollutant is at its minimum? Justify your answer.
 - An investigator uses the tangent line approximation to $P(t)$ at $t = 0$ as a model for the amount of pollutant in the lake. At what time t does this model predict that the lake becomes safe?

3. A particle moves along the x -axis so that its velocity v at any time t , for $0 \leq t \leq 16$, is given by $v(t) = e^{2 \sin t} - 1$. At time $t = 0$, the particle is at the origin.

(a) On the axes provided, sketch the graph of $v(t)$ for $0 \leq t \leq 16$.

(Note: Use the axes provided in the test booklet.)



(b) During what intervals of time is the particle moving to the left? Give a reason for your answer.

(c) Find the total distance traveled by the particle from $t = 0$ to $t = 4$.

(d) Is there any time t , $0 < t \leq 16$, at which the particle returns to the origin? Justify your answer.

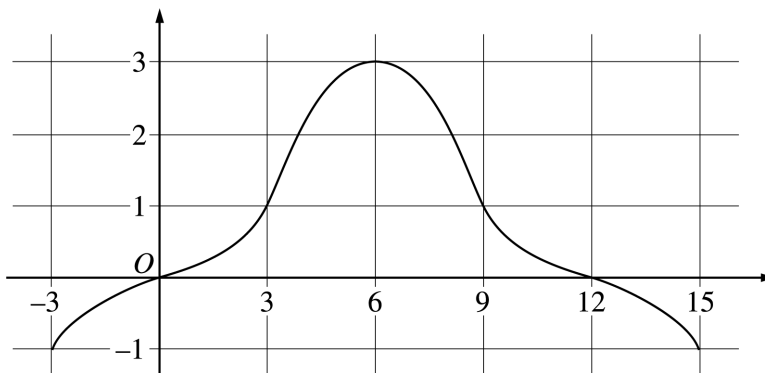
END OF PART A OF SECTION II

CALCULUS AB
SECTION II, Part B

Time—45 minutes

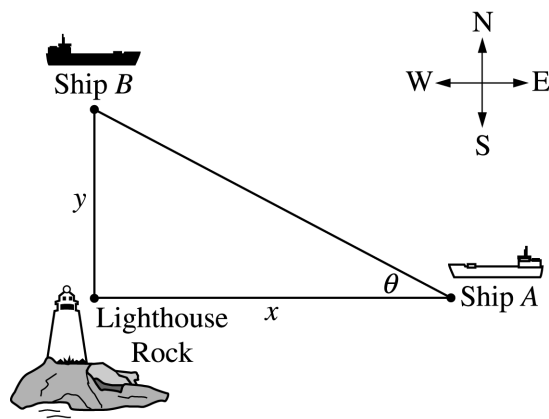
Number of problems—3

No calculator is allowed for these problems.



Graph of f

4. The graph of a differentiable function f on the closed interval $[-3, 15]$ is shown in the figure above. The graph of f has a horizontal tangent line at $x = 6$. Let $g(x) = 5 + \int_6^x f(t)dt$ for $-3 \leq x \leq 15$.
- Find $g(6)$, $g'(6)$, and $g''(6)$.
 - On what intervals is g decreasing? Justify your answer.
 - On what intervals is the graph of g concave down? Justify your answer.
 - Find a trapezoidal approximation of $\int_{-3}^{15} f(t)dt$ using six subintervals of length $\Delta t = 3$.
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5. Consider the differential equation $\frac{dy}{dx} = \frac{3-x}{y}$.
- Let $y = f(x)$ be the particular solution to the given differential equation for $1 < x < 5$ such that the line $y = -2$ is tangent to the graph of f . Find the x -coordinate of the point of tangency, and determine whether f has a local maximum, local minimum, or neither at this point. Justify your answer.
 - Let $y = g(x)$ be the particular solution to the given differential equation for $-2 < x < 8$, with the initial condition $g(6) = -4$. Find $y = g(x)$.



6. Ship A is traveling due west toward Lighthouse Rock at a speed of 15 kilometers per hour (km/hr). Ship B is traveling due north away from Lighthouse Rock at a speed of 10 km/hr. Let x be the distance between Ship A and Lighthouse Rock at time t , and let y be the distance between Ship B and Lighthouse Rock at time t , as shown in the figure above.
- Find the distance, in kilometers, between Ship A and Ship B when $x = 4$ km and $y = 3$ km.
 - Find the rate of change, in km/hr, of the distance between the two ships when $x = 4$ km and $y = 3$ km.
 - Let θ be the angle shown in the figure. Find the rate of change of θ , in radians per hour, when $x = 4$ km and $y = 3$ km.
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END OF EXAMINATION