



AP[®] Calculus AB 2003 Free-Response Questions Form B

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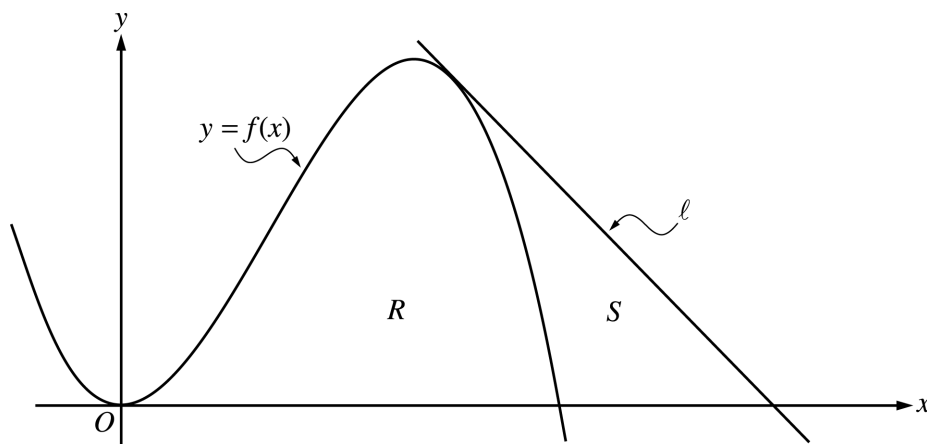
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CALCULUS AB
SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let f be the function given by $f(x) = 4x^2 - x^3$, and let ℓ be the line $y = 18 - 3x$, where ℓ is tangent to the graph of f . Let R be the region bounded by the graph of f and the x -axis, and let S be the region bounded by the graph of f , the line ℓ , and the x -axis, as shown above.
- (a) Show that ℓ is tangent to the graph of $y = f(x)$ at the point $x = 3$.
- (b) Find the area of S .
- (c) Find the volume of the solid generated when R is revolved about the x -axis.
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2. A tank contains 125 gallons of heating oil at time $t = 0$. During the time interval $0 \leq t \leq 12$ hours, heating oil is pumped into the tank at the rate

$$H(t) = 2 + \frac{10}{(1 + \ln(t + 1))} \text{ gallons per hour.}$$

During the same time interval, heating oil is removed from the tank at the rate

$$R(t) = 12 \sin\left(\frac{t^2}{47}\right) \text{ gallons per hour.}$$

- (a) How many gallons of heating oil are pumped into the tank during the time interval $0 \leq t \leq 12$ hours?
- (b) Is the level of heating oil in the tank rising or falling at time $t = 6$ hours? Give a reason for your answer.
- (c) How many gallons of heating oil are in the tank at time $t = 12$ hours?
- (d) At what time t , for $0 \leq t \leq 12$, is the volume of heating oil in the tank the least? Show the analysis that leads to your conclusion.
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Distance x (mm)	0	60	120	180	240	300	360
Diameter $B(x)$ (mm)	24	30	28	30	26	24	26

3. A blood vessel is 360 millimeters (mm) long with circular cross sections of varying diameter. The table above gives the measurements of the diameter of the blood vessel at selected points along the length of the blood vessel, where x represents the distance from one end of the blood vessel and $B(x)$ is a twice-differentiable function that represents the diameter at that point.
- (a) Write an integral expression in terms of $B(x)$ that represents the average radius, in mm, of the blood vessel between $x = 0$ and $x = 360$.
- (b) Approximate the value of your answer from part (a) using the data from the table and a midpoint Riemann sum with three subintervals of equal length. Show the computations that lead to your answer.
- (c) Using correct units, explain the meaning of $\pi \int_{125}^{275} \left(\frac{B(x)}{2}\right)^2 dx$ in terms of the blood vessel.
- (d) Explain why there must be at least one value x , for $0 < x < 360$, such that $B''(x) = 0$.
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END OF PART A OF SECTION II

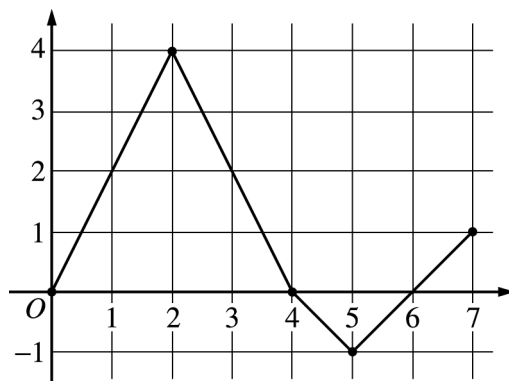
CALCULUS AB
SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

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4. A particle moves along the x -axis with velocity at time $t \geq 0$ given by $v(t) = -1 + e^{1-t}$.
- Find the acceleration of the particle at time $t = 3$.
 - Is the speed of the particle increasing at time $t = 3$? Give a reason for your answer.
 - Find all values of t at which the particle changes direction. Justify your answer.
 - Find the total distance traveled by the particle over the time interval $0 \leq t \leq 3$.
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Graph of f

5. Let f be a function defined on the closed interval $[0, 7]$. The graph of f , consisting of four line segments, is shown above. Let g be the function given by $g(x) = \int_2^x f(t) dt$.
- Find $g(3)$, $g'(3)$, and $g''(3)$.
 - Find the average rate of change of g on the interval $0 \leq x \leq 3$.
 - For how many values c , where $0 < c < 3$, is $g'(c)$ equal to the average rate found in part (b)? Explain your reasoning.
 - Find the x -coordinate of each point of inflection of the graph of g on the interval $0 < x < 7$. Justify your answer.
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6. Let f be the function satisfying $f'(x) = x\sqrt{f(x)}$ for all real numbers x , where $f(3) = 25$.
- (a) Find $f''(3)$.
- (b) Write an expression for $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = x\sqrt{y}$ with the initial condition $f(3) = 25$.
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END OF EXAMINATION