



AP[®] Calculus AB 2003 Scoring Guidelines

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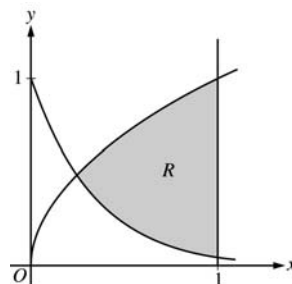
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Question 1

Let R be the shaded region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-3x}$ and the vertical line $x = 1$, as shown in the figure above.

- (a) Find the area of R .
- (b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 1$.
- (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x -axis is a rectangle whose height is 5 times the length of its base in region R . Find the volume of this solid.



Point of intersection

$$e^{-3x} = \sqrt{x} \text{ at } (T, S) = (0.238734, 0.488604)$$

$$\begin{aligned} \text{(a) Area} &= \int_T^1 (\sqrt{x} - e^{-3x}) dx \\ &= 0.442 \text{ or } 0.443 \end{aligned}$$

$$\begin{aligned} \text{(b) Volume} &= \pi \int_T^1 \left((1 - e^{-3x})^2 - (1 - \sqrt{x})^2 \right) dx \\ &= 0.453\pi \text{ or } 1.423 \text{ or } 1.424 \end{aligned}$$

$$\begin{aligned} \text{(c) Length} &= \sqrt{x} - e^{-3x} \\ \text{Height} &= 5(\sqrt{x} - e^{-3x}) \end{aligned}$$

$$\text{Volume} = \int_T^1 5(\sqrt{x} - e^{-3x})^2 dx = 1.554$$

1: Correct limits in an integral in
(a), (b), or (c)

2: $\left\{ \begin{array}{l} 1 : \text{integrand} \\ 1 : \text{answer} \end{array} \right.$

3: $\left\{ \begin{array}{l} 2 : \text{integrand} \\ \quad < -1 > \text{ reversal} \\ \quad < -1 > \text{ error with constant} \\ \quad < -1 > \text{ omits 1 in one radius} \\ \quad < -2 > \text{ other errors} \\ 1 : \text{answer} \end{array} \right.$

3: $\left\{ \begin{array}{l} 2 : \text{integrand} \\ \quad < -1 > \text{ incorrect but has} \\ \quad \quad \sqrt{x} - e^{-3x} \\ \quad \quad \text{as a factor} \\ 1 : \text{answer} \end{array} \right.$

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Question 2

A particle moves along the x -axis so that its velocity at time t is given by

$$v(t) = -(t + 1)\sin\left(\frac{t^2}{2}\right).$$

At time $t = 0$, the particle is at position $x = 1$.

- (a) Find the acceleration of the particle at time $t = 2$. Is the speed of the particle increasing at $t = 2$? Why or why not?
- (b) Find all times t in the open interval $0 < t < 3$ when the particle changes direction. Justify your answer.
- (c) Find the total distance traveled by the particle from time $t = 0$ until time $t = 3$.
- (d) During the time interval $0 \leq t \leq 3$, what is the greatest distance between the particle and the origin? Show the work that leads to your answer.

- (a) $a(2) = v'(2) = 1.587$ or 1.588
 $v(2) = -3\sin(2) < 0$
 Speed is decreasing since $a(2) > 0$ and $v(2) < 0$.

- 2 : $\left\{ \begin{array}{l} 1 : a(2) \\ 1 : \text{speed decreasing} \\ \text{with reason} \end{array} \right.$

- (b) $v(t) = 0$ when $\frac{t^2}{2} = \pi$
 $t = \sqrt{2\pi}$ or 2.506 or 2.507
 Since $v(t) < 0$ for $0 < t < \sqrt{2\pi}$ and $v(t) > 0$ for $\sqrt{2\pi} < t < 3$, the particle changes directions at $t = \sqrt{2\pi}$.

- 2 : $\left\{ \begin{array}{l} 1 : t = \sqrt{2\pi} \text{ only} \\ 1 : \text{justification} \end{array} \right.$

- (c) Distance = $\int_0^3 |v(t)| dt = 4.333$ or 4.334

- 3 : $\left\{ \begin{array}{l} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{array} \right.$

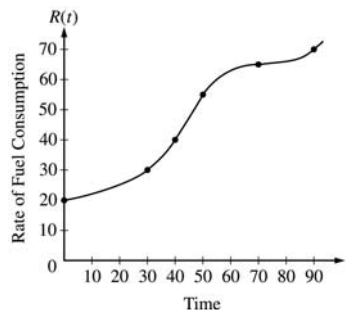
- (d) $\int_0^{\sqrt{2\pi}} v(t) dt = -3.265$
 $x(\sqrt{2\pi}) = x(0) + \int_0^{\sqrt{2\pi}} v(t) dt = -2.265$
 Since the total distance from $t = 0$ to $t = 3$ is 4.334, the particle is still to the left of the origin at $t = 3$. Hence the greatest distance from the origin is 2.265.

- 2 : $\left\{ \begin{array}{l} 1 : \pm \text{ (distance particle travels} \\ \text{while velocity is negative)} \\ 1 : \text{answer} \end{array} \right.$

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Question 3

The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function R of time t . The graph of R and a table of selected values of $R(t)$, for the time interval $0 \leq t \leq 90$ minutes, are shown above.



t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

- (a) Use data from the table to find an approximation for $R'(45)$. Show the computations that lead to your answer. Indicate units of measure.
- (b) The rate of fuel consumption is increasing fastest at time $t = 45$ minutes. What is the value of $R''(45)$? Explain your reasoning.
- (c) Approximate the value of $\int_0^{90} R(t) dt$ using a left Riemann sum with the five subintervals indicated by the data in the table. Is this numerical approximation less than the value of $\int_0^{90} R(t) dt$? Explain your reasoning.
- (d) For $0 < b \leq 90$ minutes, explain the meaning of $\int_0^b R(t) dt$ in terms of fuel consumption for the plane. Explain the meaning of $\frac{1}{b} \int_0^b R(t) dt$ in terms of fuel consumption for the plane. Indicate units of measure in both answers.

(a)
$$R'(45) \approx \frac{R(50) - R(40)}{50 - 40} = \frac{55 - 40}{10}$$

$$= 1.5 \text{ gal/min}^2$$

2 : { 1 : a difference quotient using numbers from table and interval that contains 45
1 : 1.5 gal/min²

(b) $R''(45) = 0$ since $R'(t)$ has a maximum at $t = 45$.

2 : { 1 : $R''(45) = 0$
1 : reason

(c)
$$\int_0^{90} R(t) dt \approx (30)(20) + (10)(30) + (10)(40)$$

$$+ (20)(55) + (20)(65) = 3700$$

2 : { 1 : value of left Riemann sum
1 : "less" with reason

Yes, this approximation is less because the graph of R is increasing on the interval.

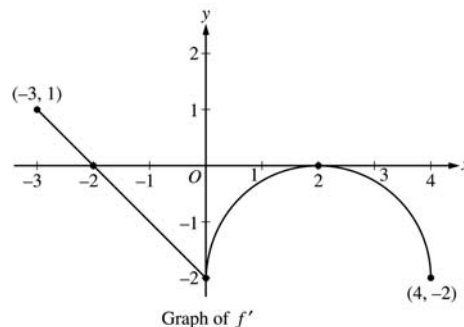
- (d) $\int_0^b R(t) dt$ is the total amount of fuel in gallons consumed for the first b minutes.
 $\frac{1}{b} \int_0^b R(t) dt$ is the average value of the rate of fuel consumption in gallons/min during the first b minutes.

3 : { 2 : meanings
1 : meaning of $\int_0^b R(t) dt$
1 : meaning of $\frac{1}{b} \int_0^b R(t) dt$
< - 1 > if no reference to time b
1 : units in both answers

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Question 4

Let f be a function defined on the closed interval $-3 \leq x \leq 4$ with $f(0) = 3$. The graph of f' , the derivative of f , consists of one line segment and a semicircle, as shown above.



- On what intervals, if any, is f increasing? Justify your answer.
- Find the x -coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$. Justify your answer.
- Find an equation for the line tangent to the graph of f at the point $(0, 3)$.
- Find $f(-3)$ and $f(4)$. Show the work that leads to your answers.

(a) The function f is increasing on $[-3, -2]$ since $f' > 0$ for $-3 \leq x < -2$.

2 : $\left\{ \begin{array}{l} 1 : \text{interval} \\ 1 : \text{reason} \end{array} \right.$

(b) $x = 0$ and $x = 2$
 f' changes from decreasing to increasing at $x = 0$ and from increasing to decreasing at $x = 2$

2 : $\left\{ \begin{array}{l} 1 : x = 0 \text{ and } x = 2 \text{ only} \\ 1 : \text{justification} \end{array} \right.$

(c) $f'(0) = -2$
 Tangent line is $y = -2x + 3$.

1 : equation

$$\begin{aligned} \text{(d)} \quad f(0) - f(-3) &= \int_{-3}^0 f'(t) dt \\ &= \frac{1}{2}(1)(1) - \frac{1}{2}(2)(2) = -\frac{3}{2} \end{aligned}$$

$$f(-3) = f(0) + \frac{3}{2} = \frac{9}{2}$$

$$\begin{aligned} f(4) - f(0) &= \int_0^4 f'(t) dt \\ &= -\left(8 - \frac{1}{2}(2)^2\pi\right) = -8 + 2\pi \end{aligned}$$

$$f(4) = f(0) - 8 + 2\pi = -5 + 2\pi$$

1 : $\pm \left(\frac{1}{2} - 2\right)$
 (difference of areas of triangles)

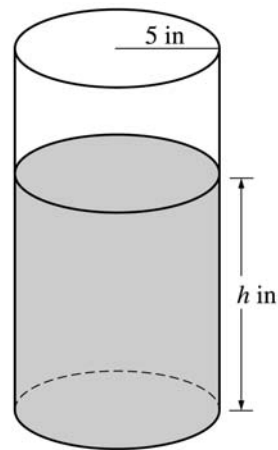
1 : answer for $f(-3)$ using FTC

4 : $\left\{ \begin{array}{l} 1 : \pm \left(8 - \frac{1}{2}(2)^2\pi\right)$
 (area of rectangle - area of semicircle) \\ 1 : answer for $f(4)$ using FTC \end{array} \right.

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Question 5

A coffeepot has the shape of a cylinder with radius 5 inches, as shown in the figure above. Let h be the depth of the coffee in the pot, measured in inches, where h is a function of time t , measured in seconds. The volume V of coffee in the pot is changing at the rate of $-5\pi\sqrt{h}$ cubic inches per second. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)



- (a) Show that $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$.
- (b) Given that $h = 17$ at time $t = 0$, solve the differential equation $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$ for h as a function of t .
- (c) At what time t is the coffeepot empty?

(a) $V = 25\pi h$

$$\frac{dV}{dt} = 25\pi \frac{dh}{dt} = -5\pi\sqrt{h}$$

$$\frac{dh}{dt} = \frac{-5\pi\sqrt{h}}{25\pi} = -\frac{\sqrt{h}}{5}$$

(b) $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$

$$\frac{1}{\sqrt{h}} dh = -\frac{1}{5} dt$$

$$2\sqrt{h} = -\frac{1}{5}t + C$$

$$2\sqrt{17} = 0 + C$$

$$h = \left(-\frac{1}{10}t + \sqrt{17}\right)^2$$

(c) $\left(-\frac{1}{10}t + \sqrt{17}\right)^2 = 0$

$$t = 10\sqrt{17}$$

$$3 : \left\{ \begin{array}{l} 1 : \frac{dV}{dt} = -5\pi\sqrt{h} \\ 1 : \text{computes } \frac{dV}{dt} \\ 1 : \text{shows result} \end{array} \right.$$

$$5 : \left\{ \begin{array}{l} 1 : \text{separates variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition } h = 17 \\ \quad \text{when } t = 0 \\ 1 : \text{solves for } h \end{array} \right.$$

Note: max 2/5 [1-1-0-0-0] if no constant of integration

Note: 0/5 if no separation of variables

1 : answer

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Question 6

Let f be the function defined by

$$f(x) = \begin{cases} \sqrt{x+1} & \text{for } 0 \leq x \leq 3 \\ 5-x & \text{for } 3 < x \leq 5. \end{cases}$$

- (a) Is f continuous at $x = 3$? Explain why or why not.
 (b) Find the average value of $f(x)$ on the closed interval $0 \leq x \leq 5$.
 (c) Suppose the function g is defined by

$$g(x) = \begin{cases} k\sqrt{x+1} & \text{for } 0 \leq x \leq 3 \\ mx+2 & \text{for } 3 < x \leq 5, \end{cases}$$

where k and m are constants. If g is differentiable at $x = 3$, what are the values of k and m ?

- (a) f is continuous at $x = 3$ because

$$\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^+} f(x) = 2.$$

Therefore, $\lim_{x \rightarrow 3} f(x) = 2 = f(3)$.

2 : $\left\{ \begin{array}{l} 1 : \text{answers "yes" and equates the} \\ \text{values of the left- and right-hand} \\ \text{limits} \\ 1 : \text{explanation involving limits} \end{array} \right.$

(b)
$$\int_0^5 f(x) dx = \int_0^3 f(x) dx + \int_3^5 f(x) dx$$

$$= \frac{2}{3}(x+1)^{3/2} \Big|_0^3 + \left(5x - \frac{1}{2}x^2\right) \Big|_3^5$$

$$= \left(\frac{16}{3} - \frac{2}{3}\right) + \left(\frac{25}{2} - \frac{21}{2}\right) = \frac{20}{3}$$

4 : $\left\{ \begin{array}{l} 1 : k \int_0^3 f(x) dx + k \int_3^5 f(x) dx \\ \text{(where } k \neq 0) \\ 1 : \text{antiderivative of } \sqrt{x+1} \\ 1 : \text{antiderivative of } 5-x \\ 1 : \text{evaluation and answer} \end{array} \right.$

Average value: $\frac{1}{5} \int_0^5 f(x) dx = \frac{4}{3}$

- (c) Since g is continuous at $x = 3$, $2k = 3m + 2$.

$$g'(x) = \begin{cases} \frac{k}{2\sqrt{x+1}} & \text{for } 0 < x < 3 \\ m & \text{for } 3 < x < 5 \end{cases}$$

$$\lim_{x \rightarrow 3^-} g'(x) = \frac{k}{4} \text{ and } \lim_{x \rightarrow 3^+} g'(x) = m$$

Since these two limits exist and g is differentiable at $x = 3$, the two limits are equal. Thus $\frac{k}{4} = m$.

3 : $\left\{ \begin{array}{l} 1 : 2k = 3m + 2 \\ 1 : \frac{k}{4} = m \\ 1 : \text{values for } k \text{ and } m \end{array} \right.$

$$8m = 3m + 2; m = \frac{2}{5} \text{ and } k = \frac{8}{5}$$