



## AP<sup>®</sup> Calculus AB 2003 Free-Response Questions

**The materials included in these files are intended for use by AP teachers for course and exam preparation; permission for any other use must be sought from the Advanced Placement Program<sup>®</sup>. Teachers may reproduce them, in whole or in part, in limited quantities for noncommercial, face-to-face teaching purposes. This permission does not apply to any third-party copyrights contained herein. This material may not be mass distributed, electronically or otherwise. These materials and any copies made of them may not be resold, and the copyright notices must be retained as they appear here.**

These materials were produced by Educational Testing Service<sup>®</sup> (ETS<sup>®</sup>), which develops and administers the examinations of the Advanced Placement Program for the College Board. The College Board and Educational Testing Service (ETS) are dedicated to the principle of equal opportunity, and their programs, services, and employment policies are guided by that principle.

The College Board is a national nonprofit membership association whose mission is to prepare, inspire, and connect students to college and opportunity. Founded in 1900, the association is composed of more than 4,300 schools, colleges, universities, and other educational organizations. Each year, the College Board serves over three million students and their parents, 22,000 high schools, and 3,500 colleges through major programs and services in college admissions, guidance, assessment, financial aid, enrollment, and teaching and learning. Among its best-known programs are the SAT<sup>®</sup>, the PSAT/NMSQT<sup>®</sup>, and the Advanced Placement Program<sup>®</sup> (AP<sup>®</sup>). The College Board is committed to the principles of equity and excellence, and that commitment is embodied in all of its programs, services, activities, and concerns.

For further information, visit [www.collegeboard.com](http://www.collegeboard.com)

Copyright © 2003 College Entrance Examination Board. All rights reserved. College Board, Advanced Placement Program, AP, AP Vertical Teams, APCD, Pacesetter, Pre-AP, SAT, Student Search Service, and the acorn logo are registered trademarks of the College Entrance Examination Board. AP Central is a trademark owned by the College Entrance Examination Board. PSAT/NMSQT is a registered trademark jointly owned by the College Entrance Examination Board and the National Merit Scholarship Corporation. Educational Testing Service and ETS are registered trademarks of Educational Testing Service. Other products and services may be trademarks of their respective owners.

For the College Board's online home for AP professionals, visit AP Central at [apcentral.collegeboard.com](http://apcentral.collegeboard.com).

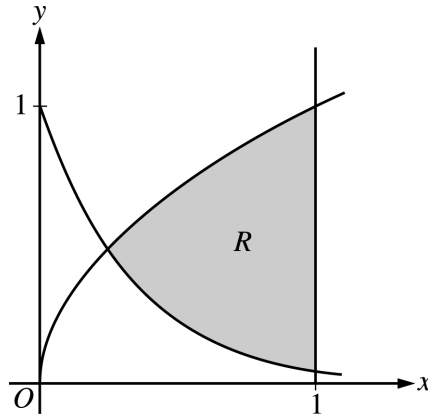
CALCULUS AB  
SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.

---



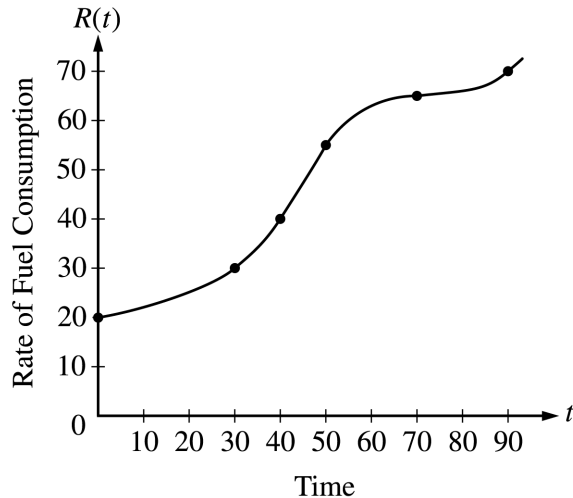
1. Let  $R$  be the shaded region bounded by the graphs of  $y = \sqrt{x}$  and  $y = e^{-3x}$  and the vertical line  $x = 1$ , as shown in the figure above.
- Find the area of  $R$ .
  - Find the volume of the solid generated when  $R$  is revolved about the horizontal line  $y = 1$ .
  - The region  $R$  is the base of a solid. For this solid, each cross section perpendicular to the  $x$ -axis is a rectangle whose height is 5 times the length of its base in region  $R$ . Find the volume of this solid.
-

2. A particle moves along the  $x$ -axis so that its velocity at time  $t$  is given by

$$v(t) = -(t + 1) \sin\left(\frac{t^2}{2}\right).$$

At time  $t = 0$ , the particle is at position  $x = 1$ .

- Find the acceleration of the particle at time  $t = 2$ . Is the speed of the particle increasing at  $t = 2$ ? Why or why not?
  - Find all times  $t$  in the open interval  $0 < t < 3$  when the particle changes direction. Justify your answer.
  - Find the total distance traveled by the particle from time  $t = 0$  until time  $t = 3$ .
  - During the time interval  $0 \leq t \leq 3$ , what is the greatest distance between the particle and the origin? Show the work that leads to your answer.
-



$t$ (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

3. The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function  $R$  of time  $t$ . The graph of  $R$  and a table of selected values of  $R(t)$ , for the time interval  $0 \leq t \leq 90$  minutes, are shown above.
- (a) Use data from the table to find an approximation for  $R'(45)$ . Show the computations that lead to your answer. Indicate units of measure.
- (b) The rate of fuel consumption is increasing fastest at time  $t = 45$  minutes. What is the value of  $R''(45)$ ? Explain your reasoning.
- (c) Approximate the value of  $\int_0^{90} R(t) dt$  using a left Riemann sum with the five subintervals indicated by the data in the table. Is this numerical approximation less than the value of  $\int_0^{90} R(t) dt$ ? Explain your reasoning.
- (d) For  $0 < b \leq 90$  minutes, explain the meaning of  $\int_0^b R(t) dt$  in terms of fuel consumption for the plane. Explain the meaning of  $\frac{1}{b} \int_0^b R(t) dt$  in terms of fuel consumption for the plane. Indicate units of measure in both answers.

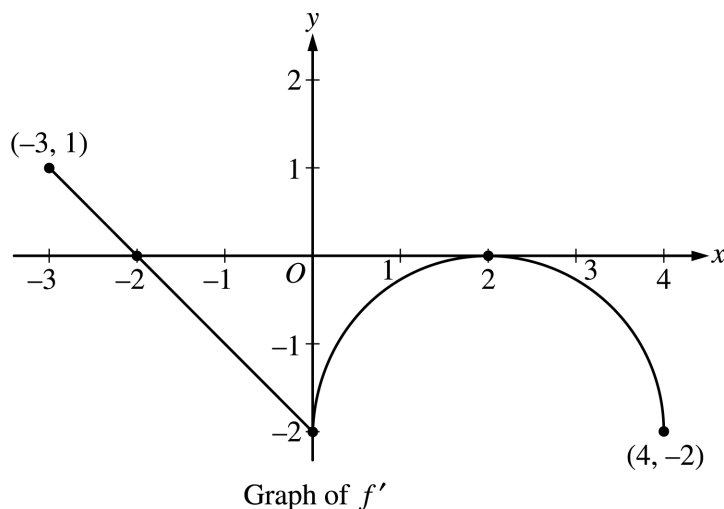
**END OF PART A OF SECTION II**

**CALCULUS AB**  
**SECTION II, Part B**

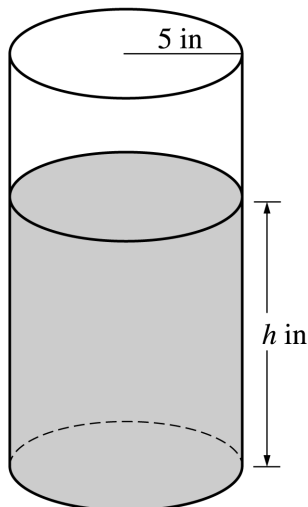
**Time—45 minutes**

**Number of problems—3**

**No calculator is allowed for these problems.**



4. Let  $f$  be a function defined on the closed interval  $-3 \leq x \leq 4$  with  $f(0) = 3$ . The graph of  $f'$ , the derivative of  $f$ , consists of one line segment and a semicircle, as shown above.
- On what intervals, if any, is  $f$  increasing? Justify your answer.
  - Find the  $x$ -coordinate of each point of inflection of the graph of  $f$  on the open interval  $-3 < x < 4$ . Justify your answer.
  - Find an equation for the line tangent to the graph of  $f$  at the point  $(0, 3)$ .
  - Find  $f(-3)$  and  $f(4)$ . Show the work that leads to your answers.



5. A coffeepot has the shape of a cylinder with radius 5 inches, as shown in the figure above. Let  $h$  be the depth of the coffee in the pot, measured in inches, where  $h$  is a function of time  $t$ , measured in seconds. The volume  $V$  of coffee in the pot is changing at the rate of  $-5\pi\sqrt{h}$  cubic inches per second. (The volume  $V$  of a cylinder with radius  $r$  and height  $h$  is  $V = \pi r^2 h$ .)
- (a) Show that  $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$ .
- (b) Given that  $h = 17$  at time  $t = 0$ , solve the differential equation  $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$  for  $h$  as a function of  $t$ .
- (c) At what time  $t$  is the coffeepot empty?
- 

6. Let  $f$  be the function defined by

$$f(x) = \begin{cases} \sqrt{x+1} & \text{for } 0 \leq x \leq 3 \\ 5-x & \text{for } 3 < x \leq 5. \end{cases}$$

- (a) Is  $f$  continuous at  $x = 3$ ? Explain why or why not.
- (b) Find the average value of  $f(x)$  on the closed interval  $0 \leq x \leq 5$ .
- (c) Suppose the function  $g$  is defined by

$$g(x) = \begin{cases} k\sqrt{x+1} & \text{for } 0 \leq x \leq 3 \\ mx+2 & \text{for } 3 < x \leq 5, \end{cases}$$

where  $k$  and  $m$  are constants. If  $g$  is differentiable at  $x = 3$ , what are the values of  $k$  and  $m$ ?

---

### END OF EXAMINATION