



## **AP<sup>®</sup> Calculus AB**

### **2004 Scoring Guidelines**

### **Form B**

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**AP<sup>®</sup> CALCULUS AB**  
**2004 SCORING GUIDELINES (Form B)**

**Question 1**

Let  $R$  be the region enclosed by the graph of  $y = \sqrt{x-1}$ , the vertical line  $x = 10$ , and the  $x$ -axis.

- (a) Find the area of  $R$ .  
 (b) Find the volume of the solid generated when  $R$  is revolved about the horizontal line  $y = 3$ .  
 (c) Find the volume of the solid generated when  $R$  is revolved about the vertical line  $x = 10$ .

(a) Area =  $\int_1^{10} \sqrt{x-1} \, dx = 18$

3 :  $\left\{ \begin{array}{l} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{array} \right.$

(b) Volume =  $\pi \int_1^{10} (9 - (3 - \sqrt{x-1})^2) \, dx$   
 = 212.057 or 212.058

3 :  $\left\{ \begin{array}{l} 1 : \text{limits and constant} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{array} \right.$

(c) Volume =  $\pi \int_0^3 (10 - (y^2 + 1))^2 \, dy$   
 = 407.150

3 :  $\left\{ \begin{array}{l} 1 : \text{limits and constant} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{array} \right.$

**AP<sup>®</sup> CALCULUS AB**  
**2004 SCORING GUIDELINES (Form B)**

**Question 2**

For  $0 \leq t \leq 31$ , the rate of change of the number of mosquitoes on Tropical Island at time  $t$  days is modeled by  $R(t) = 5\sqrt{t} \cos\left(\frac{t}{5}\right)$  mosquitoes per day. There are 1000 mosquitoes on Tropical Island at time  $t = 0$ .

- Show that the number of mosquitoes is increasing at time  $t = 6$ .
- At time  $t = 6$ , is the number of mosquitoes increasing at an increasing rate, or is the number of mosquitoes increasing at a decreasing rate? Give a reason for your answer.
- According to the model, how many mosquitoes will be on the island at time  $t = 31$ ? Round your answer to the nearest whole number.
- To the nearest whole number, what is the maximum number of mosquitoes for  $0 \leq t \leq 31$ ? Show the analysis that leads to your conclusion.

(a) Since  $R(6) = 4.438 > 0$ , the number of mosquitoes is increasing at  $t = 6$ .

1 : shows that  $R(6) > 0$

(b)  $R'(6) = -1.913$   
Since  $R'(6) < 0$ , the number of mosquitoes is increasing at a decreasing rate at  $t = 6$ .

2 :  $\left\{ \begin{array}{l} 1 : \text{considers } R'(6) \\ 1 : \text{answer with reason} \end{array} \right.$

(c)  $1000 + \int_0^{31} R(t) dt = 964.335$   
To the nearest whole number, there are 964 mosquitoes.

2 :  $\left\{ \begin{array}{l} 1 : \text{integral} \\ 1 : \text{answer} \end{array} \right.$

(d)  $R(t) = 0$  when  $t = 0$ ,  $t = 2.5\pi$ , or  $t = 7.5\pi$   
 $R(t) > 0$  on  $0 < t < 2.5\pi$   
 $R(t) < 0$  on  $2.5\pi < t < 7.5\pi$   
 $R(t) > 0$  on  $7.5\pi < t < 31$   
The absolute maximum number of mosquitoes occurs at  $t = 2.5\pi$  or at  $t = 31$ .

4 :  $\left\{ \begin{array}{l} 2 : \text{absolute maximum value} \\ 1 : \text{integral} \\ 1 : \text{answer} \\ 2 : \text{analysis} \\ 1 : \text{computes interior critical points} \\ 1 : \text{completes analysis} \end{array} \right.$

$$1000 + \int_0^{2.5\pi} R(t) dt = 1039.357,$$

There are 964 mosquitoes at  $t = 31$ , so the maximum number of mosquitoes is 1039, to the nearest whole number.

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**2004 SCORING GUIDELINES (Form B)**

**Question 3**

A test plane flies in a straight line with positive velocity  $v(t)$ , in miles per minute at time  $t$  minutes, where  $v$  is a differentiable function of  $t$ . Selected values of  $v(t)$  for  $0 \leq t \leq 40$  are shown in the table above.

|              |     |     |     |     |     |     |     |     |     |
|--------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $t$ (min)    | 0   | 5   | 10  | 15  | 20  | 25  | 30  | 35  | 40  |
| $v(t)$ (mpm) | 7.0 | 9.2 | 9.5 | 7.0 | 4.5 | 2.4 | 2.4 | 4.3 | 7.3 |

- (a) Use a midpoint Riemann sum with four subintervals of equal length and values from the table to approximate  $\int_0^{40} v(t) dt$ . Show the computations that lead to your answer. Using correct units, explain the meaning of  $\int_0^{40} v(t) dt$  in terms of the plane's flight.
- (b) Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval  $0 < t < 40$ ? Justify your answer.
- (c) The function  $f$ , defined by  $f(t) = 6 + \cos\left(\frac{t}{10}\right) + 3\sin\left(\frac{7t}{40}\right)$ , is used to model the velocity of the plane, in miles per minute, for  $0 \leq t \leq 40$ . According to this model, what is the acceleration of the plane at  $t = 23$ ? Indicate units of measure.
- (d) According to the model  $f$ , given in part (c), what is the average velocity of the plane, in miles per minute, over the time interval  $0 \leq t \leq 40$ ?

(a) Midpoint Riemann sum is  
 $10 \cdot [v(5) + v(15) + v(25) + v(35)]$   
 $= 10 \cdot [9.2 + 7.0 + 2.4 + 4.3] = 229$   
 The integral gives the total distance in miles that the plane flies during the 40 minutes.

3 :  $\begin{cases} 1 : v(5) + v(15) + v(25) + v(35) \\ 1 : \text{answer} \\ 1 : \text{meaning with units} \end{cases}$

(b) By the Mean Value Theorem,  $v'(t) = 0$  somewhere in the interval  $(0, 15)$  and somewhere in the interval  $(25, 30)$ . Therefore the acceleration will equal 0 for at least two values of  $t$ .

2 :  $\begin{cases} 1 : \text{two instances} \\ 1 : \text{justification} \end{cases}$

(c)  $f'(23) = -0.407$  or  $-0.408$  miles per minute<sup>2</sup>

1 : answer with units

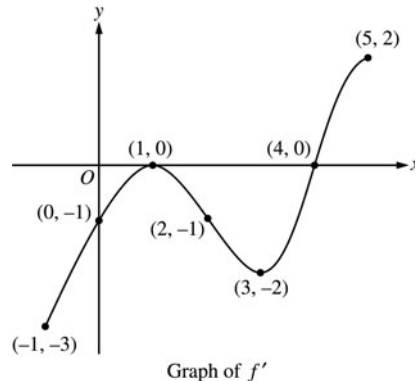
(d) Average velocity  $= \frac{1}{40} \int_0^{40} f(t) dt$   
 $= 5.916$  miles per minute

3 :  $\begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

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**2004 SCORING GUIDELINES (Form B)**

**Question 4**

The figure above shows the graph of  $f'$ , the derivative of the function  $f$ , on the closed interval  $-1 \leq x \leq 5$ . The graph of  $f'$  has horizontal tangent lines at  $x = 1$  and  $x = 3$ . The function  $f$  is twice differentiable with  $f(2) = 6$ .



- (a) Find the  $x$ -coordinate of each of the points of inflection of the graph of  $f$ . Give a reason for your answer.
- (b) At what value of  $x$  does  $f$  attain its absolute minimum value on the closed interval  $-1 \leq x \leq 5$ ? At what value of  $x$  does  $f$  attain its absolute maximum value on the closed interval  $-1 \leq x \leq 5$ ? Show the analysis that leads to your answers.
- (c) Let  $g$  be the function defined by  $g(x) = xf(x)$ . Find an equation for the line tangent to the graph of  $g$  at  $x = 2$ .

- (a)  $x = 1$  and  $x = 3$  because the graph of  $f'$  changes from increasing to decreasing at  $x = 1$ , and changes from decreasing to increasing at  $x = 3$ .

$$2 : \begin{cases} 1 : x = 1, x = 3 \\ 1 : \text{reason} \end{cases}$$

- (b) The function  $f$  decreases from  $x = -1$  to  $x = 4$ , then increases from  $x = 4$  to  $x = 5$ . Therefore, the absolute minimum value for  $f$  is at  $x = 4$ . The absolute maximum value must occur at  $x = -1$  or at  $x = 5$ .

$$4 : \begin{cases} 1 : \text{indicates } f \text{ decreases then increases} \\ 1 : \text{eliminates } x = 5 \text{ for maximum} \\ 1 : \text{absolute minimum at } x = 4 \\ 1 : \text{absolute maximum at } x = -1 \end{cases}$$

$$f(5) - f(-1) = \int_{-1}^5 f'(t) dt < 0$$

Since  $f(5) < f(-1)$ , the absolute maximum value occurs at  $x = -1$ .

- (c)  $g'(x) = f(x) + xf'(x)$   
 $g'(2) = f(2) + 2f'(2) = 6 + 2(-1) = 4$   
 $g(2) = 2f(2) = 12$

$$3 : \begin{cases} 2 : g'(x) \\ 1 : \text{tangent line} \end{cases}$$

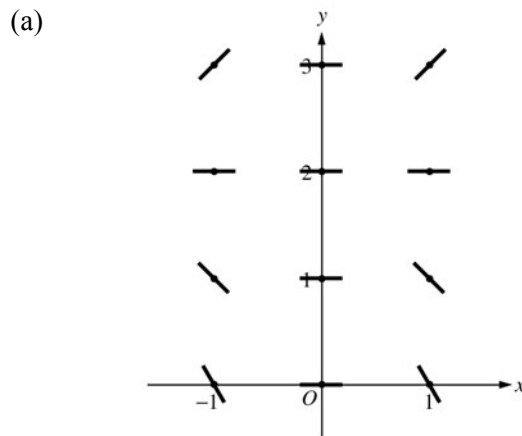
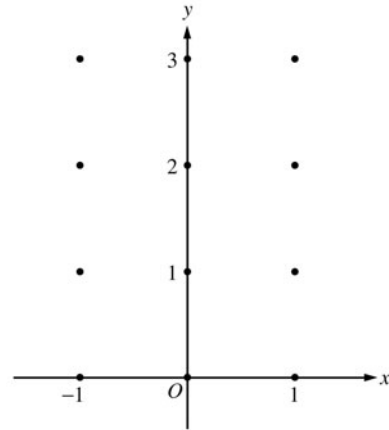
Tangent line is  $y = 4(x - 2) + 12$

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**2004 SCORING GUIDELINES (Form B)**

**Question 5**

Consider the differential equation  $\frac{dy}{dx} = x^4(y - 2)$ .

- (a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.  
**(Note: Use the axes provided in the test booklet.)**
- (b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the  $xy$ -plane. Describe all points in the  $xy$ -plane for which the slopes are negative.
- (c) Find the particular solution  $y = f(x)$  to the given differential equation with the initial condition  $f(0) = 0$ .



- (b) Slopes are negative at points  $(x, y)$  where  $x \neq 0$  and  $y < 2$ .

(c)

$$\frac{1}{y-2} dy = x^4 dx$$

$$\ln|y-2| = \frac{1}{5}x^5 + C$$

$$|y-2| = e^C e^{\frac{1}{5}x^5}$$

$$y-2 = Ke^{\frac{1}{5}x^5}, \quad K = \pm e^C$$

$$-2 = Ke^0 = K$$

$$y = 2 - 2e^{\frac{1}{5}x^5}$$

- 1 : zero slope at each point  $(x, y)$  where  $x = 0$  or  $y = 2$
- 2 : { positive slope at each point  $(x, y)$  where  $x \neq 0$  and  $y > 2$
- 1 : { negative slope at each point  $(x, y)$  where  $x \neq 0$  and  $y < 2$

1 : description

- 6 : { 1 : separates variables  
2 : antiderivatives  
1 : constant of integration  
1 : uses initial condition  
1 : solves for  $y$   
0/1 if  $y$  is not exponential

Note: max 3/6 [1-2-0-0-0] if no constant of integration

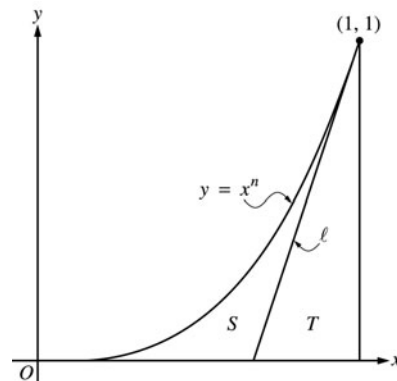
Note: 0/6 if no separation of variables

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**2004 SCORING GUIDELINES (Form B)**

**Question 6**

Let  $\ell$  be the line tangent to the graph of  $y = x^n$  at the point  $(1, 1)$ , where  $n > 1$ , as shown above.

- (a) Find  $\int_0^1 x^n dx$  in terms of  $n$ .
- (b) Let  $T$  be the triangular region bounded by  $\ell$ , the  $x$ -axis, and the line  $x = 1$ . Show that the area of  $T$  is  $\frac{1}{2n}$ .
- (c) Let  $S$  be the region bounded by the graph of  $y = x^n$ , the line  $\ell$ , and the  $x$ -axis. Express the area of  $S$  in terms of  $n$  and determine the value of  $n$  that maximizes the area of  $S$ .



(a)  $\int_0^1 x^n dx = \frac{x^{n+1}}{n+1} \Big|_0^1 = \frac{1}{n+1}$

2 :  $\left\{ \begin{array}{l} 1 : \text{antiderivative of } x^n \\ 1 : \text{answer} \end{array} \right.$

(b) Let  $b$  be the length of the base of triangle  $T$ .

$\frac{1}{b}$  is the slope of line  $\ell$ , which is  $n$

3 :  $\left\{ \begin{array}{l} 1 : \text{slope of line } \ell \text{ is } n \\ 1 : \text{base of } T \text{ is } \frac{1}{n} \\ 1 : \text{shows area is } \frac{1}{2n} \end{array} \right.$

$$\text{Area}(T) = \frac{1}{2}b(1) = \frac{1}{2n}$$

(c)  $\text{Area}(S) = \int_0^1 x^n dx - \text{Area}(T)$   
 $= \frac{1}{n+1} - \frac{1}{2n}$

4 :  $\left\{ \begin{array}{l} 1 : \text{area of } S \text{ in terms of } n \\ 1 : \text{derivative} \\ 1 : \text{sets derivative equal to } 0 \\ 1 : \text{solves for } n \end{array} \right.$

$$\frac{d}{dn} \text{Area}(S) = -\frac{1}{(n+1)^2} + \frac{1}{2n^2} = 0$$

$$2n^2 = (n+1)^2$$

$$\sqrt{2}n = (n+1)$$

$$n = \frac{1}{\sqrt{2}-1} = 1 + \sqrt{2}$$