



AP[®] Calculus AB
2004 Free-Response Questions
Form B

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CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

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1. Let R be the region enclosed by the graph of $y = \sqrt{x-1}$, the vertical line $x = 10$, and the x -axis.
- (a) Find the area of R .
 - (b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 3$.
 - (c) Find the volume of the solid generated when R is revolved about the vertical line $x = 10$.
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2. For $0 \leq t \leq 31$, the rate of change of the number of mosquitoes on Tropical Island at time t days is modeled by $R(t) = 5\sqrt{t} \cos\left(\frac{t}{5}\right)$ mosquitoes per day. There are 1000 mosquitoes on Tropical Island at time $t = 0$.
- (a) Show that the number of mosquitoes is increasing at time $t = 6$.
 - (b) At time $t = 6$, is the number of mosquitoes increasing at an increasing rate, or is the number of mosquitoes increasing at a decreasing rate? Give a reason for your answer.
 - (c) According to the model, how many mosquitoes will be on the island at time $t = 31$? Round your answer to the nearest whole number.
 - (d) To the nearest whole number, what is the maximum number of mosquitoes for $0 \leq t \leq 31$? Show the analysis that leads to your conclusion.
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t (minutes)	0	5	10	15	20	25	30	35	40
$v(t)$ (miles per minute)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

3. A test plane flies in a straight line with positive velocity $v(t)$, in miles per minute at time t minutes, where v is a differentiable function of t . Selected values of $v(t)$ for $0 \leq t \leq 40$ are shown in the table above.
- (a) Use a midpoint Riemann sum with four subintervals of equal length and values from the table to approximate $\int_0^{40} v(t) dt$. Show the computations that lead to your answer. Using correct units, explain the meaning of $\int_0^{40} v(t) dt$ in terms of the plane's flight.
- (b) Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval $0 < t < 40$? Justify your answer.
- (c) The function f , defined by $f(t) = 6 + \cos\left(\frac{t}{10}\right) + 3 \sin\left(\frac{7t}{40}\right)$, is used to model the velocity of the plane, in miles per minute, for $0 \leq t \leq 40$. According to this model, what is the acceleration of the plane at $t = 23$? Indicate units of measure.
- (d) According to the model f , given in part (c), what is the average velocity of the plane, in miles per minute, over the time interval $0 \leq t \leq 40$?

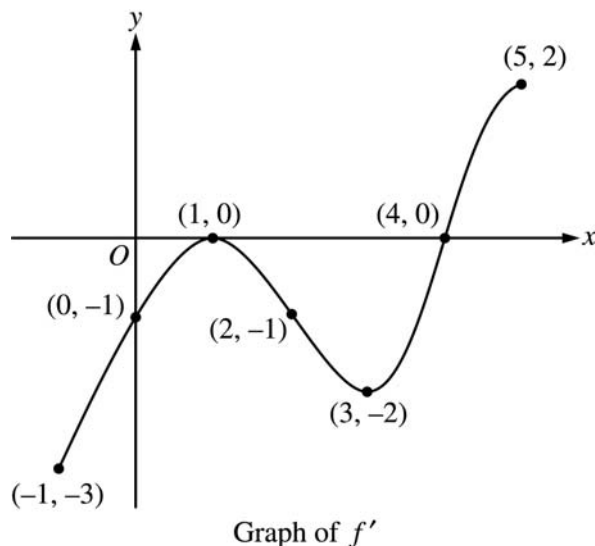
END OF PART A OF SECTION II

CALCULUS AB
SECTION II, Part B

Time—45 minutes

Number of problems—3

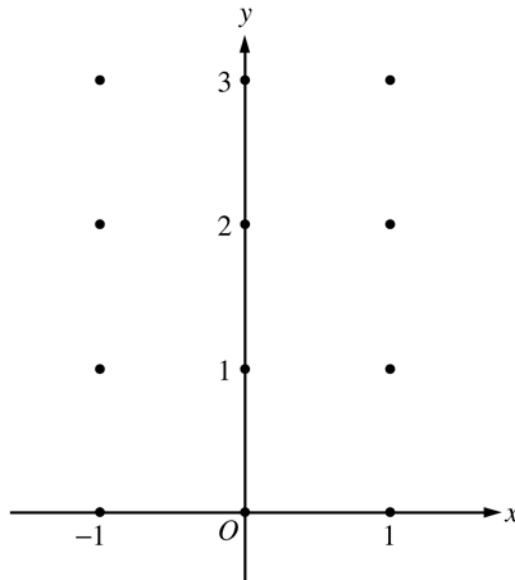
No calculator is allowed for these problems.



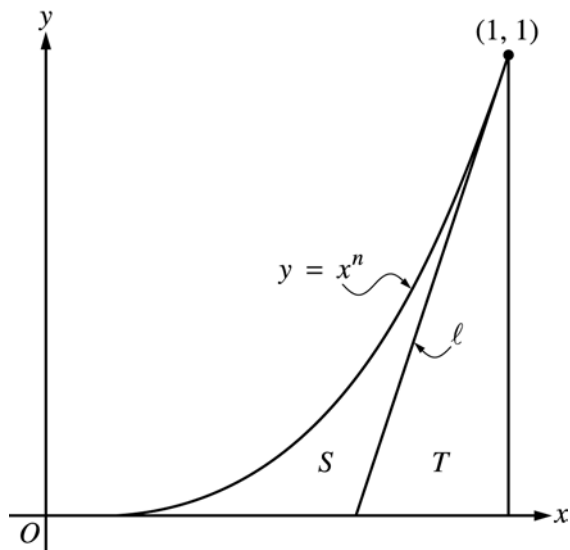
4. The figure above shows the graph of f' , the derivative of the function f , on the closed interval $-1 \leq x \leq 5$. The graph of f' has horizontal tangent lines at $x = 1$ and $x = 3$. The function f is twice differentiable with $f(2) = 6$.
- Find the x -coordinate of each of the points of inflection of the graph of f . Give a reason for your answer.
 - At what value of x does f attain its absolute minimum value on the closed interval $-1 \leq x \leq 5$? At what value of x does f attain its absolute maximum value on the closed interval $-1 \leq x \leq 5$? Show the analysis that leads to your answers.
 - Let g be the function defined by $g(x) = xf(x)$. Find an equation for the line tangent to the graph of g at $x = 2$.

5. Consider the differential equation $\frac{dy}{dx} = x^4(y - 2)$.

- (a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.
(Note: Use the axes provided in the test booklet.)



- (b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy -plane. Describe all points in the xy -plane for which the slopes are negative.
- (c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 0$.
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6. Let ℓ be the line tangent to the graph of $y = x^n$ at the point $(1, 1)$, where $n > 1$, as shown above.

(a) Find $\int_0^1 x^n dx$ in terms of n .

(b) Let T be the triangular region bounded by ℓ , the x -axis, and the line $x = 1$. Show that the area of T is $\frac{1}{2n}$.

(c) Let S be the region bounded by the graph of $y = x^n$, the line ℓ , and the x -axis. Express the area of S in terms of n and determine the value of n that maximizes the area of S .

END OF EXAMINATION