



AP[®] Calculus AB

2004 Free-Response Questions

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CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

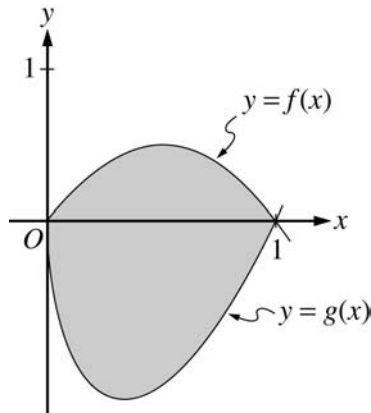
A graphing calculator is required for some problems or parts of problems.

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1. Traffic flow is defined as the rate at which cars pass through an intersection, measured in cars per minute. The traffic flow at a particular intersection is modeled by the function F defined by

$$F(t) = 82 + 4 \sin\left(\frac{t}{2}\right) \text{ for } 0 \leq t \leq 30,$$

where $F(t)$ is measured in cars per minute and t is measured in minutes.

- (a) To the nearest whole number, how many cars pass through the intersection over the 30-minute period?
- (b) Is the traffic flow increasing or decreasing at $t = 7$? Give a reason for your answer.
- (c) What is the average value of the traffic flow over the time interval $10 \leq t \leq 15$? Indicate units of measure.
- (d) What is the average rate of change of the traffic flow over the time interval $10 \leq t \leq 15$? Indicate units of measure.
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2. Let f and g be the functions given by $f(x) = 2x(1 - x)$ and $g(x) = 3(x - 1)\sqrt{x}$ for $0 \leq x \leq 1$. The graphs of f and g are shown in the figure above.
- Find the area of the shaded region enclosed by the graphs of f and g .
 - Find the volume of the solid generated when the shaded region enclosed by the graphs of f and g is revolved about the horizontal line $y = 2$.
 - Let h be the function given by $h(x) = kx(1 - x)$ for $0 \leq x \leq 1$. For each $k > 0$, the region (not shown) enclosed by the graphs of h and g is the base of a solid with square cross sections perpendicular to the x -axis. There is a value of k for which the volume of this solid is equal to 15. Write, but do not solve, an equation involving an integral expression that could be used to find the value of k .
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3. A particle moves along the y -axis so that its velocity v at time $t \geq 0$ is given by $v(t) = 1 - \tan^{-1}(e^t)$. At time $t = 0$, the particle is at $y = -1$. (Note: $\tan^{-1} x = \arctan x$)
- (a) Find the acceleration of the particle at time $t = 2$.
 - (b) Is the speed of the particle increasing or decreasing at time $t = 2$? Give a reason for your answer.
 - (c) Find the time $t \geq 0$ at which the particle reaches its highest point. Justify your answer.
 - (d) Find the position of the particle at time $t = 2$. Is the particle moving toward the origin or away from the origin at time $t = 2$? Justify your answer.
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END OF PART A OF SECTION II

CALCULUS AB
SECTION II, Part B

Time—45 minutes

Number of problems—3

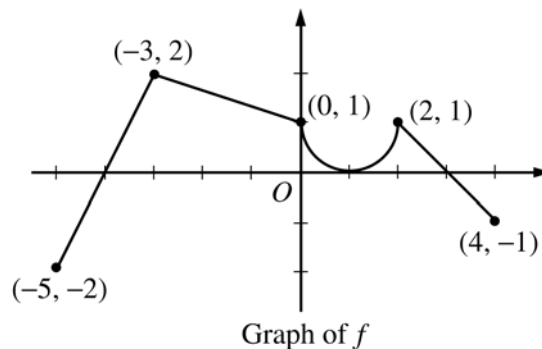
No calculator is allowed for these problems.

4. Consider the curve given by $x^2 + 4y^2 = 7 + 3xy$.

(a) Show that $\frac{dy}{dx} = \frac{3y - 2x}{8y - 3x}$.

(b) Show that there is a point P with x -coordinate 3 at which the line tangent to the curve at P is horizontal. Find the y -coordinate of P .

(c) Find the value of $\frac{d^2y}{dx^2}$ at the point P found in part (b). Does the curve have a local maximum, a local minimum, or neither at the point P ? Justify your answer.



5. The graph of the function f shown above consists of a semicircle and three line segments. Let g be the function

given by $g(x) = \int_{-3}^x f(t) dt$.

(a) Find $g(0)$ and $g'(0)$.

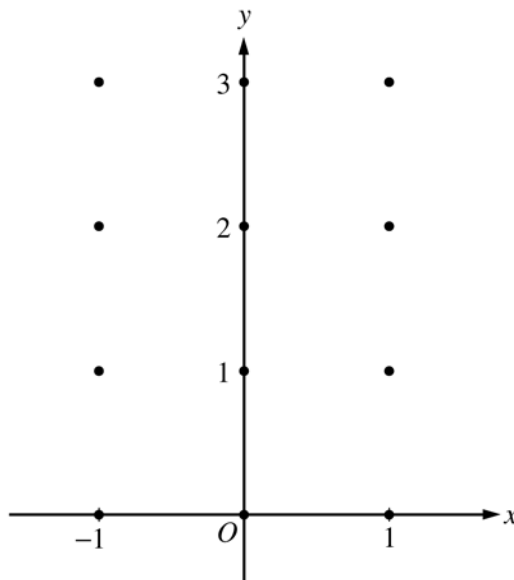
(b) Find all values of x in the open interval $(-5, 4)$ at which g attains a relative maximum. Justify your answer.

(c) Find the absolute minimum value of g on the closed interval $[-5, 4]$. Justify your answer.

(d) Find all values of x in the open interval $(-5, 4)$ at which the graph of g has a point of inflection.

6. Consider the differential equation $\frac{dy}{dx} = x^2(y - 1)$.

- (a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.
(Note: Use the axes provided in the pink test booklet.)



- (b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy -plane. Describe all points in the xy -plane for which the slopes are positive.
- (c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$.
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END OF EXAMINATION