



AP[®] Calculus AB
2005 Free-Response Questions
Form B

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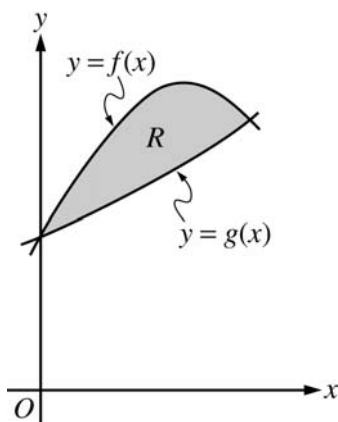
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CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let f and g be the functions given by $f(x) = 1 + \sin(2x)$ and $g(x) = e^{x/2}$. Let R be the shaded region in the first quadrant enclosed by the graphs of f and g as shown in the figure above.
- Find the area of R .
 - Find the volume of the solid generated when R is revolved about the x -axis.
 - The region R is the base of a solid. For this solid, the cross sections perpendicular to the x -axis are semicircles with diameters extending from $y = f(x)$ to $y = g(x)$. Find the volume of this solid.
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WRITE ALL WORK IN THE TEST BOOKLET.

2. A water tank at Camp Newton holds 1200 gallons of water at time $t = 0$. During the time interval $0 \leq t \leq 18$ hours, water is pumped into the tank at the rate

$$W(t) = 95\sqrt{t} \sin^2\left(\frac{t}{6}\right) \text{ gallons per hour.}$$

During the same time interval, water is removed from the tank at the rate

$$R(t) = 275 \sin^2\left(\frac{t}{3}\right) \text{ gallons per hour.}$$

- (a) Is the amount of water in the tank increasing at time $t = 15$? Why or why not?
- (b) To the nearest whole number, how many gallons of water are in the tank at time $t = 18$?
- (c) At what time t , for $0 \leq t \leq 18$, is the amount of water in the tank at an absolute minimum? Show the work that leads to your conclusion.
- (d) For $t > 18$, no water is pumped into the tank, but water continues to be removed at the rate $R(t)$ until the tank becomes empty. Let k be the time at which the tank becomes empty. Write, but do not solve, an equation involving an integral expression that can be used to find the value of k .
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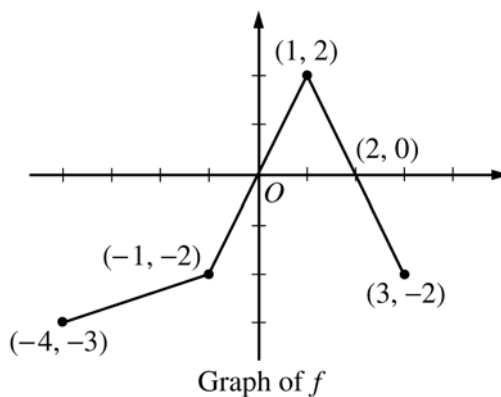
3. A particle moves along the x -axis so that its velocity v at time t , for $0 \leq t \leq 5$, is given by $v(t) = \ln(t^2 - 3t + 3)$. The particle is at position $x = 8$ at time $t = 0$.
- (a) Find the acceleration of the particle at time $t = 4$.
 - (b) Find all times t in the open interval $0 < t < 5$ at which the particle changes direction. During which time intervals, for $0 \leq t \leq 5$, does the particle travel to the left?
 - (c) Find the position of the particle at time $t = 2$.
 - (d) Find the average speed of the particle over the interval $0 \leq t \leq 2$.
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WRITE ALL WORK IN THE TEST BOOKLET.

END OF PART A OF SECTION II

CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



4. The graph of the function f above consists of three line segments.
- (a) Let g be the function given by $g(x) = \int_{-4}^x f(t) dt$. For each of $g(-1)$, $g'(-1)$, and $g''(-1)$, find the value or state that it does not exist.
- (b) For the function g defined in part (a), find the x -coordinate of each point of inflection of the graph of g on the open interval $-4 < x < 3$. Explain your reasoning.
- (c) Let h be the function given by $h(x) = \int_x^3 f(t) dt$. Find all values of x in the closed interval $-4 \leq x \leq 3$ for which $h(x) = 0$.
- (d) For the function h defined in part (c), find all intervals on which h is decreasing. Explain your reasoning.

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5. Consider the curve given by $y^2 = 2 + xy$.

(a) Show that $\frac{dy}{dx} = \frac{y}{2y - x}$.

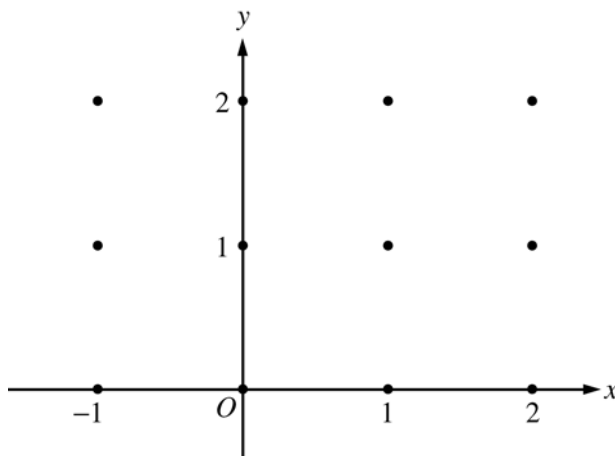
(b) Find all points (x, y) on the curve where the line tangent to the curve has slope $\frac{1}{2}$.

(c) Show that there are no points (x, y) on the curve where the line tangent to the curve is horizontal.

(d) Let x and y be functions of time t that are related by the equation $y^2 = 2 + xy$. At time $t = 5$, the value of y is 3 and $\frac{dy}{dt} = 6$. Find the value of $\frac{dx}{dt}$ at time $t = 5$.

6. Consider the differential equation $\frac{dy}{dx} = \frac{-xy^2}{2}$. Let $y = f(x)$ be the particular solution to this differential equation with the initial condition $f(-1) = 2$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated. (Note: Use the axes provided in the test booklet.)



(b) Write an equation for the line tangent to the graph of f at $x = -1$.

(c) Find the solution $y = f(x)$ to the given differential equation with the initial condition $f(-1) = 2$.

WRITE ALL WORK IN THE TEST BOOKLET.

END OF EXAM