



## AP<sup>®</sup> Calculus AB 2005 Scoring Guidelines

### **The College Board: Connecting Students to College Success**

The College Board is a not-for-profit membership association whose mission is to connect students to college success and opportunity. Founded in 1900, the association is composed of more than 4,700 schools, colleges, universities, and other educational organizations. Each year, the College Board serves over three and a half million students and their parents, 23,000 high schools, and 3,500 colleges through major programs and services in college admissions, guidance, assessment, financial aid, enrollment, and teaching and learning. Among its best-known programs are the SAT<sup>®</sup>, the PSAT/NMSQT<sup>®</sup>, and the Advanced Placement Program<sup>®</sup> (AP<sup>®</sup>). The College Board is committed to the principles of excellence and equity, and that commitment is embodied in all of its programs, services, activities, and concerns.

Copyright © 2005 by College Board. All rights reserved. College Board, AP Central, APCD, Advanced Placement Program, AP, AP Vertical Teams, Pre-AP, SAT, and the acorn logo are registered trademarks of the College Entrance Examination Board. Admitted Class Evaluation Service, CollegeEd, Connect to college success, MyRoad, SAT Professional Development, SAT Readiness Program, and Setting the Cornerstones are trademarks owned by the College Entrance Examination Board. PSAT/NMSQT is a registered trademark of the College Entrance Examination Board and National Merit Scholarship Corporation. Other products and services may be trademarks of their respective owners. Permission to use copyrighted College Board materials may be requested online at: <http://www.collegeboard.com/inquiry/cbpermit.html>.

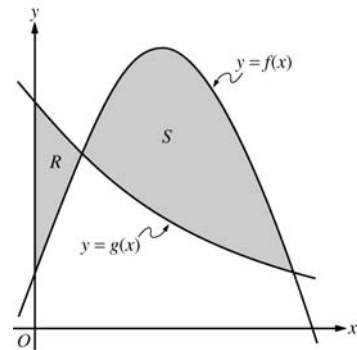
**Visit the College Board on the Web: [www.collegeboard.com](http://www.collegeboard.com).**

**AP Central is the official online home for the AP Program and Pre-AP: [apcentral.collegeboard.com](http://apcentral.collegeboard.com).**

**AP<sup>®</sup> CALCULUS AB  
2005 SCORING GUIDELINES**

**Question 1**

Let  $f$  and  $g$  be the functions given by  $f(x) = \frac{1}{4} + \sin(\pi x)$  and  $g(x) = 4^{-x}$ . Let  $R$  be the shaded region in the first quadrant enclosed by the  $y$ -axis and the graphs of  $f$  and  $g$ , and let  $S$  be the shaded region in the first quadrant enclosed by the graphs of  $f$  and  $g$ , as shown in the figure above.



- (a) Find the area of  $R$ .  
 (b) Find the area of  $S$ .  
 (c) Find the volume of the solid generated when  $S$  is revolved about the horizontal line  $y = -1$ .

$$f(x) = g(x) \text{ when } \frac{1}{4} + \sin(\pi x) = 4^{-x}.$$

$f$  and  $g$  intersect when  $x = 0.178218$  and when  $x = 1$ .  
 Let  $a = 0.178218$ .

(a)  $\int_0^a (g(x) - f(x)) dx = 0.064$  or  $0.065$

3 :  $\begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(b)  $\int_a^1 (f(x) - g(x)) dx = 0.410$

3 :  $\begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(c)  $\pi \int_a^1 ((f(x) + 1)^2 - (g(x) + 1)^2) dx = 4.558$  or  $4.559$

3 :  $\begin{cases} 2 : \text{integrand} \\ 1 : \text{limits, constant, and answer} \end{cases}$

**AP<sup>®</sup> CALCULUS AB  
2005 SCORING GUIDELINES**

**Question 2**

The tide removes sand from Sandy Point Beach at a rate modeled by the function  $R$ , given by

$$R(t) = 2 + 5 \sin\left(\frac{4\pi t}{25}\right).$$

A pumping station adds sand to the beach at a rate modeled by the function  $S$ , given by

$$S(t) = \frac{15t}{1 + 3t}.$$

Both  $R(t)$  and  $S(t)$  have units of cubic yards per hour and  $t$  is measured in hours for  $0 \leq t \leq 6$ . At time  $t = 0$ , the beach contains 2500 cubic yards of sand.

- How much sand will the tide remove from the beach during this 6-hour period? Indicate units of measure.
- Write an expression for  $Y(t)$ , the total number of cubic yards of sand on the beach at time  $t$ .
- Find the rate at which the total amount of sand on the beach is changing at time  $t = 4$ .
- For  $0 \leq t \leq 6$ , at what time  $t$  is the amount of sand on the beach a minimum? What is the minimum value? Justify your answers.

(a)  $\int_0^6 R(t) dt = 31.815$  or  $31.816 \text{ yd}^3$

2 :  $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer with units} \end{cases}$

(b)  $Y(t) = 2500 + \int_0^t (S(x) - R(x)) dx$

3 :  $\begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$

(c)  $Y'(t) = S(t) - R(t)$

$Y'(4) = S(4) - R(4) = -1.908$  or  $-1.909 \text{ yd}^3/\text{hr}$

1 : answer

(d)  $Y'(t) = 0$  when  $S(t) - R(t) = 0$ .

The only value in  $[0, 6]$  to satisfy  $S(t) = R(t)$  is  $a = 5.117865$ .

3 :  $\begin{cases} 1 : \text{sets } Y'(t) = 0 \\ 1 : \text{critical } t\text{-value} \\ 1 : \text{answer with justification} \end{cases}$

$t$	$Y(t)$
0	2500
$a$	2492.3694
6	2493.2766

The amount of sand is a minimum when  $t = 5.117$  or 5.118 hours. The minimum value is 2492.369 cubic yards.

**AP<sup>®</sup> CALCULUS AB  
2005 SCORING GUIDELINES**

**Question 3**

Distance $x$ (cm)	0	1	5	6	8
Temperature $T(x)$ ( $^{\circ}\text{C}$ )	100	93	70	62	55

A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature  $T(x)$ , in degrees Celsius ( $^{\circ}\text{C}$ ), of the wire  $x$  cm from the heated end. The function  $T$  is decreasing and twice differentiable.

- (a) Estimate  $T'(7)$ . Show the work that leads to your answer. Indicate units of measure.
- (b) Write an integral expression in terms of  $T(x)$  for the average temperature of the wire. Estimate the average temperature of the wire using a trapezoidal sum with the four subintervals indicated by the data in the table. Indicate units of measure.
- (c) Find  $\int_0^8 T'(x) dx$ , and indicate units of measure. Explain the meaning of  $\int_0^8 T'(x) dx$  in terms of the temperature of the wire.
- (d) Are the data in the table consistent with the assertion that  $T''(x) > 0$  for every  $x$  in the interval  $0 < x < 8$ ? Explain your answer.

(a) 
$$\frac{T(8) - T(6)}{8 - 6} = \frac{55 - 62}{2} = -\frac{7}{2}^{\circ}\text{C/cm}$$

1 : answer

(b) 
$$\frac{1}{8} \int_0^8 T(x) dx$$

Trapezoidal approximation for  $\int_0^8 T(x) dx$ :

$$A = \frac{100 + 93}{2} \cdot 1 + \frac{93 + 70}{2} \cdot 4 + \frac{70 + 62}{2} \cdot 1 + \frac{62 + 55}{2} \cdot 2$$

Average temperature  $\approx \frac{1}{8} A = 75.6875^{\circ}\text{C}$

3 :  $\left\{ \begin{array}{l} 1 : \frac{1}{8} \int_0^8 T(x) dx \\ 1 : \text{trapezoidal sum} \\ 1 : \text{answer} \end{array} \right.$

(c) 
$$\int_0^8 T'(x) dx = T(8) - T(0) = 55 - 100 = -45^{\circ}\text{C}$$

The temperature drops  $45^{\circ}\text{C}$  from the heated end of the wire to the other end of the wire.

2 :  $\left\{ \begin{array}{l} 1 : \text{value} \\ 1 : \text{meaning} \end{array} \right.$

(d) Average rate of change of temperature on  $[1, 5]$  is  $\frac{70 - 93}{5 - 1} = -5.75$ .

Average rate of change of temperature on  $[5, 6]$  is  $\frac{62 - 70}{6 - 5} = -8$ .

No. By the MVT,  $T'(c_1) = -5.75$  for some  $c_1$  in the interval  $(1, 5)$  and  $T'(c_2) = -8$  for some  $c_2$  in the interval  $(5, 6)$ . It follows that  $T'$  must decrease somewhere in the interval  $(c_1, c_2)$ . Therefore  $T''$  is not positive for every  $x$  in  $[0, 8]$ .

2 :  $\left\{ \begin{array}{l} 1 : \text{two slopes of secant lines} \\ 1 : \text{answer with explanation} \end{array} \right.$

Units of  $^{\circ}\text{C/cm}$  in (a), and  $^{\circ}\text{C}$  in (b) and (c)

1 : units in (a), (b), and (c)

**AP<sup>®</sup> CALCULUS AB  
2005 SCORING GUIDELINES**

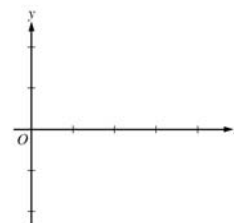
**Question 4**

$x$	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3	$3 < x < 4$
$f(x)$	-1	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	4	Positive	0	Positive	DNE	Negative	-3	Negative
$f''(x)$	-2	Negative	0	Positive	DNE	Negative	0	Positive

Let  $f$  be a function that is continuous on the interval  $[0, 4)$ . The function  $f$  is twice differentiable except at  $x = 2$ . The function  $f$  and its derivatives have the properties indicated in the table above, where DNE indicates that the derivatives of  $f$  do not exist at  $x = 2$ .

(a) For  $0 < x < 4$ , find all values of  $x$  at which  $f$  has a relative extremum. Determine whether  $f$  has a relative maximum or a relative minimum at each of these values. Justify your answer.

(b) On the axes provided, sketch the graph of a function that has all the characteristics of  $f$ .  
(Note: Use the axes provided in the pink test booklet.)

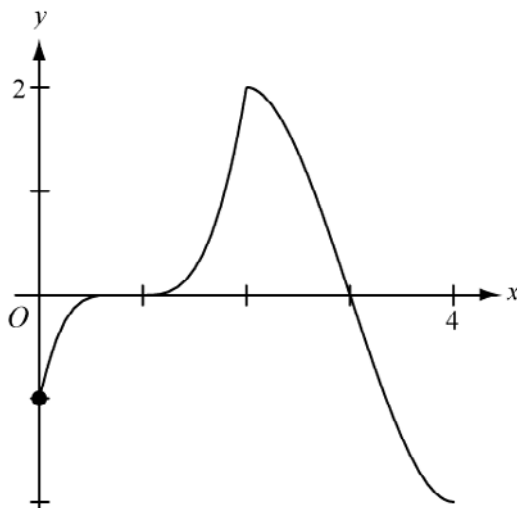


(c) Let  $g$  be the function defined by  $g(x) = \int_1^x f(t) dt$  on the open interval  $(0, 4)$ . For  $0 < x < 4$ , find all values of  $x$  at which  $g$  has a relative extremum. Determine whether  $g$  has a relative maximum or a relative minimum at each of these values. Justify your answer.

(d) For the function  $g$  defined in part (c), find all values of  $x$ , for  $0 < x < 4$ , at which the graph of  $g$  has a point of inflection. Justify your answer.

(a)  $f$  has a relative maximum at  $x = 2$  because  $f'$  changes from positive to negative at  $x = 2$ .

(b)



(c)  $g'(x) = f(x) = 0$  at  $x = 1, 3$ .  
 $g'$  changes from negative to positive at  $x = 1$  so  $g$  has a relative minimum at  $x = 1$ .  $g'$  changes from positive to negative at  $x = 3$  so  $g$  has a relative maximum at  $x = 3$ .

(d) The graph of  $g$  has a point of inflection at  $x = 2$  because  $g'' = f'$  changes sign at  $x = 2$ .

2 : { 1 : relative extremum at  $x = 2$   
1 : relative maximum with justification

2 : { 1 : points at  $x = 0, 1, 2, 3$   
and behavior at  $(2, 2)$   
1 : appropriate increasing/decreasing  
and concavity behavior

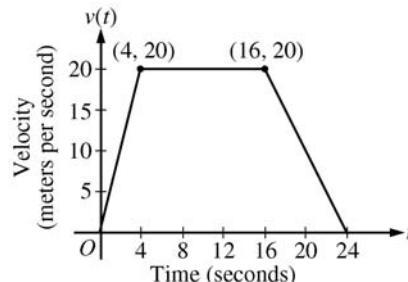
3 : { 1 :  $g'(x) = f(x)$   
1 : critical points  
1 : answer with justification

2 : { 1 :  $x = 2$   
1 : answer with justification

**AP<sup>®</sup> CALCULUS AB  
2005 SCORING GUIDELINES**

**Question 5**

A car is traveling on a straight road. For  $0 \leq t \leq 24$  seconds, the car's velocity  $v(t)$ , in meters per second, is modeled by the piecewise-linear function defined by the graph above.



- (a) Find  $\int_0^{24} v(t) dt$ . Using correct units, explain the meaning of  $\int_0^{24} v(t) dt$ .
- (b) For each of  $v'(4)$  and  $v'(20)$ , find the value or explain why it does not exist. Indicate units of measure.
- (c) Let  $a(t)$  be the car's acceleration at time  $t$ , in meters per second per second. For  $0 < t < 24$ , write a piecewise-defined function for  $a(t)$ .
- (d) Find the average rate of change of  $v$  over the interval  $8 \leq t \leq 20$ . Does the Mean Value Theorem guarantee a value of  $c$ , for  $8 < c < 20$ , such that  $v'(c)$  is equal to this average rate of change? Why or why not?

- (a)  $\int_0^{24} v(t) dt = \frac{1}{2}(4)(20) + (12)(20) + \frac{1}{2}(8)(20) = 360$   
The car travels 360 meters in these 24 seconds.

2 :  $\begin{cases} 1 : \text{value} \\ 1 : \text{meaning with units} \end{cases}$

- (b)  $v'(4)$  does not exist because
- $$\lim_{t \rightarrow 4^-} \left( \frac{v(t) - v(4)}{t - 4} \right) = 5 \neq 0 = \lim_{t \rightarrow 4^+} \left( \frac{v(t) - v(4)}{t - 4} \right).$$
- $$v'(20) = \frac{20 - 0}{16 - 24} = -\frac{5}{2} \text{ m/sec}^2$$

3 :  $\begin{cases} 1 : v'(4) \text{ does not exist, with explanation} \\ 1 : v'(20) \\ 1 : \text{units} \end{cases}$

- (c) 
$$a(t) = \begin{cases} 5 & \text{if } 0 < t < 4 \\ 0 & \text{if } 4 < t < 16 \\ -\frac{5}{2} & \text{if } 16 < t < 24 \end{cases}$$
  
 $a(t)$  does not exist at  $t = 4$  and  $t = 16$ .

2 :  $\begin{cases} 1 : \text{finds the values } 5, 0, -\frac{5}{2} \\ 1 : \text{identifies constants with correct intervals} \end{cases}$

- (d) The average rate of change of  $v$  on  $[8, 20]$  is
- $$\frac{v(20) - v(8)}{20 - 8} = -\frac{5}{6} \text{ m/sec}^2.$$
- No, the Mean Value Theorem does not apply to  $v$  on  $[8, 20]$  because  $v$  is not differentiable at  $t = 16$ .

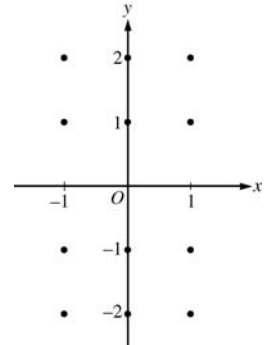
2 :  $\begin{cases} 1 : \text{average rate of change of } v \text{ on } [8, 20] \\ 1 : \text{answer with explanation} \end{cases}$

**AP<sup>®</sup> CALCULUS AB  
2005 SCORING GUIDELINES**

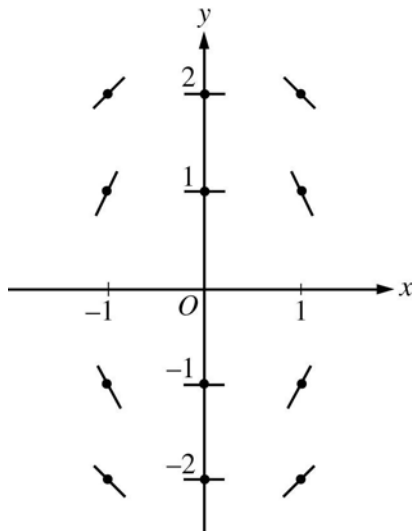
**Question 6**

Consider the differential equation  $\frac{dy}{dx} = -\frac{2x}{y}$ .

- (a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.  
**(Note: Use the axes provided in the pink test booklet.)**
- (b) Let  $y = f(x)$  be the particular solution to the differential equation with the initial condition  $f(1) = -1$ . Write an equation for the line tangent to the graph of  $f$  at  $(1, -1)$  and use it to approximate  $f(1.1)$ .
- (c) Find the particular solution  $y = f(x)$  to the given differential equation with the initial condition  $f(1) = -1$ .



(a)



2 :  $\begin{cases} 1 : \text{zero slopes} \\ 1 : \text{nonzero slopes} \end{cases}$

- (b) The line tangent to  $f$  at  $(1, -1)$  is  $y + 1 = 2(x - 1)$ .  
Thus,  $f(1.1)$  is approximately  $-0.8$ .

2 :  $\begin{cases} 1 : \text{equation of the tangent line} \\ 1 : \text{approximation for } f(1.1) \end{cases}$

- (c)  $\frac{dy}{dx} = -\frac{2x}{y}$   
 $y \, dy = -2x \, dx$   
 $\frac{y^2}{2} = -x^2 + C$   
 $\frac{1}{2} = -1 + C; C = \frac{3}{2}$   
 $y^2 = -2x^2 + 3$   
 Since the particular solution goes through  $(1, -1)$ ,  
 $y$  must be negative.  
 Thus the particular solution is  $y = -\sqrt{3 - 2x^2}$ .

5 :  $\begin{cases} 1 : \text{separates variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: max 2/5 [1-1-0-0-0] if no constant of integration  
 Note: 0/5 if no separation of variables