# AP® Calculus AB
# 2005 Free-Response Questions

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## CALCULUS AB
## SECTION II, Part A
**Time—45 minutes**
**Number of problems—3**

**A graphing calculator is required for some problems or parts of problems.**

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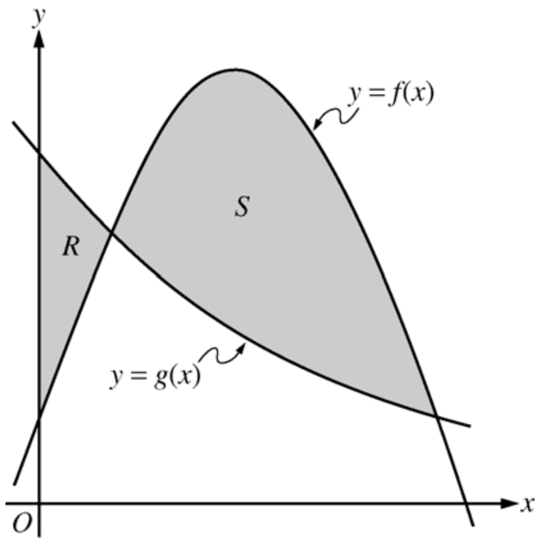


1. Let $f$ and $g$ be the functions given by $f(x) = \frac{1}{4} + \sin(\pi x)$ and $g(x) = 4^{-x}$. Let $R$ be the shaded region in the first quadrant enclosed by the $y$-axis and the graphs of $f$ and $g$, and let $S$ be the shaded region in the first quadrant enclosed by the graphs of $f$ and $g$, as shown in the figure above.

   (a) Find the area of $R$.

   (b) Find the area of $S$.

   (c) Find the volume of the solid generated when $S$ is revolved about the horizontal line $y = -1$.

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**WRITE ALL WORK IN THE TEST BOOKLET.**

2. The tide removes sand from Sandy Point Beach at a rate modeled by the function $R$, given by

$$R(t) = 2 + 5\sin\left(\frac{4\pi t}{25}\right).$$

A pumping station adds sand to the beach at a rate modeled by the function $S$, given by

$$S(t) = \frac{15t}{1 + 3t}.$$

Both $R(t)$ and $S(t)$ have units of cubic yards per hour and $t$ is measured in hours for $0 \le t \le 6$. At time $t = 0$, the beach contains 2500 cubic yards of sand.

(a) How much sand will the tide remove from the beach during this 6-hour period? Indicate units of measure.

(b) Write an expression for $Y(t)$, the total number of cubic yards of sand on the beach at time $t$.

(c) Find the rate at which the total amount of sand on the beach is changing at time $t = 4$.

(d) For $0 \le t \le 6$, at what time $t$ is the amount of sand on the beach a minimum? What is the minimum value? Justify your answers.

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| Distance $x$ (cm) | 0 | 1 | 5 | 6 | 8 |
|---|---|---|---|---|---|
| Temperature $T(x)$ (°C) | 100 | 93 | 70 | 62 | 55 |

3. A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature $T(x)$, in degrees Celsius (°C), of the wire $x$ cm from the heated end. The function $T$ is decreasing and twice differentiable.

   (a) Estimate $T'(7)$. Show the work that leads to your answer. Indicate units of measure.

   (b) Write an integral expression in terms of $T(x)$ for the average temperature of the wire. Estimate the average temperature of the wire using a trapezoidal sum with the four subintervals indicated by the data in the table. Indicate units of measure.

   (c) Find $\int_0^8 T'(x)\,dx$, and indicate units of measure. Explain the meaning of $\int_0^8 T'(x)\,dx$ in terms of the temperature of the wire.

   (d) Are the data in the table consistent with the assertion that $T''(x) > 0$ for every $x$ in the interval $0 < x < 8$? Explain your answer.

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**WRITE ALL WORK IN THE TEST BOOKLET.**

**END OF PART A OF SECTION II**

## CALCULUS AB
## SECTION II, Part B
**Time—45 minutes**
**Number of problems—3**

**No calculator is allowed for these problems.**

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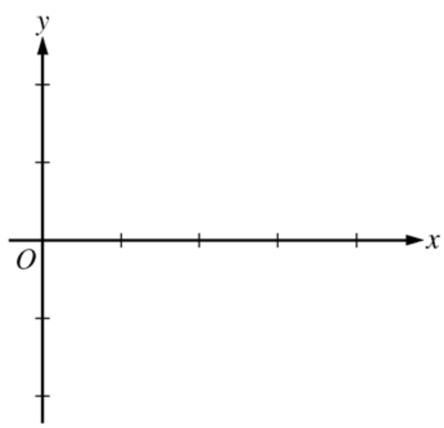
| $x$ | 0 | $0 < x < 1$ | 1 | $1 < x < 2$ | 2 | $2 < x < 3$ | 3 | $3 < x < 4$ |
|---|---|---|---|---|---|---|---|---|
| $f(x)$ | –1 | Negative | 0 | Positive | 2 | Positive | 0 | Negative |
| $f'(x)$ | 4 | Positive | 0 | Positive | DNE | Negative | –3 | Negative |
| $f''(x)$ | –2 | Negative | 0 | Positive | DNE | Negative | 0 | Positive |

4. Let $f$ be a function that is continuous on the interval $[0, 4)$. The function $f$ is twice differentiable except at $x = 2$. The function $f$ and its derivatives have the properties indicated in the table above, where DNE indicates that the derivatives of $f$ do not exist at $x = 2$.

(a) For $0 < x < 4$, find all values of $x$ at which $f$ has a relative extremum. Determine whether $f$ has a relative maximum or a relative minimum at each of these values. Justify your answer.

(b) On the axes provided, sketch the graph of a function that has all the characteristics of $f$.

**(Note: Use the axes provided in the pink test booklet.)**




(c) Let $g$ be the function defined by $g(x) = \int_1^x f(t)\,dt$ on the open interval $(0, 4)$. For $0 < x < 4$, find all values of $x$ at which $g$ has a relative extremum. Determine whether $g$ has a relative maximum or a relative minimum at each of these values. Justify your answer.

(d) For the function $g$ defined in part (c), find all values of $x$, for $0 < x < 4$, at which the graph of $g$ has a point of inflection. Justify your answer.

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**WRITE ALL WORK IN THE TEST BOOKLET.**



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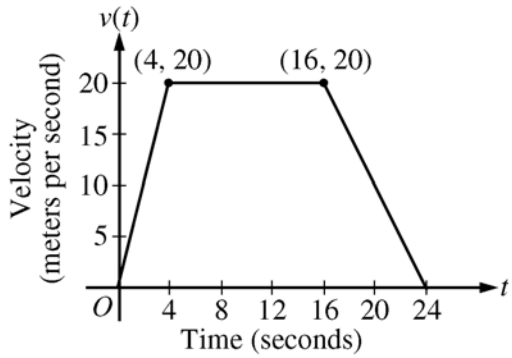


5. A car is traveling on a straight road. For $0 \le t \le 24$ seconds, the car's velocity $v(t)$, in meters per second, is modeled by the piecewise-linear function defined by the graph above.

(a) Find $\int_0^{24} v(t)\,dt$. Using correct units, explain the meaning of $\int_0^{24} v(t)\,dt$.

(b) For each of $v'(4)$ and $v'(20)$, find the value or explain why it does not exist. Indicate units of measure.

(c) Let $a(t)$ be the car's acceleration at time $t$, in meters per second per second. For $0 < t < 24$, write a piecewise-defined function for $a(t)$.

(d) Find the average rate of change of $v$ over the interval $8 \le t \le 20$. Does the Mean Value Theorem guarantee a value of $c$, for $8 < c < 20$, such that $v'(c)$ is equal to this average rate of change? Why or why not?

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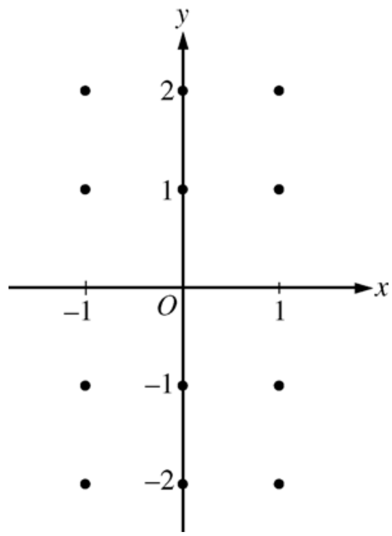
**WRITE ALL WORK IN THE TEST BOOKLET.**

6. Consider the differential equation $\frac{dy}{dx} = -\frac{2x}{y}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated. (**Note: Use the axes provided in the pink test booklet.**)




(b) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = -1$. Write an equation for the line tangent to the graph of $f$ at $(1, -1)$ and use it to approximate $f(1.1)$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(1) = -1$.

**WRITE ALL WORK IN THE TEST BOOKLET.**

**END OF EXAM**