CollegeBoard
Advanced Placement Program

AP® Calculus AB
2006 Scoring Guidelines
Form B

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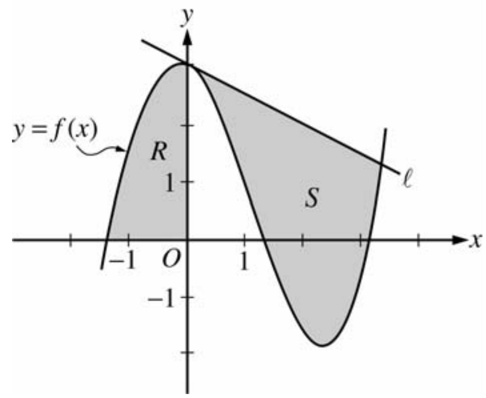
Question 1



Let f be the function given by $f(x) = \frac{x^3}{4} - \frac{x^2}{3} - \frac{x}{2} + 3\cos x$. Let R be the shaded region in the second quadrant bounded by the graph of f, and let S be the shaded region bounded by the graph of f and line ℓ, the line tangent to the graph of f at $x = 0$, as shown above.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

(c) Write, but do not evaluate, an integral expression that can be used to find the area of S.

Solution	Scoring
For $x < 0$, $f(x) = 0$ when $x = -1.37312$. Let $P = -1.37312$.	
(a) Area of $R = \int_P^0 f(x)\,dx = 2.903$	2 : { 1 : integral 1 : answer
(b) Volume $= \pi\int_P^0 \left((f(x) + 2)^2 - 4\right) dx = 59.361$	4 : { 1 : limits and constant 2 : integrand 1 : answer
(c) The equation of the tangent line ℓ is $y = 3 - \frac{1}{2}x$. The graph of f and line ℓ intersect at $A = 3.38987$. Area of $S = \int_0^A \left(\left(3 - \frac{1}{2}x\right) - f(x)\right) dx$	3 : { 1 : tangent line 1 : integrand 1 : limits

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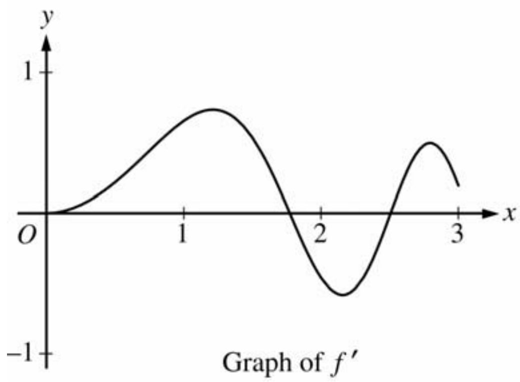
Question 2



Graph of f'

Let f be the function defined for $x \geq 0$ with $f(0) = 5$ and f', the first derivative of f, given by $f'(x) = e^{(-x/4)} \sin(x^2)$. The graph of $y = f'(x)$ is shown above.

(a) Use the graph of f' to determine whether the graph of f is concave up, concave down, or neither on the interval $1.7 < x < 1.9$. Explain your reasoning.

(b) On the interval $0 \leq x \leq 3$, find the value of x at which f has an absolute maximum. Justify your answer.

(c) Write an equation for the line tangent to the graph of f at $x = 2$.

(a) On the interval $1.7 < x < 1.9$, f' is decreasing and thus f is concave down on this interval.

$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$

(b) $f'(x) = 0$ when $x = 0, \sqrt{\pi}, \sqrt{2\pi}, \sqrt{3\pi}, \ldots$

On $[0, 3]$ f' changes from positive to negative only at $\sqrt{\pi}$. The absolute maximum must occur at $x = \sqrt{\pi}$ or at an endpoint.

$f(0) = 5$

$f(\sqrt{\pi}) = f(0) + \int_0^{\sqrt{\pi}} f'(x)\,dx = 5.67911$

$f(3) = f(0) + \int_0^{3} f'(x)\,dx = 5.57893$

This shows that f has an absolute maximum at $x = \sqrt{\pi}$.

3 :
- 1 : identifies $\sqrt{\pi}$ and 3 as candidates
  - or -
  indicates that the graph of f increases, decreases, then increases
- 1 : justifies $f(\sqrt{\pi}) > f(3)$
- 1 : answer

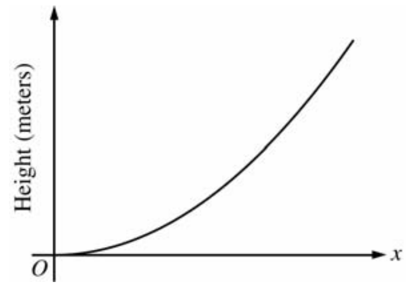
(c) $f(2) = f(0) + \int_0^{2} f'(x)\,dx = 5.62342$

$f'(2) = e^{-0.5} \sin(4) = -0.45902$

$y - 5.623 = (-0.459)(x - 2)$

4 :
- 2 : $f(2)$ expression
  - 1 : integral
  - 1 : including $f(0)$ term
- 1 : $f'(2)$
- 1 : equation

Question 3



The figure above is the graph of a function of x, which models the height of a skateboard ramp. The function meets the following requirements.

(i) At $x = 0$, the value of the function is 0, and the slope of the graph of the function is 0.
(ii) At $x = 4$, the value of the function is 1, and the slope of the graph of the function is 1.
(iii) Between $x = 0$ and $x = 4$, the function is increasing.

(a) Let $f(x) = ax^2$, where a is a nonzero constant. Show that it is not possible to find a value for a so that f meets requirement (ii) above.

(b) Let $g(x) = cx^3 - \frac{x^2}{16}$, where c is a nonzero constant. Find the value of c so that g meets requirement (ii) above. Show the work that leads to your answer.

(c) Using the function g and your value of c from part (b), show that g does not meet requirement (iii) above.

(d) Let $h(x) = \frac{x^n}{k}$, where k is a nonzero constant and n is a positive integer. Find the values of k and n so that h meets requirement (ii) above. Show that h also meets requirements (i) and (iii) above.

(a) $f(4) = 1$ implies that $a = \frac{1}{16}$ and $f'(4) = 2a(4) = 1$ implies that $a = \frac{1}{8}$. Thus, f cannot satisfy (ii).

$2: \begin{cases} 1: a = \frac{1}{16} \text{ or } a = \frac{1}{8} \\ 1: \text{shows } a \text{ does not work} \end{cases}$

(b) $g(4) = 64c - 1 = 1$ implies that $c = \frac{1}{32}$.

When $c = \frac{1}{32}$, $g'(4) = 3c(4)^2 - \frac{2(4)}{16} = 3\left(\frac{1}{32}\right)(16) - \frac{1}{2} = 1$

$1:$ value of c

(c) $g'(x) = \frac{3}{32}x^2 - \frac{x}{8} = \frac{1}{32}x(3x - 4)$

$g'(x) < 0$ for $0 < x < \frac{4}{3}$, so g does not satisfy (iii).

$2: \begin{cases} 1: g'(x) \\ 1: \text{explanation} \end{cases}$

(d) $h(4) = \frac{4^n}{k} = 1$ implies that $4^n = k$.

$h'(4) = \frac{n4^{n-1}}{k} = \frac{n4^{n-1}}{4^n} = \frac{n}{4} = 1$ gives $n = 4$ and $k = 4^4 = 256$.

$h(x) = \frac{x^4}{256} \Rightarrow h(0) = 0.$

$h'(x) = \frac{4x^3}{256} \Rightarrow h'(0) = 0$ and $h'(x) > 0$ for $0 < x < 4$.

$4: \begin{cases} 1: \frac{4^n}{k} = 1 \\ 1: \frac{n4^{n-1}}{k} = 1 \\ 1: \text{values for } k \text{ and } n \\ 1: \text{verifications} \end{cases}$

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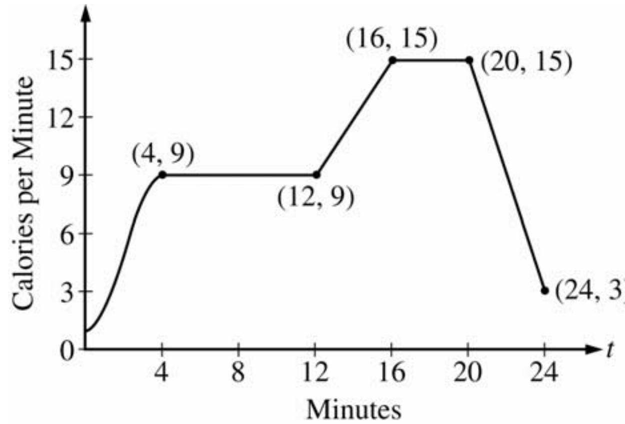
Question 4



The rate, in calories per minute, at which a person using an exercise machine burns calories is modeled by the function f. In the figure above, $f(t) = -\frac{1}{4}t^3 + \frac{3}{2}t^2 + 1$ for $0 \le t \le 4$ and f is piecewise linear for $4 \le t \le 24$.

(a) Find $f'(22)$. Indicate units of measure.

(b) For the time interval $0 \le t \le 24$, at what time t is f increasing at its greatest rate? Show the reasoning that supports your answer.

(c) Find the total number of calories burned over the time interval $6 \le t \le 18$ minutes.

(d) The setting on the machine is now changed so that the person burns $f(t) + c$ calories per minute. For this setting, find c so that an average of 15 calories per minute is burned during the time interval $6 \le t \le 18$.

(a) $f'(22) = \frac{15-3}{20-24} = -3$ calories/min/min

1 : $f'(22)$ and units

(b) f is increasing on $[0, 4]$ and on $[12, 16]$.

On $(12, 16)$, $f'(t) = \frac{15-9}{16-12} = \frac{3}{2}$ since f has constant slope on this interval.

On $(0, 4)$, $f'(t) = -\frac{3}{4}t^2 + 3t$ and

$f''(t) = -\frac{3}{2}t + 3 = 0$ when $t = 2$. This is where f' has a maximum on $[0, 4]$ since $f'' > 0$ on $(0, 2)$ and $f'' < 0$ on $(2, 4)$.

On $[0, 24]$, f is increasing at its greatest rate when $t = 2$ because $f'(2) = 3 > \frac{3}{2}$.

4 :
- 1 : f' on $(0, 4)$
- 1 : shows f' has a max at $t = 2$ on $(0, 4)$
- 1 : shows for $12 < t < 16$, $f'(t) < f'(2)$
- 1 : answer

(c) $\int_6^{18} f(t)\,dt = 6(9) + \frac{1}{2}(4)(9+15) + 2(15)$

$= 132$ calories

2 :
- 1 : method
- 1 : answer

(d) We want $\frac{1}{12}\int_6^{18} (f(t) + c)\,dt = 15$.

This means $132 + 12c = 15(12)$. So, $c = 4$.

OR

Currently, the average is $\frac{132}{12} = 11$ calories/min.

Adding c to $f(t)$ will shift the average by c.

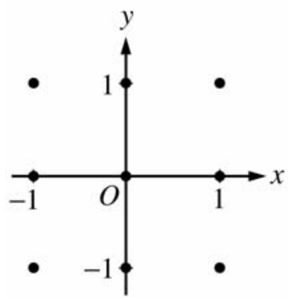
So $c = 4$ to get an average of 15 calories/min.

2 :
- 1 : setup
- 1 : value of c

Question 5

Consider the differential equation $\frac{dy}{dx} = (y-1)^2 \cos(\pi x)$.

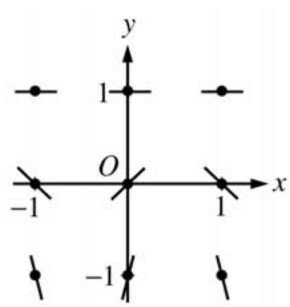
(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.
(Note: Use the axes provided in the exam booklet.)



(b) There is a horizontal line with equation $y = c$ that satisfies this differential equation. Find the value of c.

(c) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = 0$.

(a)



$2 : \begin{cases} 1 : \text{zero slopes} \\ 1 : \text{all other slopes} \end{cases}$

(b) The line $y = 1$ satisfies the differential equation, so $c = 1$.

$1 : c = 1$

(c)

$$\frac{1}{(y-1)^2}\, dy = \cos(\pi x)\, dx$$

$$-(y-1)^{-1} = \frac{1}{\pi}\sin(\pi x) + C$$

$$\frac{1}{1-y} = \frac{1}{\pi}\sin(\pi x) + C$$

$$1 = \frac{1}{\pi}\sin(\pi) + C = C$$

$$\frac{1}{1-y} = \frac{1}{\pi}\sin(\pi x) + 1$$

$$\frac{\pi}{1-y} = \sin(\pi x) + \pi$$

$$y = 1 - \frac{\pi}{\sin(\pi x) + \pi} \quad \text{for } -\infty < x < \infty$$

$6 : \begin{cases} 1 : \text{separates variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$

Note: max $3/6$ [1-2-0-0-0] if no constant of integration
Note: $0/6$ if no separation of variables

Question 6

t (sec)	0	15	25	30	35	50	60
$v(t)$ (ft/sec)	–20	–30	–20	–14	–10	0	10
$a(t)$ $\left(\text{ft}/\text{sec}^2\right)$	1	5	2	1	2	4	2

A car travels on a straight track. During the time interval $0 \le t \le 60$ seconds, the car's velocity v, measured in feet per second, and acceleration a, measured in feet per second per second, are continuous functions. The table above shows selected values of these functions.

(a) Using appropriate units, explain the meaning of $\int_{30}^{60} |v(t)|\, dt$ in terms of the car's motion. Approximate $\int_{30}^{60} |v(t)|\, dt$ using a trapezoidal approximation with the three subintervals determined by the table.

(b) Using appropriate units, explain the meaning of $\int_{0}^{30} a(t)\, dt$ in terms of the car's motion. Find the exact value of $\int_{0}^{30} a(t)\, dt$.

(c) For $0 < t < 60$, must there be a time t when $v(t) = -5$? Justify your answer.

(d) For $0 < t < 60$, must there be a time t when $a(t) = 0$? Justify your answer.

(a) $\int_{30}^{60} |v(t)|\, dt$ is the distance in feet that the car travels from $t = 30$ sec to $t = 60$ sec.

Trapezoidal approximation for $\int_{30}^{60} |v(t)|\, dt$:

$$A = \frac{1}{2}(14 + 10)5 + \frac{1}{2}(10)(15) + \frac{1}{2}(10)(10) = 185 \text{ ft}$$

$2 : \begin{cases} 1 : \text{explanation} \\ 1 : \text{value} \end{cases}$

(b) $\int_{0}^{30} a(t)\, dt$ is the car's change in velocity in ft/sec from $t = 0$ sec to $t = 30$ sec.

$$\int_{0}^{30} a(t)\, dt = \int_{0}^{30} v'(t)\, dt = v(30) - v(0)$$
$$= -14 - (-20) = 6 \text{ ft/sec}$$

$2 : \begin{cases} 1 : \text{explanation} \\ 1 : \text{value} \end{cases}$

(c) Yes. Since $v(35) = -10 < -5 < 0 = v(50)$, the IVT guarantees a t in $(35, 50)$ so that $v(t) = -5$.

$2 : \begin{cases} 1 : v(35) < -5 < v(50) \\ 1 : \text{Yes; refers to IVT or hypotheses} \end{cases}$

(d) Yes. Since $v(0) = v(25)$, the MVT guarantees a t in $(0, 25)$ so that $a(t) = v'(t) = 0$.

$2 : \begin{cases} 1 : v(0) = v(25) \\ 1 : \text{Yes; refers to MVT or hypotheses} \end{cases}$

Units of ft in (a) and ft/sec in (b)

1 : units in (a) and (b)