



AP[®] Calculus AB 2006 Free-Response Questions

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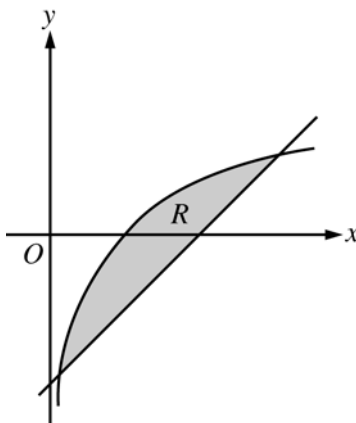
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2006 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

**CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3**

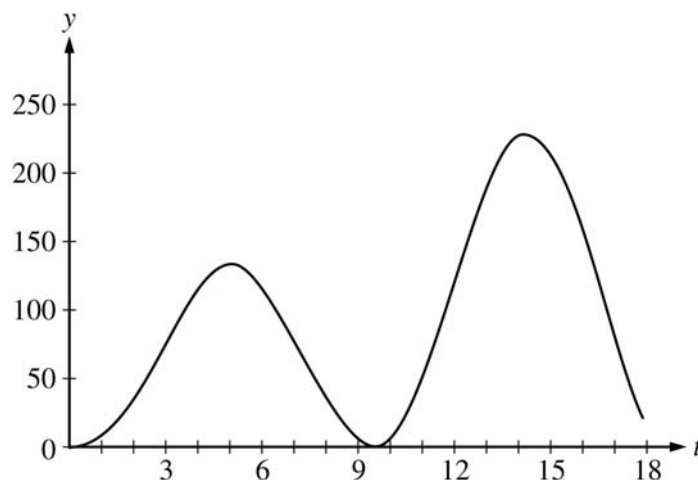
A graphing calculator is required for some problems or parts of problems.



1. Let R be the shaded region bounded by the graph of $y = \ln x$ and the line $y = x - 2$, as shown above.
 - (a) Find the area of R .
 - (b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -3$.
 - (c) Write, but do not evaluate, an integral expression that can be used to find the volume of the solid generated when R is rotated about the y -axis.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

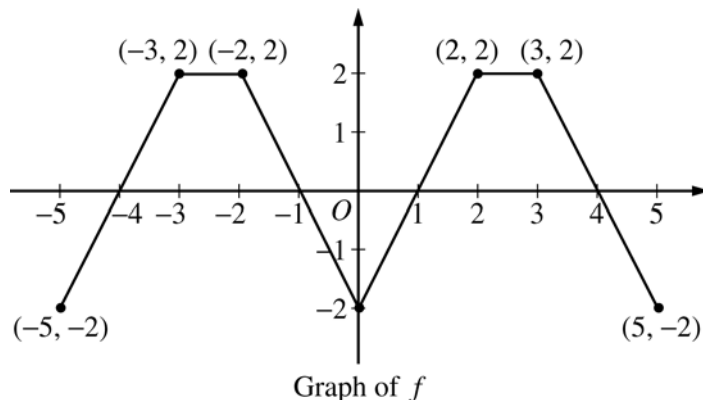
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2. At an intersection in Thomasville, Oregon, cars turn left at the rate $L(t) = 60\sqrt{t} \sin^2\left(\frac{t}{3}\right)$ cars per hour over the time interval $0 \leq t \leq 18$ hours. The graph of $y = L(t)$ is shown above.
- To the nearest whole number, find the total number of cars turning left at the intersection over the time interval $0 \leq t \leq 18$ hours.
 - Traffic engineers will consider turn restrictions when $L(t) \geq 150$ cars per hour. Find all values of t for which $L(t) \geq 150$ and compute the average value of L over this time interval. Indicate units of measure.
 - Traffic engineers will install a signal if there is any two-hour time interval during which the product of the total number of cars turning left and the total number of oncoming cars traveling straight through the intersection is greater than 200,000. In every two-hour time interval, 500 oncoming cars travel straight through the intersection. Does this intersection require a traffic signal? Explain the reasoning that leads to your conclusion.

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3. The graph of the function f shown above consists of six line segments. Let g be the function given by $g(x) = \int_0^x f(t) dt$.
- Find $g(4)$, $g'(4)$, and $g''(4)$.
 - Does g have a relative minimum, a relative maximum, or neither at $x = 1$? Justify your answer.
 - Suppose that f is defined for all real numbers x and is periodic with a period of length 5. The graph above shows two periods of f . Given that $g(5) = 2$, find $g(10)$ and write an equation for the line tangent to the graph of g at $x = 108$.

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END OF PART A OF SECTION II

CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

t (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

4. Rocket A has positive velocity $v(t)$ after being launched upward from an initial height of 0 feet at time $t = 0$ seconds. The velocity of the rocket is recorded for selected values of t over the interval $0 \leq t \leq 80$ seconds, as shown in the table above.
- (a) Find the average acceleration of rocket A over the time interval $0 \leq t \leq 80$ seconds. Indicate units of measure.
- (b) Using correct units, explain the meaning of $\int_{10}^{70} v(t) dt$ in terms of the rocket's flight. Use a midpoint Riemann sum with 3 subintervals of equal length to approximate $\int_{10}^{70} v(t) dt$.
- (c) Rocket B is launched upward with an acceleration of $a(t) = \frac{3}{\sqrt{t+1}}$ feet per second per second. At time $t = 0$ seconds, the initial height of the rocket is 0 feet, and the initial velocity is 2 feet per second. Which of the two rockets is traveling faster at time $t = 80$ seconds? Explain your answer.

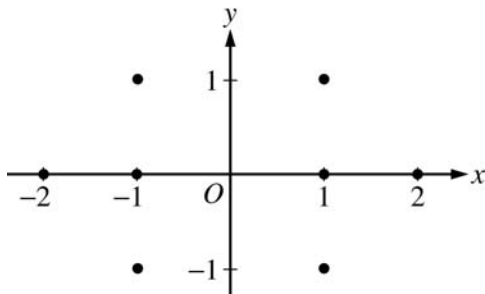
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5. Consider the differential equation $\frac{dy}{dx} = \frac{1+y}{x}$, where $x \neq 0$.

(a) On the axes provided, sketch a slope field for the given differential equation at the eight points indicated.

(Note: Use the axes provided in the pink exam booklet.)



(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(-1) = 1$ and state its domain.

6. The twice-differentiable function f is defined for all real numbers and satisfies the following conditions:

$$f(0) = 2, \quad f'(0) = -4, \quad \text{and} \quad f''(0) = 3.$$

(a) The function g is given by $g(x) = e^{ax} + f(x)$ for all real numbers, where a is a constant. Find $g'(0)$ and $g''(0)$ in terms of a . Show the work that leads to your answers.

(b) The function h is given by $h(x) = \cos(kx)f(x)$ for all real numbers, where k is a constant. Find $h'(x)$ and write an equation for the line tangent to the graph of h at $x = 0$.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

END OF EXAM