



AP[®] Calculus AB 2007 Scoring Guidelines Form B

The College Board: Connecting Students to College Success

The College Board is a not-for-profit membership association whose mission is to connect students to college success and opportunity. Founded in 1900, the association is composed of more than 5,000 schools, colleges, universities, and other educational organizations. Each year, the College Board serves seven million students and their parents, 23,000 high schools, and 3,500 colleges through major programs and services in college admissions, guidance, assessment, financial aid, enrollment, and teaching and learning. Among its best-known programs are the SAT[®], the PSAT/NMSQT[®], and the Advanced Placement Program[®] (AP[®]). The College Board is committed to the principles of excellence and equity, and that commitment is embodied in all of its programs, services, activities, and concerns.

© 2007 The College Board. All rights reserved. College Board, Advanced Placement Program, AP, AP Central, SAT, and the acorn logo are registered trademarks of the College Board. PSAT/NMSQT is a registered trademark of the College Board and National Merit Scholarship Corporation.

Permission to use copyrighted College Board materials may be requested online at:
www.collegeboard.com/inquiry/cbpermit.html.

Visit the College Board on the Web: www.collegeboard.com.

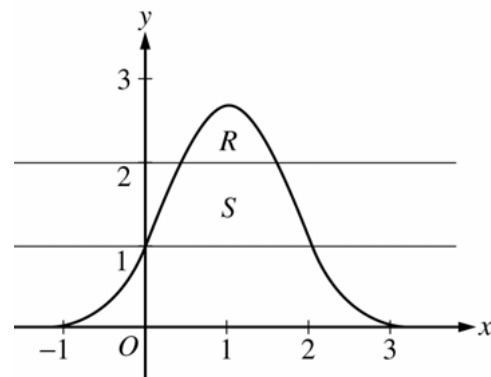
AP Central is the official online home for the AP Program: apcentral.collegeboard.com.

AP[®] CALCULUS AB
2007 SCORING GUIDELINES (Form B)

Question 1

Let R be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal line $y = 2$, and let S be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal lines $y = 1$ and $y = 2$, as shown above.

- (a) Find the area of R .
 (b) Find the area of S .
 (c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 1$.



$e^{2x-x^2} = 2$ when $x = 0.446057, 1.553943$
 Let $P = 0.446057$ and $Q = 1.553943$

(a) Area of $R = \int_P^Q (e^{2x-x^2} - 2) dx = 0.514$

3 : { 1 : integrand
 1 : limits
 1 : answer

(b) $e^{2x-x^2} = 1$ when $x = 0, 2$

Area of $S = \int_0^2 (e^{2x-x^2} - 1) dx - \text{Area of } R$
 $= 2.06016 - \text{Area of } R = 1.546$

3 : { 1 : integrand
 1 : limits
 1 : answer

OR

$\int_0^P (e^{2x-x^2} - 1) dx + (Q - P) \cdot 1 + \int_Q^2 (e^{2x-x^2} - 1) dx$
 $= 0.219064 + 1.107886 + 0.219064 = 1.546$

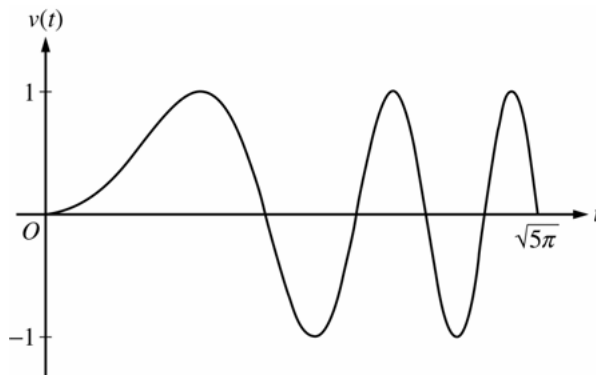
(c) Volume = $\pi \int_P^Q \left((e^{2x-x^2} - 1)^2 - (2 - 1)^2 \right) dx$

3 : { 2 : integrand
 1 : constant and limits

AP[®] CALCULUS AB
2007 SCORING GUIDELINES (Form B)

Question 2

A particle moves along the x -axis so that its velocity v at time $t \geq 0$ is given by $v(t) = \sin(t^2)$. The graph of v is shown above for $0 \leq t \leq \sqrt{5\pi}$. The position of the particle at time t is $x(t)$ and its position at time $t = 0$ is $x(0) = 5$.



- Find the acceleration of the particle at time $t = 3$.
- Find the total distance traveled by the particle from time $t = 0$ to $t = 3$.
- Find the position of the particle at time $t = 3$.
- For $0 \leq t \leq \sqrt{5\pi}$, find the time t at which the particle is farthest to the right. Explain your answer.

(a) $a(3) = v'(3) = 6\cos 9 = -5.466$ or -5.467

1 : $a(3)$

(b) Distance = $\int_0^3 |v(t)| dt = 1.702$

OR

For $0 < t < 3$, $v(t) = 0$ when $t = \sqrt{\pi} = 1.77245$ and

$t = \sqrt{2\pi} = 2.50663$

$x(0) = 5$

$x(\sqrt{\pi}) = 5 + \int_0^{\sqrt{\pi}} v(t) dt = 5.89483$

$x(\sqrt{2\pi}) = 5 + \int_0^{\sqrt{2\pi}} v(t) dt = 5.43041$

$x(3) = 5 + \int_0^3 v(t) dt = 5.77356$

$|x(\sqrt{\pi}) - x(0)| + |x(\sqrt{2\pi}) - x(\sqrt{\pi})| + |x(3) - x(\sqrt{2\pi})| = 1.702$

2 : $\begin{cases} 1 : \text{setup} \\ 1 : \text{answer} \end{cases}$

(c) $x(3) = 5 + \int_0^3 v(t) dt = 5.773$ or 5.774

3 : $\begin{cases} 2 \begin{cases} 1 : \text{integrand} \\ 1 : \text{uses } x(0) = 5 \end{cases} \\ 1 : \text{answer} \end{cases}$

(d) The particle's rightmost position occurs at time $t = \sqrt{\pi} = 1.772$.

The particle changes from moving right to moving left at those times t for which $v(t) = 0$ with $v(t)$ changing from positive to negative, namely at $t = \sqrt{\pi}, \sqrt{3\pi}, \sqrt{5\pi}$ ($t = 1.772, 3.070, 3.963$).

Using $x(T) = 5 + \int_0^T v(t) dt$, the particle's positions at the times it changes from rightward to leftward movement are:

$T:$ 0 $\sqrt{\pi}$ $\sqrt{3\pi}$ $\sqrt{5\pi}$

$x(T):$ 5 5.895 5.788 5.752

The particle is farthest to the right when $T = \sqrt{\pi}$.

3 : $\begin{cases} 1 : \text{sets } v(t) = 0 \\ 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$

AP[®] CALCULUS AB
2007 SCORING GUIDELINES (Form B)

Question 3

The wind chill is the temperature, in degrees Fahrenheit ($^{\circ}\text{F}$), a human feels based on the air temperature, in degrees Fahrenheit, and the wind velocity v , in miles per hour (mph). If the air temperature is 32°F , then the wind chill is given by $W(v) = 55.6 - 22.1v^{0.16}$ and is valid for $5 \leq v \leq 60$.

- (a) Find $W'(20)$. Using correct units, explain the meaning of $W'(20)$ in terms of the wind chill.
- (b) Find the average rate of change of W over the interval $5 \leq v \leq 60$. Find the value of v at which the instantaneous rate of change of W is equal to the average rate of change of W over the interval $5 \leq v \leq 60$.
- (c) Over the time interval $0 \leq t \leq 4$ hours, the air temperature is a constant 32°F . At time $t = 0$, the wind velocity is $v = 20$ mph. If the wind velocity increases at a constant rate of 5 mph per hour, what is the rate of change of the wind chill with respect to time at $t = 3$ hours? Indicate units of measure.

(a) $W'(20) = -22.1 \cdot 0.16 \cdot 20^{-0.84} = -0.285$ or -0.286

When $v = 20$ mph, the wind chill is decreasing at $0.286^{\circ}\text{F}/\text{mph}$.

(b) The average rate of change of W over the interval $5 \leq v \leq 60$ is $\frac{W(60) - W(5)}{60 - 5} = -0.253$ or -0.254 .
 $W'(v) = \frac{W(60) - W(5)}{60 - 5}$ when $v = 23.011$.

(c) $\left. \frac{dW}{dt} \right|_{t=3} = \left(\frac{dW}{dv} \cdot \frac{dv}{dt} \right) \Big|_{t=3} = W'(35) \cdot 5 = -0.892^{\circ}\text{F}/\text{hr}$

OR

$$W = 55.6 - 22.1(20 + 5t)^{0.16}$$

$$\left. \frac{dW}{dt} \right|_{t=3} = -0.892^{\circ}\text{F}/\text{hr}$$

Units of $^{\circ}\text{F}/\text{mph}$ in (a) and $^{\circ}\text{F}/\text{hr}$ in (c)

$$2 : \begin{cases} 1 : \text{value} \\ 1 : \text{explanation} \end{cases}$$

$$3 : \begin{cases} 1 : \text{average rate of change} \\ 1 : W'(v) = \text{average rate of change} \\ 1 : \text{value of } v \end{cases}$$

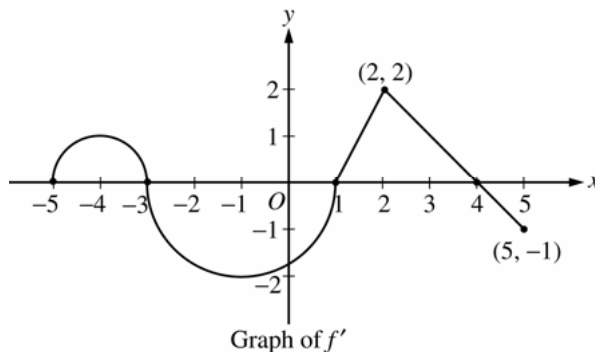
$$3 : \begin{cases} 1 : \frac{dv}{dt} = 5 \\ 1 : \text{uses } v(3) = 35, \\ \text{or} \\ \text{uses } v(t) = 20 + 5t \\ 1 : \text{answer} \end{cases}$$

1 : units in (a) and (c)

AP[®] CALCULUS AB
2007 SCORING GUIDELINES (Form B)

Question 4

Let f be a function defined on the closed interval $-5 \leq x \leq 5$ with $f(1) = 3$. The graph of f' , the derivative of f , consists of two semicircles and two line segments, as shown above.



- (a) For $-5 < x < 5$, find all values x at which f has a relative maximum. Justify your answer.
- (b) For $-5 < x < 5$, find all values x at which the graph of f has a point of inflection. Justify your answer.
- (c) Find all intervals on which the graph of f is concave up and also has positive slope. Explain your reasoning.
- (d) Find the absolute minimum value of $f(x)$ over the closed interval $-5 \leq x \leq 5$. Explain your reasoning.

- (a) $f'(x) = 0$ at $x = -3, 1, 4$
 f' changes from positive to negative at -3 and 4 .
 Thus, f has a relative maximum at $x = -3$ and at $x = 4$.

2 : $\left\{ \begin{array}{l} 1 : x\text{-values} \\ 1 : \text{justification} \end{array} \right.$

- (b) f' changes from increasing to decreasing, or vice versa, at $x = -4, -1$, and 2 . Thus, the graph of f has points of inflection when $x = -4, -1$, and 2 .

2 : $\left\{ \begin{array}{l} 1 : x\text{-values} \\ 1 : \text{justification} \end{array} \right.$

- (c) The graph of f is concave up with positive slope where f' is increasing and positive: $-5 < x < -4$ and $1 < x < 2$.

2 : $\left\{ \begin{array}{l} 1 : \text{intervals} \\ 1 : \text{explanation} \end{array} \right.$

- (d) Candidates for the absolute minimum are where f' changes from negative to positive (at $x = 1$) and at the endpoints ($x = -5, 5$).

3 : $\left\{ \begin{array}{l} 1 : \text{identifies } x = 1 \text{ as a candidate} \\ 1 : \text{considers endpoints} \\ 1 : \text{value and explanation} \end{array} \right.$

$$f(-5) = 3 + \int_1^{-5} f'(x) dx = 3 - \frac{\pi}{2} + 2\pi > 3$$

$$f(1) = 3$$

$$f(5) = 3 + \int_1^5 f'(x) dx = 3 + \frac{3 \cdot 2}{2} - \frac{1}{2} > 3$$

The absolute minimum value of f on $[-5, 5]$ is $f(1) = 3$.

AP[®] CALCULUS AB
2007 SCORING GUIDELINES (Form B)

Question 5

Consider the differential equation $\frac{dy}{dx} = \frac{1}{2}x + y - 1$.

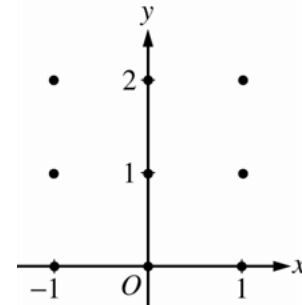
- (a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

(Note: Use the axes provided in the exam booklet.)

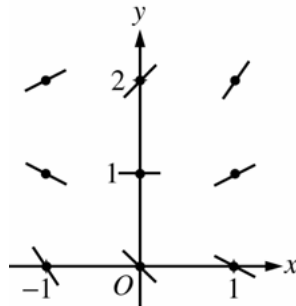
- (b) Find $\frac{d^2y}{dx^2}$ in terms of x and y . Describe the region in the xy -plane in which all solution curves to the differential equation are concave up.

- (c) Let $y = f(x)$ be a particular solution to the differential equation with the initial condition $f(0) = 1$. Does f have a relative minimum, a relative maximum, or neither at $x = 0$? Justify your answer.

- (d) Find the values of the constants m and b , for which $y = mx + b$ is a solution to the differential equation.



(a)



(b) $\frac{d^2y}{dx^2} = \frac{1}{2} + \frac{dy}{dx} = \frac{1}{2}x + y - \frac{1}{2}$

Solution curves will be concave up on the half-plane above the line

$$y = -\frac{1}{2}x + \frac{1}{2}.$$

(c) $\left. \frac{dy}{dx} \right|_{(0,1)} = 0 + 1 - 1 = 0$ and $\left. \frac{d^2y}{dx^2} \right|_{(0,1)} = 0 + 1 - \frac{1}{2} > 0$

Thus, f has a relative minimum at $(0, 1)$.

- (d) Substituting $y = mx + b$ into the differential equation:

$$m = \frac{1}{2}x + (mx + b) - 1 = \left(m + \frac{1}{2}\right)x + (b - 1)$$

Then $0 = m + \frac{1}{2}$ and $m = b - 1$: $m = -\frac{1}{2}$ and $b = \frac{1}{2}$.

2 : Sign of slope at each point and relative steepness of slope lines in rows and columns.

3 : $\begin{cases} 2 : \frac{d^2y}{dx^2} \\ 1 : \text{description} \end{cases}$

2 : $\begin{cases} 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

2 : $\begin{cases} 1 : \text{value for } m \\ 1 : \text{value for } b \end{cases}$

AP[®] CALCULUS AB
2007 SCORING GUIDELINES (Form B)

Question 6

Let f be a twice-differentiable function such that $f(2) = 5$ and $f(5) = 2$. Let g be the function given by $g(x) = f(f(x))$.

- (a) Explain why there must be a value c for $2 < c < 5$ such that $f'(c) = -1$.
- (b) Show that $g'(2) = g'(5)$. Use this result to explain why there must be a value k for $2 < k < 5$ such that $g''(k) = 0$.
- (c) Show that if $f''(x) = 0$ for all x , then the graph of g does not have a point of inflection.
- (d) Let $h(x) = f(x) - x$. Explain why there must be a value r for $2 < r < 5$ such that $h(r) = 0$.

- (a) The Mean Value Theorem guarantees that there is a value c , with $2 < c < 5$, so that

$$f'(c) = \frac{f(5) - f(2)}{5 - 2} = \frac{2 - 5}{5 - 2} = -1.$$

$$2 : \begin{cases} 1 : \frac{f(5) - f(2)}{5 - 2} \\ 1 : \text{conclusion, using MVT} \end{cases}$$

- (b) $g'(x) = f'(f(x)) \cdot f'(x)$
 $g'(2) = f'(f(2)) \cdot f'(2) = f'(5) \cdot f'(2)$
 $g'(5) = f'(f(5)) \cdot f'(5) = f'(2) \cdot f'(5)$
 Thus, $g'(2) = g'(5)$.

$$3 : \begin{cases} 1 : g'(x) \\ 1 : g'(2) = f'(5) \cdot f'(2) = g'(5) \\ 1 : \text{uses MVT with } g' \end{cases}$$

Since f is twice-differentiable, g' is differentiable everywhere, so the Mean Value Theorem applied to g' on $[2, 5]$ guarantees there is a value k , with $2 < k < 5$, such that $g''(k) = \frac{g'(5) - g'(2)}{5 - 2} = 0$.

- (c) $g''(x) = f''(f(x)) \cdot f'(x) \cdot f'(x) + f'(f(x)) \cdot f''(x)$
 If $f''(x) = 0$ for all x , then
 $g''(x) = 0 \cdot f'(x) \cdot f'(x) + f'(f(x)) \cdot 0 = 0$ for all x .
 Thus, there is no x -value at which $g''(x)$ changes sign, so the graph of g has no inflection points.

$$2 : \begin{cases} 1 : \text{considers } g'' \\ 1 : g''(x) = 0 \text{ for all } x \end{cases}$$

OR

If $f''(x) = 0$ for all x , then f is linear, so $g = f \circ f$ is linear and the graph of g has no inflection points.

OR

$$2 : \begin{cases} 1 : f \text{ is linear} \\ 1 : g \text{ is linear} \end{cases}$$

- (d) Let $h(x) = f(x) - x$.
 $h(2) = f(2) - 2 = 5 - 2 = 3$
 $h(5) = f(5) - 5 = 2 - 5 = -3$
 Since $h(2) > 0 > h(5)$, the Intermediate Value Theorem guarantees that there is a value r , with $2 < r < 5$, such that $h(r) = 0$.

$$2 : \begin{cases} 1 : h(2) \text{ and } h(5) \\ 1 : \text{conclusion, using IVT} \end{cases}$$