AP® Calculus AB
2007 Free-Response Questions
Form B

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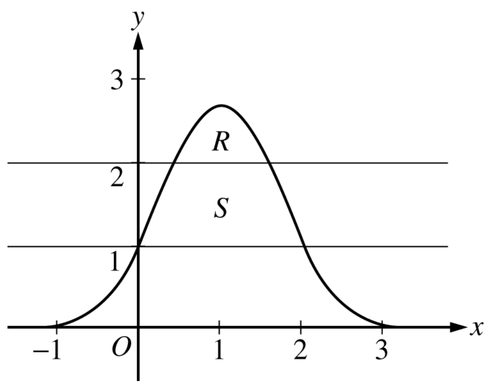
CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



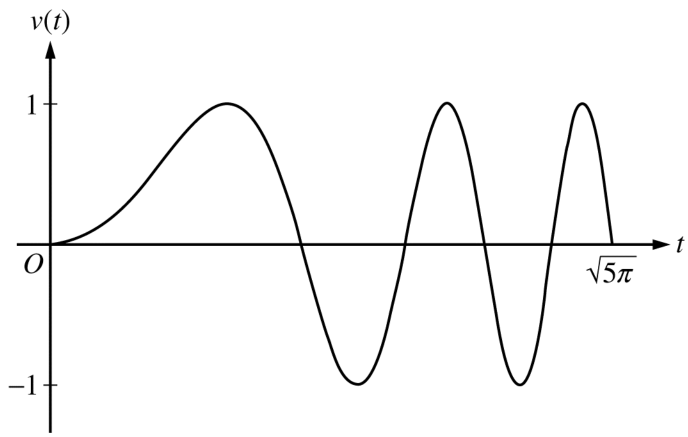
1. Let R be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal line $y = 2$, and let S be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal lines $y = 1$ and $y = 2$, as shown above.

   (a) Find the area of R.

   (b) Find the area of S.

   (c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 1$.

WRITE ALL WORK IN THE EXAM BOOKLET.



2. A particle moves along the x-axis so that its velocity v at time $t \geq 0$ is given by $v(t) = \sin\left(t^2\right)$. The graph of v is shown above for $0 \leq t \leq \sqrt{5\pi}$. The position of the particle at time t is $x(t)$ and its position at time $t = 0$ is $x(0) = 5$.

(a) Find the acceleration of the particle at time $t = 3$.

(b) Find the total distance traveled by the particle from time $t = 0$ to $t = 3$.

(c) Find the position of the particle at time $t = 3$.

(d) For $0 \leq t \leq \sqrt{5\pi}$, find the time t at which the particle is farthest to the right. Explain your answer.

WRITE ALL WORK IN THE EXAM BOOKLET.

3. The wind chill is the temperature, in degrees Fahrenheit $(°\text{F})$, a human feels based on the air temperature, in degrees Fahrenheit, and the wind velocity v, in miles per hour (mph). If the air temperature is $32°\text{F}$, then the wind chill is given by $W(v) = 55.6 - 22.1v^{0.16}$ and is valid for $5 \le v \le 60$.

   (a) Find $W'(20)$. Using correct units, explain the meaning of $W'(20)$ in terms of the wind chill.

   (b) Find the average rate of change of W over the interval $5 \le v \le 60$. Find the value of v at which the instantaneous rate of change of W is equal to the average rate of change of W over the interval $5 \le v \le 60$.

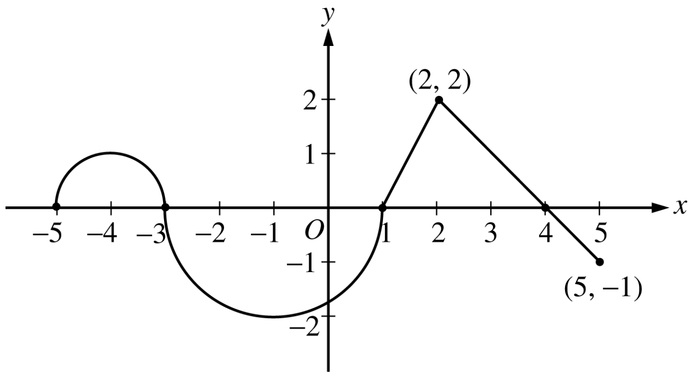
   (c) Over the time interval $0 \le t \le 4$ hours, the air temperature is a constant $32°\text{F}$. At time $t = 0$, the wind velocity is $v = 20$ mph. If the wind velocity increases at a constant rate of 5 mph per hour, what is the rate of change of the wind chill with respect to time at $t = 3$ hours? Indicate units of measure.

WRITE ALL WORK IN THE EXAM BOOKLET.

END OF PART A OF SECTION II

CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



Graph of f'

4. Let f be a function defined on the closed interval $-5 \le x \le 5$ with $f(1) = 3$. The graph of f', the derivative of f, consists of two semicircles and two line segments, as shown above.

   (a) For $-5 < x < 5$, find all values x at which f has a relative maximum. Justify your answer.

   (b) For $-5 < x < 5$, find all values x at which the graph of f has a point of inflection. Justify your answer.

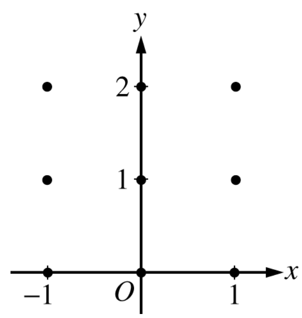
   (c) Find all intervals on which the graph of f is concave up and also has positive slope. Explain your reasoning.

   (d) Find the absolute minimum value of $f(x)$ over the closed interval $-5 \le x \le 5$. Explain your reasoning.

WRITE ALL WORK IN THE EXAM BOOKLET.

5. Consider the differential equation $\frac{dy}{dx} = \frac{1}{2}x + y - 1$.

   (a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.
   (Note: Use the axes provided in the exam booklet.)



   (b) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Describe the region in the xy-plane in which all solution curves to the differential equation are concave up.

   (c) Let $y = f(x)$ be a particular solution to the differential equation with the initial condition $f(0) = 1$. Does f have a relative minimum, a relative maximum, or neither at $x = 0$? Justify your answer.

   (d) Find the values of the constants m and b, for which $y = mx + b$ is a solution to the differential equation.

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6. Let f be a twice-differentiable function such that $f(2) = 5$ and $f(5) = 2$. Let g be the function given by $g(x) = f(f(x))$.

   (a) Explain why there must be a value c for $2 < c < 5$ such that $f'(c) = -1$.

   (b) Show that $g'(2) = g'(5)$. Use this result to explain why there must be a value k for $2 < k < 5$ such that $g''(k) = 0$.

   (c) Show that if $f''(x) = 0$ for all x, then the graph of g does not have a point of inflection.

   (d) Let $h(x) = f(x) - x$. Explain why there must be a value r for $2 < r < 5$ such that $h(r) = 0$.

WRITE ALL WORK IN THE EXAM BOOKLET.

END OF EXAM