



## **AP<sup>®</sup> Calculus AB 2008 Free-Response Questions**

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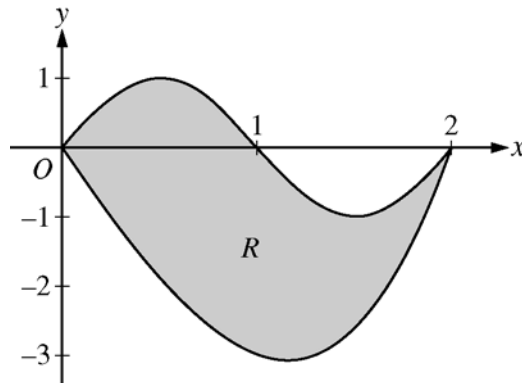
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**CALCULUS AB**  
**SECTION II, Part A**  
**Time—45 minutes**  
**Number of problems—3**

A graphing calculator is required for some problems or parts of problems.



1. Let  $R$  be the region bounded by the graphs of  $y = \sin(\pi x)$  and  $y = x^3 - 4x$ , as shown in the figure above.
- Find the area of  $R$ .
  - The horizontal line  $y = -2$  splits the region  $R$  into two parts. Write, but do not evaluate, an integral expression for the area of the part of  $R$  that is below this horizontal line.
  - The region  $R$  is the base of a solid. For this solid, each cross section perpendicular to the  $x$ -axis is a square. Find the volume of this solid.
  - The region  $R$  models the surface of a small pond. At all points in  $R$  at a distance  $x$  from the  $y$ -axis, the depth of the water is given by  $h(x) = 3 - x$ . Find the volume of water in the pond.
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**WRITE ALL WORK IN THE PINK EXAM BOOKLET.**

$t$ (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

2. Concert tickets went on sale at noon ( $t = 0$ ) and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time  $t$  is modeled by a twice-differentiable function  $L$  for  $0 \leq t \leq 9$ . Values of  $L(t)$  at various times  $t$  are shown in the table above.
- (a) Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. ( $t = 5.5$ ). Show the computations that lead to your answer. Indicate units of measure.
- (b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.
- (c) For  $0 \leq t \leq 9$ , what is the fewest number of times at which  $L'(t)$  must equal 0? Give a reason for your answer.
- (d) The rate at which tickets were sold for  $0 \leq t \leq 9$  is modeled by  $r(t) = 550te^{-t/2}$  tickets per hour. Based on the model, how many tickets were sold by 3 P.M. ( $t = 3$ ), to the nearest whole number?
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3. Oil is leaking from a pipeline on the surface of a lake and forms an oil slick whose volume increases at a constant rate of 2000 cubic centimeters per minute. The oil slick takes the form of a right circular cylinder with both its radius and height changing with time. (Note: The volume  $V$  of a right circular cylinder with radius  $r$  and height  $h$  is given by  $V = \pi r^2 h$ .)
- (a) At the instant when the radius of the oil slick is 100 centimeters and the height is 0.5 centimeter, the radius is increasing at the rate of 2.5 centimeters per minute. At this instant, what is the rate of change of the height of the oil slick with respect to time, in centimeters per minute?
- (b) A recovery device arrives on the scene and begins removing oil. The rate at which oil is removed is  $R(t) = 400\sqrt{t}$  cubic centimeters per minute, where  $t$  is the time in minutes since the device began working. Oil continues to leak at the rate of 2000 cubic centimeters per minute. Find the time  $t$  when the oil slick reaches its maximum volume. Justify your answer.
- (c) By the time the recovery device began removing oil, 60,000 cubic centimeters of oil had already leaked. Write, but do not evaluate, an expression involving an integral that gives the volume of oil at the time found in part (b).
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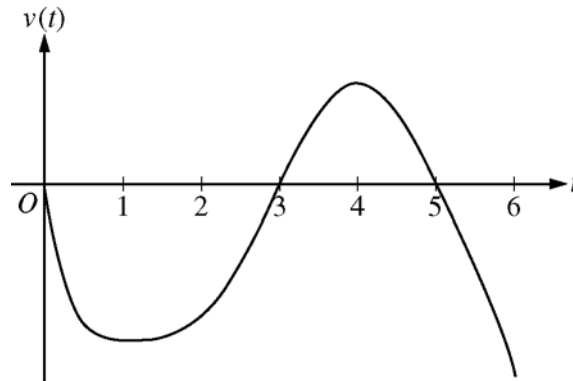
**END OF PART A OF SECTION II**

**CALCULUS AB**  
**SECTION II, Part B**

**Time—45 minutes**

**Number of problems—3**

**No calculator is allowed for these problems.**



Graph of  $v$

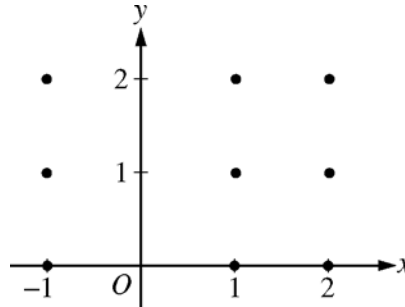
4. A particle moves along the  $x$ -axis so that its velocity at time  $t$ , for  $0 \leq t \leq 6$ , is given by a differentiable function  $v$  whose graph is shown above. The velocity is 0 at  $t = 0$ ,  $t = 3$ , and  $t = 5$ , and the graph has horizontal tangents at  $t = 1$  and  $t = 4$ . The areas of the regions bounded by the  $t$ -axis and the graph of  $v$  on the intervals  $[0, 3]$ ,  $[3, 5]$ , and  $[5, 6]$  are 8, 3, and 2, respectively. At time  $t = 0$ , the particle is at  $x = -2$ .
- For  $0 \leq t \leq 6$ , find both the time and the position of the particle when the particle is farthest to the left. Justify your answer.
  - For how many values of  $t$ , where  $0 \leq t \leq 6$ , is the particle at  $x = -8$ ? Explain your reasoning.
  - On the interval  $2 < t < 3$ , is the speed of the particle increasing or decreasing? Give a reason for your answer.
  - During what time intervals, if any, is the acceleration of the particle negative? Justify your answer.

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5. Consider the differential equation  $\frac{dy}{dx} = \frac{y-1}{x^2}$ , where  $x \neq 0$ .

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

(Note: Use the axes provided in the exam booklet.)



(b) Find the particular solution  $y = f(x)$  to the differential equation with the initial condition  $f(2) = 0$ .

(c) For the particular solution  $y = f(x)$  described in part (b), find  $\lim_{x \rightarrow \infty} f(x)$ .

6. Let  $f$  be the function given by  $f(x) = \frac{\ln x}{x}$  for all  $x > 0$ . The derivative of  $f$  is given by  $f'(x) = \frac{1 - \ln x}{x^2}$ .

(a) Write an equation for the line tangent to the graph of  $f$  at  $x = e^2$ .

(b) Find the  $x$ -coordinate of the critical point of  $f$ . Determine whether this point is a relative minimum, a relative maximum, or neither for the function  $f$ . Justify your answer.

(c) The graph of the function  $f$  has exactly one point of inflection. Find the  $x$ -coordinate of this point.

(d) Find  $\lim_{x \rightarrow 0^+} f(x)$ .

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**END OF EXAM**