



## **AP<sup>®</sup> Calculus AB 2010 Free-Response Questions**

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**CALCULUS AB**  
**SECTION II, Part A**  
**Time—45 minutes**  
**Number of problems—3**

**A graphing calculator is required for some problems or parts of problems.**

1. There is no snow on Janet's driveway when snow begins to fall at midnight. From midnight to 9 A.M., snow accumulates on the driveway at a rate modeled by  $f(t) = 7te^{\cos t}$  cubic feet per hour, where  $t$  is measured in hours since midnight. Janet starts removing snow at 6 A.M. ( $t = 6$ ). The rate  $g(t)$ , in cubic feet per hour, at which Janet removes snow from the driveway at time  $t$  hours after midnight is modeled by

$$g(t) = \begin{cases} 0 & \text{for } 0 \leq t < 6 \\ 125 & \text{for } 6 \leq t < 7 \\ 108 & \text{for } 7 \leq t \leq 9. \end{cases}$$

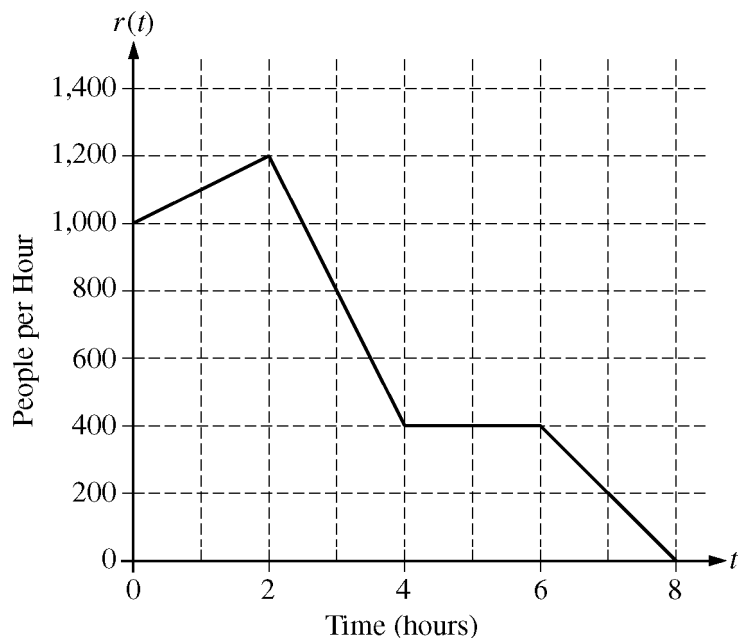
- (a) How many cubic feet of snow have accumulated on the driveway by 6 A.M.?
- (b) Find the rate of change of the volume of snow on the driveway at 8 A.M.
- (c) Let  $h(t)$  represent the total amount of snow, in cubic feet, that Janet has removed from the driveway at time  $t$  hours after midnight. Express  $h$  as a piecewise-defined function with domain  $0 \leq t \leq 9$ .
- (d) How many cubic feet of snow are on the driveway at 9 A.M.?
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$t$ (hours)	0	2	5	7	8
$E(t)$ (hundreds of entries)	0	4	13	21	23

2. A zoo sponsored a one-day contest to name a new baby elephant. Zoo visitors deposited entries in a special box between noon ( $t = 0$ ) and 8 P.M. ( $t = 8$ ). The number of entries in the box  $t$  hours after noon is modeled by a differentiable function  $E$  for  $0 \leq t \leq 8$ . Values of  $E(t)$ , in hundreds of entries, at various times  $t$  are shown in the table above.
- (a) Use the data in the table to approximate the rate, in hundreds of entries per hour, at which entries were being deposited at time  $t = 6$ . Show the computations that lead to your answer.
- (b) Use a trapezoidal sum with the four subintervals given by the table to approximate the value of  $\frac{1}{8} \int_0^8 E(t) dt$ .  
Using correct units, explain the meaning of  $\frac{1}{8} \int_0^8 E(t) dt$  in terms of the number of entries.
- (c) At 8 P.M., volunteers began to process the entries. They processed the entries at a rate modeled by the function  $P$ , where  $P(t) = t^3 - 30t^2 + 298t - 976$  hundreds of entries per hour for  $8 \leq t \leq 12$ . According to the model, how many entries had not yet been processed by midnight ( $t = 12$ )?
- (d) According to the model from part (c), at what time were the entries being processed most quickly? Justify your answer.

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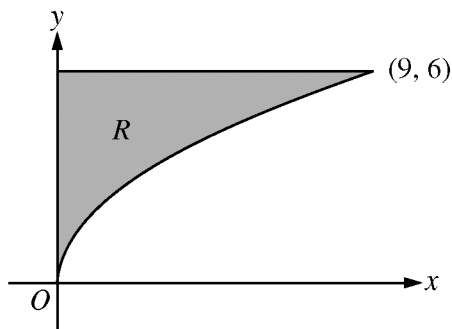
3. There are 700 people in line for a popular amusement-park ride when the ride begins operation in the morning. Once it begins operation, the ride accepts passengers until the park closes 8 hours later. While there is a line, people move onto the ride at a rate of 800 people per hour. The graph above shows the rate,  $r(t)$ , at which people arrive at the ride throughout the day. Time  $t$  is measured in hours from the time the ride begins operation.
- How many people arrive at the ride between  $t = 0$  and  $t = 3$ ? Show the computations that lead to your answer.
  - Is the number of people waiting in line to get on the ride increasing or decreasing between  $t = 2$  and  $t = 3$ ? Justify your answer.
  - At what time  $t$  is the line for the ride the longest? How many people are in line at that time? Justify your answers.
  - Write, but do not solve, an equation involving an integral expression of  $r$  whose solution gives the earliest time  $t$  at which there is no longer a line for the ride.

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**END OF PART A OF SECTION II**

**CALCULUS AB**  
**SECTION II, Part B**  
**Time—45 minutes**  
**Number of problems—3**

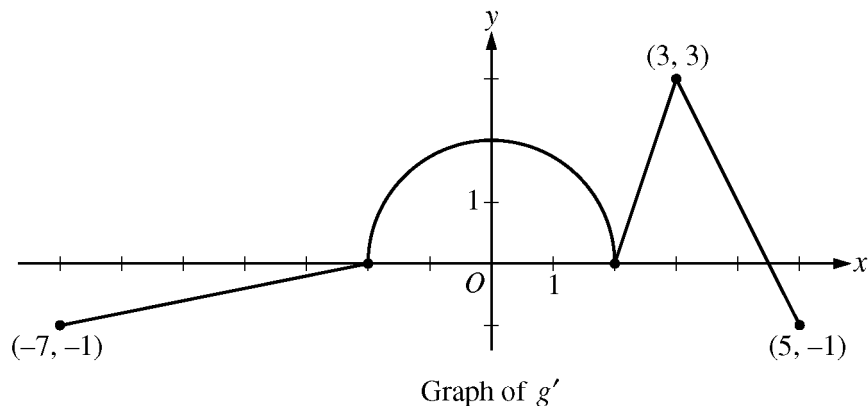
No calculator is allowed for these problems.



4. Let  $R$  be the region in the first quadrant bounded by the graph of  $y = 2\sqrt{x}$ , the horizontal line  $y = 6$ , and the  $y$ -axis, as shown in the figure above.
- (a) Find the area of  $R$ .
- (b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when  $R$  is rotated about the horizontal line  $y = 7$ .
- (c) Region  $R$  is the base of a solid. For each  $y$ , where  $0 \leq y \leq 6$ , the cross section of the solid taken perpendicular to the  $y$ -axis is a rectangle whose height is 3 times the length of its base in region  $R$ . Write, but do not evaluate, an integral expression that gives the volume of the solid.

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5. The function  $g$  is defined and differentiable on the closed interval  $[-7, 5]$  and satisfies  $g(0) = 5$ . The graph of  $y = g'(x)$ , the derivative of  $g$ , consists of a semicircle and three line segments, as shown in the figure above.
- Find  $g(3)$  and  $g(-2)$ .
  - Find the  $x$ -coordinate of each point of inflection of the graph of  $y = g(x)$  on the interval  $-7 < x < 5$ . Explain your reasoning.
  - The function  $h$  is defined by  $h(x) = g(x) - \frac{1}{2}x^2$ . Find the  $x$ -coordinate of each critical point of  $h$ , where  $-7 < x < 5$ , and classify each critical point as the location of a relative minimum, relative maximum, or neither a minimum nor a maximum. Explain your reasoning.
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6. Solutions to the differential equation  $\frac{dy}{dx} = xy^3$  also satisfy  $\frac{d^2y}{dx^2} = y^3(1 + 3x^2y^2)$ . Let  $y = f(x)$  be a particular solution to the differential equation  $\frac{dy}{dx} = xy^3$  with  $f(1) = 2$ .
- Write an equation for the line tangent to the graph of  $y = f(x)$  at  $x = 1$ .
  - Use the tangent line equation from part (a) to approximate  $f(1.1)$ . Given that  $f(x) > 0$  for  $1 < x < 1.1$ , is the approximation for  $f(1.1)$  greater than or less than  $f(1.1)$ ? Explain your reasoning.
  - Find the particular solution  $y = f(x)$  with initial condition  $f(1) = 2$ .
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**END OF EXAM**