



## AP<sup>®</sup> Calculus AB 2011 Scoring Guidelines Form B

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**AP<sup>®</sup> CALCULUS AB**  
**2011 SCORING GUIDELINES (Form B)**

**Question 1**

A cylindrical can of radius 10 millimeters is used to measure rainfall in Stormville. The can is initially empty, and rain enters the can during a 60-day period. The height of water in the can is modeled by the function  $S$ , where  $S(t)$  is measured in millimeters and  $t$  is measured in days for  $0 \leq t \leq 60$ . The rate at which the height of the water is rising in the can is given by  $S'(t) = 2\sin(0.03t) + 1.5$ .

- (a) According to the model, what is the height of the water in the can at the end of the 60-day period?
- (b) According to the model, what is the average rate of change in the height of water in the can over the 60-day period? Show the computations that lead to your answer. Indicate units of measure.
- (c) Assuming no evaporation occurs, at what rate is the volume of water in the can changing at time  $t = 7$ ? Indicate units of measure.
- (d) During the same 60-day period, rain on Monsoon Mountain accumulates in a can identical to the one in Stormville. The height of the water in the can on Monsoon Mountain is modeled by the function  $M$ , where  $M(t) = \frac{1}{400}(3t^3 - 30t^2 + 330t)$ . The height  $M(t)$  is measured in millimeters, and  $t$  is measured in days for  $0 \leq t \leq 60$ . Let  $D(t) = M'(t) - S'(t)$ . Apply the Intermediate Value Theorem to the function  $D$  on the interval  $0 \leq t \leq 60$  to justify that there exists a time  $t$ ,  $0 < t < 60$ , at which the heights of water in the two cans are changing at the same rate.

(a)  $S(60) = \int_0^{60} S'(t) dt = 171.813 \text{ mm}$

3 :  $\begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(b)  $\frac{S(60) - S(0)}{60} = 2.863 \text{ or } 2.864 \text{ mm/day}$

1 : answer

(c)  $V(t) = 100\pi S(t)$   
 $V'(7) = 100\pi S'(7) = 602.218$

2 :  $\begin{cases} 1 : \text{relationship between } V \text{ and } S \\ 1 : \text{answer} \end{cases}$

The volume of water in the can is increasing at a rate of  $602.218 \text{ mm}^3/\text{day}$ .

(d)  $D(0) = -0.675 < 0$  and  $D(60) = 69.37730 > 0$

2 :  $\begin{cases} 1 : \text{considers } D(0) \text{ and } D(60) \\ 1 : \text{justification} \end{cases}$

Because  $D$  is continuous, the Intermediate Value Theorem implies that there is a time  $t$ ,  $0 < t < 60$ , at which  $D(t) = 0$ . At this time, the heights of water in the two cans are changing at the same rate.

1 : units in (b) or (c)

**AP<sup>®</sup> CALCULUS AB**  
**2011 SCORING GUIDELINES (Form B)**

**Question 2**

A 12,000-liter tank of water is filled to capacity. At time  $t = 0$ , water begins to drain out of the tank at a rate modeled by  $r(t)$ , measured in liters per hour, where  $r$  is given by the piecewise-defined function

$$r(t) = \begin{cases} \frac{600t}{t+3} & \text{for } 0 \leq t \leq 5 \\ 1000e^{-0.2t} & \text{for } t > 5 \end{cases}$$

- (a) Is  $r$  continuous at  $t = 5$ ? Show the work that leads to your answer.
- (b) Find the average rate at which water is draining from the tank between time  $t = 0$  and time  $t = 8$  hours.
- (c) Find  $r'(3)$ . Using correct units, explain the meaning of that value in the context of this problem.
- (d) Write, but do not solve, an equation involving an integral to find the time  $A$  when the amount of water in the tank is 9000 liters.

(a)  $\lim_{t \rightarrow 5^-} r(t) = \lim_{t \rightarrow 5^-} \left( \frac{600t}{t+3} \right) = 375 = r(5)$

$$\lim_{t \rightarrow 5^+} r(t) = \lim_{t \rightarrow 5^+} (1000e^{-0.2t}) = 367.879$$

Because the left-hand and right-hand limits are not equal,  $r$  is not continuous at  $t = 5$ .

2 : conclusion with analysis

(b)  $\frac{1}{8} \int_0^8 r(t) dt = \frac{1}{8} \left( \int_0^5 \frac{600t}{t+3} dt + \int_5^8 1000e^{-0.2t} dt \right)$   
 $= 258.052$  or  $258.053$

3 :  $\begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and constant} \\ 1 : \text{answer} \end{cases}$

(c)  $r'(3) = 50$

The rate at which water is draining out of the tank at time  $t = 3$  hours is increasing at 50 liters/hour<sup>2</sup>.

2 :  $\begin{cases} 1 : r'(3) \\ 1 : \text{meaning of } r'(3) \end{cases}$

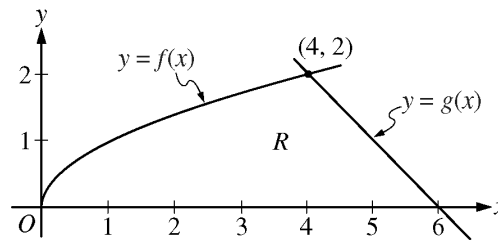
(d)  $12,000 - \int_0^A r(t) dt = 9000$

2 :  $\begin{cases} 1 : \text{integral} \\ 1 : \text{equation} \end{cases}$

**AP<sup>®</sup> CALCULUS AB  
2011 SCORING GUIDELINES (Form B)**

**Question 3**

The functions  $f$  and  $g$  are given by  $f(x) = \sqrt{x}$  and  $g(x) = 6 - x$ . Let  $R$  be the region bounded by the  $x$ -axis and the graphs of  $f$  and  $g$ , as shown in the figure above.



- (a) Find the area of  $R$ .
- (b) The region  $R$  is the base of a solid. For each  $y$ , where  $0 \leq y \leq 2$ , the cross section of the solid taken perpendicular to the  $y$ -axis is a rectangle whose base lies in  $R$  and whose height is  $2y$ . Write, but do not evaluate, an integral expression that gives the volume of the solid.
- (c) There is a point  $P$  on the graph of  $f$  at which the line tangent to the graph of  $f$  is perpendicular to the graph of  $g$ . Find the coordinates of point  $P$ .

(a) 
$$\text{Area} = \int_0^4 \sqrt{x} \, dx + \frac{1}{2} \cdot 2 \cdot 2 = \frac{2}{3} x^{3/2} \Big|_{x=0}^{x=4} + 2 = \frac{22}{3}$$

3 : { 1 : integral  
1 : antiderivative  
1 : answer

(b) 
$$y = \sqrt{x} \Rightarrow x = y^2$$
  

$$y = 6 - x \Rightarrow x = 6 - y$$

Width =  $(6 - y) - y^2$

Volume =  $\int_0^2 2y(6 - y - y^2) \, dy$

3 : { 2 : integrand  
1 : answer

(c) 
$$g'(x) = -1$$

Thus a line perpendicular to the graph of  $g$  has slope 1.

$$f'(x) = \frac{1}{2\sqrt{x}}$$

$$\frac{1}{2\sqrt{x}} = 1 \Rightarrow x = \frac{1}{4}$$

The point  $P$  has coordinates  $\left(\frac{1}{4}, \frac{1}{2}\right)$ .

3 : { 1 :  $f'(x)$   
1 : equation  
1 : answer

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**2011 SCORING GUIDELINES (Form B)**

**Question 4**

Consider a differentiable function  $f$  having domain all positive real numbers, and for which it is known that  $f'(x) = (4 - x)x^{-3}$  for  $x > 0$ .

- (a) Find the  $x$ -coordinate of the critical point of  $f$ . Determine whether the point is a relative maximum, a relative minimum, or neither for the function  $f$ . Justify your answer.
- (b) Find all intervals on which the graph of  $f$  is concave down. Justify your answer.
- (c) Given that  $f(1) = 2$ , determine the function  $f$ .

- (a)  $f'(x) = 0$  at  $x = 4$   
 $f'(x) > 0$  for  $0 < x < 4$   
 $f'(x) < 0$  for  $x > 4$   
 Therefore  $f$  has a relative maximum at  $x = 4$ .

$$3 : \begin{cases} 1 : x = 4 \\ 1 : \text{relative maximum} \\ 1 : \text{justification} \end{cases}$$

- (b)  $f''(x) = -x^{-3} + (4 - x)(-3x^{-4})$   
 $= -x^{-3} - 12x^{-4} + 3x^{-3}$   
 $= 2x^{-4}(x - 6)$   
 $= \frac{2(x - 6)}{x^4}$   
 $f''(x) < 0$  for  $0 < x < 6$

$$3 : \begin{cases} 2 : f''(x) \\ 1 : \text{answer with justification} \end{cases}$$

The graph of  $f$  is concave down on the interval  $0 < x < 6$ .

- (c)  $f(x) = 2 + \int_1^x (4t^{-3} - t^{-2}) dt$   
 $= 2 + \left[ -2t^{-2} + t^{-1} \right]_{t=1}^{t=x}$   
 $= 3 - 2x^{-2} + x^{-1}$

$$3 : \begin{cases} 1 : \text{integral} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$$

**AP<sup>®</sup> CALCULUS AB**  
**2011 SCORING GUIDELINES (Form B)**

**Question 5**

$t$ (seconds)	0	10	40	60
$B(t)$ (meters)	100	136	9	49
$v(t)$ (meters per second)	2.0	2.3	2.5	4.6

Ben rides a unicycle back and forth along a straight east-west track. The twice-differentiable function  $B$  models Ben's position on the track, measured in meters from the western end of the track, at time  $t$ , measured in seconds from the start of the ride. The table above gives values for  $B(t)$  and Ben's velocity,  $v(t)$ , measured in meters per second, at selected times  $t$ .

- (a) Use the data in the table to approximate Ben's acceleration at time  $t = 5$  seconds. Indicate units of measure.
- (b) Using correct units, interpret the meaning of  $\int_0^{60} |v(t)| dt$  in the context of this problem. Approximate  $\int_0^{60} |v(t)| dt$  using a left Riemann sum with the subintervals indicated by the data in the table.
- (c) For  $40 \leq t \leq 60$ , must there be a time  $t$  when Ben's velocity is 2 meters per second? Justify your answer.
- (d) A light is directly above the western end of the track. Ben rides so that at time  $t$ , the distance  $L(t)$  between Ben and the light satisfies  $(L(t))^2 = 12^2 + (B(t))^2$ . At what rate is the distance between Ben and the light changing at time  $t = 40$ ?

(a)  $a(5) \approx \frac{v(10) - v(0)}{10 - 0} = \frac{0.3}{10} = 0.03 \text{ meters/sec}^2$

1 : answer

- (b)  $\int_0^{60} |v(t)| dt$  is the total distance, in meters, that Ben rides over the 60-second interval  $t = 0$  to  $t = 60$ .

2 :  $\left\{ \begin{array}{l} 1 : \text{meaning of integral} \\ 1 : \text{approximation} \end{array} \right.$

$$\int_0^{60} |v(t)| dt \approx 2.0 \cdot 10 + 2.3(40 - 10) + 2.5(60 - 40) = 139 \text{ meters}$$

- (c) Because  $\frac{B(60) - B(40)}{60 - 40} = \frac{49 - 9}{20} = 2$ , the Mean Value Theorem implies there is a time  $t$ ,  $40 < t < 60$ , such that  $v(t) = 2$ .

2 :  $\left\{ \begin{array}{l} 1 : \text{difference quotient} \\ 1 : \text{conclusion with justification} \end{array} \right.$

(d)  $2L(t)L'(t) = 2B(t)B'(t)$   
 $L'(40) = \frac{B(40)v(40)}{L(40)} = \frac{9 \cdot 2.5}{\sqrt{144 + 81}} = \frac{3}{2} \text{ meters/sec}$

3 :  $\left\{ \begin{array}{l} 1 : \text{derivatives} \\ 1 : \text{uses } B'(t) = v(t) \\ 1 : \text{answer} \end{array} \right.$

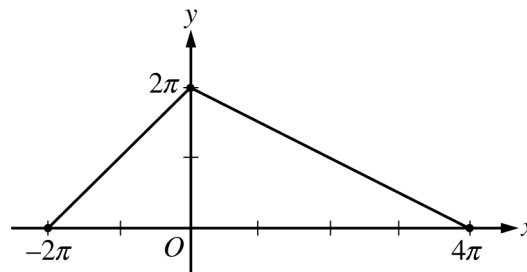
1 : units in (a) or (b)

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**2011 SCORING GUIDELINES (Form B)**

**Question 6**

Let  $g$  be the piecewise-linear function defined on  $[-2\pi, 4\pi]$

whose graph is given above, and let  $f(x) = g(x) - \cos\left(\frac{x}{2}\right)$ .



Graph of  $g$

- (a) Find  $\int_{-2\pi}^{4\pi} f(x) dx$ . Show the computations that lead to your answer.
- (b) Find all  $x$ -values in the open interval  $(-2\pi, 4\pi)$  for which  $f$  has a critical point.
- (c) Let  $h(x) = \int_0^{3x} g(t) dt$ . Find  $h'\left(-\frac{\pi}{3}\right)$ .

$$\begin{aligned} \text{(a)} \quad \int_{-2\pi}^{4\pi} f(x) dx &= \int_{-2\pi}^{4\pi} \left( g(x) - \cos\left(\frac{x}{2}\right) \right) dx \\ &= 6\pi^2 - \left[ 2\sin\left(\frac{x}{2}\right) \right]_{x=-2\pi}^{x=4\pi} \\ &= 6\pi^2 \end{aligned}$$

2 :  $\begin{cases} 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

$$\text{(b)} \quad f'(x) = g'(x) + \frac{1}{2}\sin\left(\frac{x}{2}\right) = \begin{cases} 1 + \frac{1}{2}\sin\left(\frac{x}{2}\right) & \text{for } -2\pi < x < 0 \\ -\frac{1}{2} + \frac{1}{2}\sin\left(\frac{x}{2}\right) & \text{for } 0 < x < 4\pi \end{cases}$$

4 :  $\begin{cases} 1 : \frac{d}{dx}\left(\cos\left(\frac{x}{2}\right)\right) \\ 1 : g'(x) \\ 1 : x = 0 \\ 1 : x = \pi \end{cases}$

$f'(x)$  does not exist at  $x = 0$ .

For  $-2\pi < x < 0$ ,  $f'(x) \neq 0$ .

For  $0 < x < 4\pi$ ,  $f'(x) = 0$  when  $x = \pi$ .

$f$  has critical points at  $x = 0$  and  $x = \pi$ .

$$\begin{aligned} \text{(c)} \quad h'(x) &= g(3x) \cdot 3 \\ h'\left(-\frac{\pi}{3}\right) &= 3g(-\pi) = 3\pi \end{aligned}$$

3 :  $\begin{cases} 2 : h'(x) \\ 1 : \text{answer} \end{cases}$