



**AP<sup>®</sup> Calculus AB**  
**2011 Free-Response Questions**  
**Form B**

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**CALCULUS AB**  
**SECTION II, Part A**  
**Time—30 minutes**  
**Number of problems—2**

**A graphing calculator is required for these problems.**

1. A cylindrical can of radius 10 millimeters is used to measure rainfall in Stormville. The can is initially empty, and rain enters the can during a 60-day period. The height of water in the can is modeled by the function  $S$ , where  $S(t)$  is measured in millimeters and  $t$  is measured in days for  $0 \leq t \leq 60$ . The rate at which the height of the water is rising in the can is given by  $S'(t) = 2\sin(0.03t) + 1.5$ .
- (a) According to the model, what is the height of the water in the can at the end of the 60-day period?
- (b) According to the model, what is the average rate of change in the height of water in the can over the 60-day period? Show the computations that lead to your answer. Indicate units of measure.
- (c) Assuming no evaporation occurs, at what rate is the volume of water in the can changing at time  $t = 7$ ? Indicate units of measure.
- (d) During the same 60-day period, rain on Monsoon Mountain accumulates in a can identical to the one in Stormville. The height of the water in the can on Monsoon Mountain is modeled by the function  $M$ , where  $M(t) = \frac{1}{400}(3t^3 - 30t^2 + 330t)$ . The height  $M(t)$  is measured in millimeters, and  $t$  is measured in days for  $0 \leq t \leq 60$ . Let  $D(t) = M'(t) - S'(t)$ . Apply the Intermediate Value Theorem to the function  $D$  on the interval  $0 \leq t \leq 60$  to justify that there exists a time  $t$ ,  $0 < t < 60$ , at which the heights of water in the two cans are changing at the same rate.

2. A 12,000-liter tank of water is filled to capacity. At time  $t = 0$ , water begins to drain out of the tank at a rate modeled by  $r(t)$ , measured in liters per hour, where  $r$  is given by the piecewise-defined function

$$r(t) = \begin{cases} \frac{600t}{t+3} & \text{for } 0 \leq t \leq 5 \\ 1000e^{-0.2t} & \text{for } t > 5 \end{cases}$$

- (a) Is  $r$  continuous at  $t = 5$ ? Show the work that leads to your answer.
- (b) Find the average rate at which water is draining from the tank between time  $t = 0$  and time  $t = 8$  hours.
- (c) Find  $r'(3)$ . Using correct units, explain the meaning of that value in the context of this problem.
- (d) Write, but do not solve, an equation involving an integral to find the time  $A$  when the amount of water in the tank is 9000 liters.

**WRITE ALL WORK IN THE EXAM BOOKLET.**

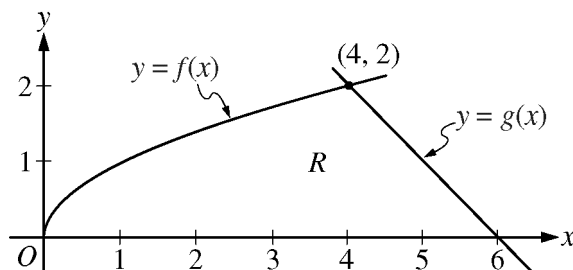
**END OF PART A OF SECTION II**

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**CALCULUS AB**  
**SECTION II, Part B**  
**Time—60 minutes**  
**Number of problems—4**

No calculator is allowed for these problems.



3. The functions  $f$  and  $g$  are given by  $f(x) = \sqrt{x}$  and  $g(x) = 6 - x$ . Let  $R$  be the region bounded by the  $x$ -axis and the graphs of  $f$  and  $g$ , as shown in the figure above.
- Find the area of  $R$ .
  - The region  $R$  is the base of a solid. For each  $y$ , where  $0 \leq y \leq 2$ , the cross section of the solid taken perpendicular to the  $y$ -axis is a rectangle whose base lies in  $R$  and whose height is  $2y$ . Write, but do not evaluate, an integral expression that gives the volume of the solid.
  - There is a point  $P$  on the graph of  $f$  at which the line tangent to the graph of  $f$  is perpendicular to the graph of  $g$ . Find the coordinates of point  $P$ .
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4. Consider a differentiable function  $f$  having domain all positive real numbers, and for which it is known that  $f'(x) = (4 - x)x^{-3}$  for  $x > 0$ .
- Find the  $x$ -coordinate of the critical point of  $f$ . Determine whether the point is a relative maximum, a relative minimum, or neither for the function  $f$ . Justify your answer.
  - Find all intervals on which the graph of  $f$  is concave down. Justify your answer.
  - Given that  $f(1) = 2$ , determine the function  $f$ .
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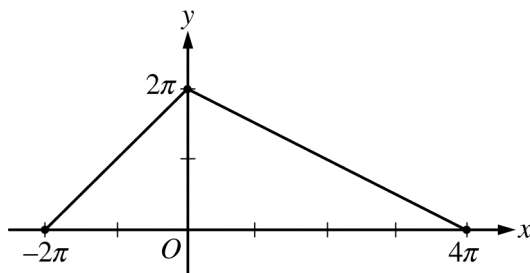
**WRITE ALL WORK IN THE EXAM BOOKLET.**

|                               |     |     |     |     |
|-------------------------------|-----|-----|-----|-----|
| $t$<br>(seconds)              | 0   | 10  | 40  | 60  |
| $B(t)$<br>(meters)            | 100 | 136 | 9   | 49  |
| $v(t)$<br>(meters per second) | 2.0 | 2.3 | 2.5 | 4.6 |

5. Ben rides a unicycle back and forth along a straight east-west track. The twice-differentiable function  $B$  models Ben's position on the track, measured in meters from the western end of the track, at time  $t$ , measured in seconds from the start of the ride. The table above gives values for  $B(t)$  and Ben's velocity,  $v(t)$ , measured in meters per second, at selected times  $t$ .
- (a) Use the data in the table to approximate Ben's acceleration at time  $t = 5$  seconds. Indicate units of measure.
- (b) Using correct units, interpret the meaning of  $\int_0^{60} |v(t)| dt$  in the context of this problem. Approximate  $\int_0^{60} |v(t)| dt$  using a left Riemann sum with the subintervals indicated by the data in the table.
- (c) For  $40 \leq t \leq 60$ , must there be a time  $t$  when Ben's velocity is 2 meters per second? Justify your answer.
- (d) A light is directly above the western end of the track. Ben rides so that at time  $t$ , the distance  $L(t)$  between Ben and the light satisfies  $(L(t))^2 = 12^2 + (B(t))^2$ . At what rate is the distance between Ben and the light changing at time  $t = 40$ ?

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**WRITE ALL WORK IN THE EXAM BOOKLET.**

Graph of  $g$ 

6. Let  $g$  be the piecewise-linear function defined on  $[-2\pi, 4\pi]$  whose graph is given above, and

$$\text{let } f(x) = g(x) - \cos\left(\frac{x}{2}\right).$$

(a) Find  $\int_{-2\pi}^{4\pi} f(x) dx$ . Show the computations that lead to your answer.

(b) Find all  $x$ -values in the open interval  $(-2\pi, 4\pi)$  for which  $f$  has a critical point.

(c) Let  $h(x) = \int_0^{3x} g(t) dt$ . Find  $h'\left(-\frac{\pi}{3}\right)$ .

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**WRITE ALL WORK IN THE EXAM BOOKLET.**

**END OF EXAM**