



AP[®] Calculus AB 2011 Scoring Guidelines

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Question 1

For $0 \leq t \leq 6$, a particle is moving along the x -axis. The particle's position, $x(t)$, is not explicitly given. The velocity of the particle is given by $v(t) = 2\sin(e^{t/4}) + 1$. The acceleration of the particle is given by $a(t) = \frac{1}{2}e^{t/4}\cos(e^{t/4})$ and $x(0) = 2$.

- (a) Is the speed of the particle increasing or decreasing at time $t = 5.5$? Give a reason for your answer.
 (b) Find the average velocity of the particle for the time period $0 \leq t \leq 6$.
 (c) Find the total distance traveled by the particle from time $t = 0$ to $t = 6$.
 (d) For $0 \leq t \leq 6$, the particle changes direction exactly once. Find the position of the particle at that time.

(a) $v(5.5) = -0.45337$, $a(5.5) = -1.35851$

The speed is increasing at time $t = 5.5$, because velocity and acceleration have the same sign.

2 : conclusion with reason

(b) Average velocity = $\frac{1}{6} \int_0^6 v(t) dt = 1.949$

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(c) Distance = $\int_0^6 |v(t)| dt = 12.573$

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(d) $v(t) = 0$ when $t = 5.19552$. Let $b = 5.19552$.
 $v(t)$ changes sign from positive to negative at time $t = b$.
 $x(b) = 2 + \int_0^b v(t) dt = 14.134$ or 14.135

3 : $\begin{cases} 1 : \text{considers } v(t) = 0 \\ 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

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Question 2

t (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

As a pot of tea cools, the temperature of the tea is modeled by a differentiable function H for $0 \leq t \leq 10$, where time t is measured in minutes and temperature $H(t)$ is measured in degrees Celsius. Values of $H(t)$ at selected values of time t are shown in the table above.

- (a) Use the data in the table to approximate the rate at which the temperature of the tea is changing at time $t = 3.5$. Show the computations that lead to your answer.
- (b) Using correct units, explain the meaning of $\frac{1}{10} \int_0^{10} H(t) dt$ in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate $\frac{1}{10} \int_0^{10} H(t) dt$.
- (c) Evaluate $\int_0^{10} H'(t) dt$. Using correct units, explain the meaning of the expression in the context of this problem.
- (d) At time $t = 0$, biscuits with temperature 100°C were removed from an oven. The temperature of the biscuits at time t is modeled by a differentiable function B for which it is known that $B'(t) = -13.84e^{-0.173t}$. Using the given models, at time $t = 10$, how much cooler are the biscuits than the tea?

(a)
$$H'(3.5) \approx \frac{H(5) - H(2)}{5 - 2}$$

$$= \frac{52 - 60}{3} = -2.666 \text{ or } -2.667 \text{ degrees Celsius per minute}$$

1 : answer

(b) $\frac{1}{10} \int_0^{10} H(t) dt$ is the average temperature of the tea, in degrees Celsius, over the 10 minutes.

$$\frac{1}{10} \int_0^{10} H(t) dt \approx \frac{1}{10} \left(2 \cdot \frac{66 + 60}{2} + 3 \cdot \frac{60 + 52}{2} + 4 \cdot \frac{52 + 44}{2} + 1 \cdot \frac{44 + 43}{2} \right)$$

$$= 52.95$$

3 : $\begin{cases} 1 : \text{meaning of expression} \\ 1 : \text{trapezoidal sum} \\ 1 : \text{estimate} \end{cases}$

(c) $\int_0^{10} H'(t) dt = H(10) - H(0) = 43 - 66 = -23$
 The temperature of the tea drops 23 degrees Celsius from time $t = 0$ to time $t = 10$ minutes.

2 : $\begin{cases} 1 : \text{value of integral} \\ 1 : \text{meaning of expression} \end{cases}$

(d) $B(10) = 100 + \int_0^{10} B'(t) dt = 34.18275$; $H(10) - B(10) = 8.817$
 The biscuits are 8.817 degrees Celsius cooler than the tea.

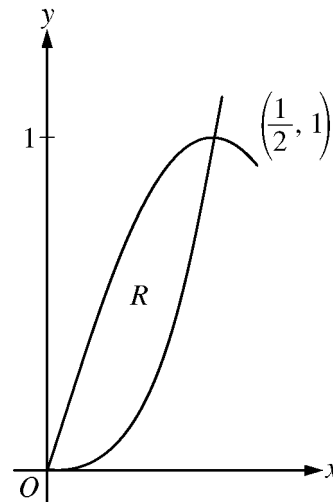
3 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{uses } B(0) = 100 \\ 1 : \text{answer} \end{cases}$

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Question 3

Let R be the region in the first quadrant enclosed by the graphs of $f(x) = 8x^3$ and $g(x) = \sin(\pi x)$, as shown in the figure above.

- (a) Write an equation for the line tangent to the graph of f at $x = \frac{1}{2}$.
- (b) Find the area of R .
- (c) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 1$.



(a) $f\left(\frac{1}{2}\right) = 1$

$$f'(x) = 24x^2, \text{ so } f'\left(\frac{1}{2}\right) = 6$$

An equation for the tangent line is $y = 1 + 6\left(x - \frac{1}{2}\right)$.

$$2 : \begin{cases} 1 : f'\left(\frac{1}{2}\right) \\ 1 : \text{answer} \end{cases}$$

(b) Area = $\int_0^{1/2} (g(x) - f(x)) dx$

$$= \int_0^{1/2} (\sin(\pi x) - 8x^3) dx$$

$$= \left[-\frac{1}{\pi} \cos(\pi x) - 2x^4 \right]_{x=0}^{x=1/2}$$

$$= -\frac{1}{8} + \frac{1}{\pi}$$

$$4 : \begin{cases} 1 : \text{integrand} \\ 2 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$$

(c) $\pi \int_0^{1/2} ((1 - f(x))^2 - (1 - g(x))^2) dx$

$$= \pi \int_0^{1/2} ((1 - 8x^3)^2 - (1 - \sin(\pi x))^2) dx$$

$$3 : \begin{cases} 1 : \text{limits and constant} \\ 2 : \text{integrand} \end{cases}$$

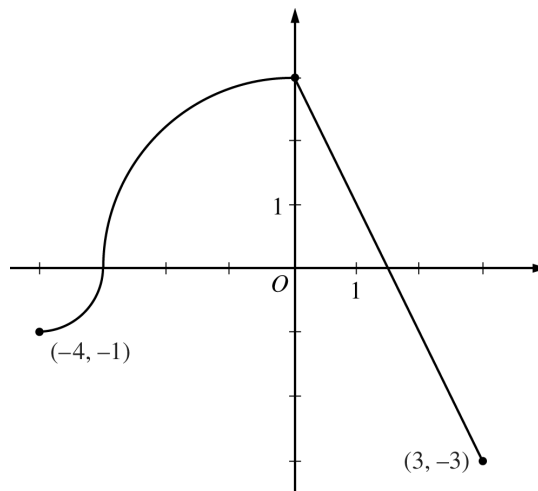
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Question 4

The continuous function f is defined on the interval $-4 \leq x \leq 3$. The graph of f consists of two quarter circles and one line segment, as shown in the figure above.

Let $g(x) = 2x + \int_0^x f(t) dt$.

- (a) Find $g(-3)$. Find $g'(x)$ and evaluate $g'(-3)$.
- (b) Determine the x -coordinate of the point at which g has an absolute maximum on the interval $-4 \leq x \leq 3$. Justify your answer.
- (c) Find all values of x on the interval $-4 < x < 3$ for which the graph of g has a point of inflection. Give a reason for your answer.
- (d) Find the average rate of change of f on the interval $-4 \leq x \leq 3$. There is no point c , $-4 < c < 3$, for which $f'(c)$ is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem.



Graph of f

(a) $g(-3) = 2(-3) + \int_0^{-3} f(t) dt = -6 - \frac{9\pi}{4}$
 $g'(x) = 2 + f(x)$
 $g'(-3) = 2 + f(-3) = 2$

$$3 : \begin{cases} 1 : g(-3) \\ 1 : g'(x) \\ 1 : g'(-3) \end{cases}$$

(b) $g'(x) = 0$ when $f(x) = -2$. This occurs at $x = \frac{5}{2}$.
 $g'(x) > 0$ for $-4 < x < \frac{5}{2}$ and $g'(x) < 0$ for $\frac{5}{2} < x < 3$.
 Therefore g has an absolute maximum at $x = \frac{5}{2}$.

$$3 : \begin{cases} 1 : \text{considers } g'(x) = 0 \\ 1 : \text{identifies interior candidate} \\ 1 : \text{answer with justification} \end{cases}$$

(c) $g''(x) = f'(x)$ changes sign only at $x = 0$. Thus the graph of g has a point of inflection at $x = 0$.

1 : answer with reason

(d) The average rate of change of f on the interval $-4 \leq x \leq 3$ is
 $\frac{f(3) - f(-4)}{3 - (-4)} = -\frac{2}{7}$.

$$2 : \begin{cases} 1 : \text{average rate of change} \\ 1 : \text{explanation} \end{cases}$$

To apply the Mean Value Theorem, f must be differentiable at each point in the interval $-4 < x < 3$. However, f is not differentiable at $x = -3$ and $x = 0$.

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Question 5

At the beginning of 2010, a landfill contained 1400 tons of solid waste. The increasing function W models the total amount of solid waste stored at the landfill. Planners estimate that W will satisfy the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ for the next 20 years. W is measured in tons, and t is measured in years from the start of 2010.

- (a) Use the line tangent to the graph of W at $t = 0$ to approximate the amount of solid waste that the landfill contains at the end of the first 3 months of 2010 (time $t = \frac{1}{4}$).
- (b) Find $\frac{d^2W}{dt^2}$ in terms of W . Use $\frac{d^2W}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the amount of solid waste that the landfill contains at time $t = \frac{1}{4}$.
- (c) Find the particular solution $W = W(t)$ to the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ with initial condition $W(0) = 1400$.

(a) $\left. \frac{dW}{dt} \right|_{t=0} = \frac{1}{25}(W(0) - 300) = \frac{1}{25}(1400 - 300) = 44$

The tangent line is $y = 1400 + 44t$.

$$W\left(\frac{1}{4}\right) \approx 1400 + 44\left(\frac{1}{4}\right) = 1411 \text{ tons}$$

(b) $\frac{d^2W}{dt^2} = \frac{1}{25} \frac{dW}{dt} = \frac{1}{625}(W - 300)$ and $W \geq 1400$

Therefore $\frac{d^2W}{dt^2} > 0$ on the interval $0 \leq t \leq \frac{1}{4}$.

The answer in part (a) is an underestimate.

(c) $\frac{dW}{dt} = \frac{1}{25}(W - 300)$

$$\int \frac{1}{W - 300} dW = \int \frac{1}{25} dt$$

$$\ln|W - 300| = \frac{1}{25}t + C$$

$$\ln(1400 - 300) = \frac{1}{25}(0) + C \Rightarrow \ln(1100) = C$$

$$W - 300 = 1100e^{\frac{1}{25}t}$$

$$W(t) = 300 + 1100e^{\frac{1}{25}t}, \quad 0 \leq t \leq 20$$

$$2 : \begin{cases} 1 : \frac{dW}{dt} \text{ at } t = 0 \\ 1 : \text{answer} \end{cases}$$

$$2 : \begin{cases} 1 : \frac{d^2W}{dt^2} \\ 1 : \text{answer with reason} \end{cases}$$

$$5 : \begin{cases} 1 : \text{separation of variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } W \end{cases}$$

Note: max 2/5 [1-1-0-0-0] if no constant of integration

Note: 0/5 if no separation of variables

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Question 6

Let f be a function defined by $f(x) = \begin{cases} 1 - 2\sin x & \text{for } x \leq 0 \\ e^{-4x} & \text{for } x > 0. \end{cases}$

- (a) Show that f is continuous at $x = 0$.
 (b) For $x \neq 0$, express $f'(x)$ as a piecewise-defined function. Find the value of x for which $f'(x) = -3$.
 (c) Find the average value of f on the interval $[-1, 1]$.

(a) $\lim_{x \rightarrow 0^-} (1 - 2\sin x) = 1$

$$\lim_{x \rightarrow 0^+} e^{-4x} = 1$$

$$f(0) = 1$$

So, $\lim_{x \rightarrow 0} f(x) = f(0)$.

Therefore f is continuous at $x = 0$.

(b) $f'(x) = \begin{cases} -2\cos x & \text{for } x < 0 \\ -4e^{-4x} & \text{for } x > 0 \end{cases}$

$$-2\cos x \neq -3 \text{ for all values of } x < 0.$$

$$-4e^{-4x} = -3 \text{ when } x = -\frac{1}{4}\ln\left(\frac{3}{4}\right) > 0.$$

Therefore $f'(x) = -3$ for $x = -\frac{1}{4}\ln\left(\frac{3}{4}\right)$.

(c)
$$\begin{aligned} \int_{-1}^1 f(x) dx &= \int_{-1}^0 f(x) dx + \int_0^1 f(x) dx \\ &= \int_{-1}^0 (1 - 2\sin x) dx + \int_0^1 e^{-4x} dx \\ &= \left[x + 2\cos x \right]_{x=-1}^{x=0} + \left[-\frac{1}{4}e^{-4x} \right]_{x=0}^{x=1} \\ &= (3 - 2\cos(-1)) + \left(-\frac{1}{4}e^{-4} + \frac{1}{4} \right) \end{aligned}$$

$$\begin{aligned} \text{Average value} &= \frac{1}{2} \int_{-1}^1 f(x) dx \\ &= \frac{13}{8} - \cos(-1) - \frac{1}{8}e^{-4} \end{aligned}$$

2 : analysis

3 : $\begin{cases} 2 : f'(x) \\ 1 : \text{value of } x \end{cases}$

4 : $\begin{cases} 1 : \int_{-1}^0 (1 - 2\sin x) dx \text{ and } \int_0^1 e^{-4x} dx \\ 2 : \text{antiderivatives} \\ 1 : \text{answer} \end{cases}$