



AP[®] Calculus AB 2012 Scoring Guidelines

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**AP[®] CALCULUS AB
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Question 1

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W , where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t = 0$, the temperature of the water is 55°F . The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

- (a) Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.
- (b) Use the data in the table to evaluate $\int_0^{20} W'(t) dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t) dt$ in the context of this problem.
- (c) For $0 \leq t \leq 20$, the average temperature of the water in the tub is $\frac{1}{20} \int_0^{20} W(t) dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20} \int_0^{20} W(t) dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.
- (d) For $20 \leq t \leq 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t} \cos(0.06t)$. Based on the model, what is the temperature of the water at time $t = 25$?

(a)
$$W'(12) \approx \frac{W(15) - W(9)}{15 - 9} = \frac{67.9 - 61.8}{6}$$

$$= 1.017 \text{ (or 1.016)}$$

The water temperature is increasing at a rate of approximately 1.017°F per minute at time $t = 12$ minutes.

(b)
$$\int_0^{20} W'(t) dt = W(20) - W(0) = 71.0 - 55.0 = 16$$

The water has warmed by 16°F over the interval from $t = 0$ to $t = 20$ minutes.

(c)
$$\frac{1}{20} \int_0^{20} W(t) dt \approx \frac{1}{20} (4 \cdot W(0) + 5 \cdot W(4) + 6 \cdot W(9) + 5 \cdot W(15))$$

$$= \frac{1}{20} (4 \cdot 55.0 + 5 \cdot 57.1 + 6 \cdot 61.8 + 5 \cdot 67.9)$$

$$= \frac{1}{20} \cdot 1215.8 = 60.79$$

This approximation is an underestimate, because a left Riemann sum is used and the function W is strictly increasing.

(d)
$$W(25) = 71.0 + \int_{20}^{25} W'(t) dt$$

$$= 71.0 + 2.043155 = 73.043$$

2 : $\begin{cases} 1 : \text{estimate} \\ 1 : \text{interpretation with units} \end{cases}$

2 : $\begin{cases} 1 : \text{value} \\ 1 : \text{interpretation with units} \end{cases}$

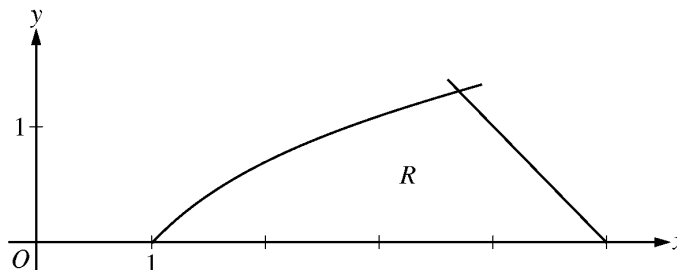
3 : $\begin{cases} 1 : \text{left Riemann sum} \\ 1 : \text{approximation} \\ 1 : \text{underestimate with reason} \end{cases}$

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

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Question 2

Let R be the region in the first quadrant bounded by the x -axis and the graphs of $y = \ln x$ and $y = 5 - x$, as shown in the figure above.



- (a) Find the area of R .
- (b) Region R is the base of a solid. For the solid, each cross section perpendicular to the x -axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.
- (c) The horizontal line $y = k$ divides R into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k .

$$\ln x = 5 - x \Rightarrow x = 3.69344$$

Therefore, the graphs of $y = \ln x$ and $y = 5 - x$ intersect in the first quadrant at the point $(A, B) = (3.69344, 1.30656)$.

(a)
$$\text{Area} = \int_0^B (5 - y - e^y) dy$$

$$= 2.986 \text{ (or } 2.985)$$

3 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$

OR

$$\text{Area} = \int_1^A \ln x \, dx + \int_A^5 (5 - x) \, dx$$

$$= 2.986 \text{ (or } 2.985)$$

(b)
$$\text{Volume} = \int_1^A (\ln x)^2 \, dx + \int_A^5 (5 - x)^2 \, dx$$

3 : $\begin{cases} 2 : \text{integrands} \\ 1 : \text{expression for total volume} \end{cases}$

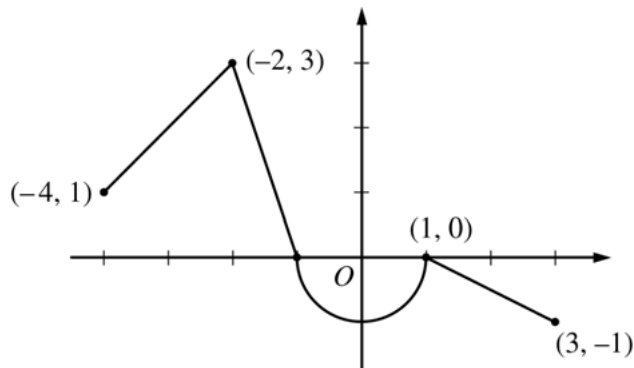
(c)
$$\int_0^k (5 - y - e^y) \, dy = \frac{1}{2} \cdot 2.986 \text{ (or } \frac{1}{2} \cdot 2.985)$$

3 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{equation} \end{cases}$

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Question 3

Let f be the continuous function defined on $[-4, 3]$ whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let g be the function given by $g(x) = \int_1^x f(t) dt$.



Graph of f

- Find the values of $g(2)$ and $g(-2)$.
- For each of $g'(-3)$ and $g''(-3)$, find the value or state that it does not exist.
- Find the x -coordinate of each point at which the graph of g has a horizontal tangent line. For each of these points, determine whether g has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.
- For $-4 < x < 3$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning.

(a) $g(2) = \int_1^2 f(t) dt = -\frac{1}{2}(1)\left(\frac{1}{2}\right) = -\frac{1}{4}$

$$g(-2) = \int_1^{-2} f(t) dt = -\int_{-2}^1 f(t) dt$$

$$= -\left(\frac{3}{2} - \frac{\pi}{2}\right) = \frac{\pi}{2} - \frac{3}{2}$$

$$2: \begin{cases} 1: g(2) \\ 1: g(-2) \end{cases}$$

(b) $g'(x) = f(x) \Rightarrow g'(-3) = f(-3) = 2$
 $g''(x) = f'(x) \Rightarrow g''(-3) = f'(-3) = 1$

$$2: \begin{cases} 1: g'(-3) \\ 1: g''(-3) \end{cases}$$

(c) The graph of g has a horizontal tangent line where $g'(x) = f(x) = 0$. This occurs at $x = -1$ and $x = 1$.

$g'(x)$ changes sign from positive to negative at $x = -1$.
Therefore, g has a relative maximum at $x = -1$.

$g'(x)$ does not change sign at $x = 1$. Therefore, g has neither a relative maximum nor a relative minimum at $x = 1$.

$$3: \begin{cases} 1: \text{considers } g'(x) = 0 \\ 1: x = -1 \text{ and } x = 1 \\ 1: \text{answers with justifications} \end{cases}$$

(d) The graph of g has a point of inflection at each of $x = -2$, $x = 0$, and $x = 1$ because $g''(x) = f'(x)$ changes sign at each of these values.

$$2: \begin{cases} 1: \text{answer} \\ 1: \text{explanation} \end{cases}$$

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Question 4

The function f is defined by $f(x) = \sqrt{25 - x^2}$ for $-5 \leq x \leq 5$.

- (a) Find $f'(x)$.
- (b) Write an equation for the line tangent to the graph of f at $x = -3$.
- (c) Let g be the function defined by $g(x) = \begin{cases} f(x) & \text{for } -5 \leq x \leq -3 \\ x + 7 & \text{for } -3 < x \leq 5. \end{cases}$
Is g continuous at $x = -3$? Use the definition of continuity to explain your answer.
- (d) Find the value of $\int_0^5 x\sqrt{25 - x^2} \, dx$.

(a) $f'(x) = \frac{1}{2}(25 - x^2)^{-1/2}(-2x) = \frac{-x}{\sqrt{25 - x^2}}, \quad -5 < x < 5$

2 : $f'(x)$

(b) $f'(-3) = \frac{3}{\sqrt{25 - 9}} = \frac{3}{4}$

$f(-3) = \sqrt{25 - 9} = 4$

An equation for the tangent line is $y = 4 + \frac{3}{4}(x + 3)$.

2 : $\begin{cases} 1 : f'(-3) \\ 1 : \text{answer} \end{cases}$

(c) $\lim_{x \rightarrow -3^-} g(x) = \lim_{x \rightarrow -3^-} f(x) = \lim_{x \rightarrow -3^-} \sqrt{25 - x^2} = 4$

$\lim_{x \rightarrow -3^+} g(x) = \lim_{x \rightarrow -3^+} (x + 7) = 4$

Therefore, $\lim_{x \rightarrow -3} g(x) = 4$.

$g(-3) = f(-3) = 4$

So, $\lim_{x \rightarrow -3} g(x) = g(-3)$.

Therefore, g is continuous at $x = -3$.

2 : $\begin{cases} 1 : \text{considers one-sided limits} \\ 1 : \text{answer with explanation} \end{cases}$

(d) Let $u = 25 - x^2 \Rightarrow du = -2x \, dx$

$\int_0^5 x\sqrt{25 - x^2} \, dx = -\frac{1}{2} \int_{25}^0 \sqrt{u} \, du$

$= \left[-\frac{1}{2} \cdot \frac{2}{3} u^{3/2} \right]_{u=25}^{u=0}$

$= -\frac{1}{3}(0 - 125) = \frac{125}{3}$

3 : $\begin{cases} 2 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

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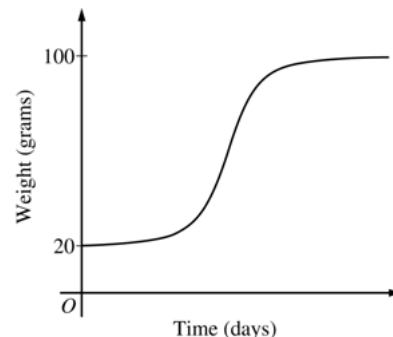
Question 5

The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20$.

- (a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.
- (b) Find $\frac{d^2B}{dt^2}$ in terms of B . Use $\frac{d^2B}{dt^2}$ to explain why the graph of B cannot resemble the following graph.
- (c) Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 20$.



(a) $\left. \frac{dB}{dt} \right|_{B=40} = \frac{1}{5}(60) = 12$

$$\left. \frac{dB}{dt} \right|_{B=70} = \frac{1}{5}(30) = 6$$

Because $\left. \frac{dB}{dt} \right|_{B=40} > \left. \frac{dB}{dt} \right|_{B=70}$, the bird is gaining weight faster when it weighs 40 grams.

(b) $\frac{d^2B}{dt^2} = -\frac{1}{5} \frac{dB}{dt} = -\frac{1}{5} \cdot \frac{1}{5}(100 - B) = -\frac{1}{25}(100 - B)$

Therefore, the graph of B is concave down for $20 \leq B < 100$. A portion of the given graph is concave up.

(c) $\frac{dB}{dt} = \frac{1}{5}(100 - B)$

$$\int \frac{1}{100 - B} dB = \int \frac{1}{5} dt$$

$$-\ln|100 - B| = \frac{1}{5}t + C$$

Because $20 \leq B < 100$, $|100 - B| = 100 - B$.

$$-\ln(100 - 20) = \frac{1}{5}(0) + C \Rightarrow -\ln(80) = C$$

$$100 - B = 80e^{-t/5}$$

$$B(t) = 100 - 80e^{-t/5}, \quad t \geq 0$$

2 : $\left\{ \begin{array}{l} 1 : \text{uses } \frac{dB}{dt} \\ 1 : \text{answer with reason} \end{array} \right.$

2 : $\left\{ \begin{array}{l} 1 : \frac{d^2B}{dt^2} \text{ in terms of } B \\ 1 : \text{explanation} \end{array} \right.$

5 : $\left\{ \begin{array}{l} 1 : \text{separation of variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } B \end{array} \right.$

Note: max 2/5 [1-1-0-0-0] if no constant of integration

Note: 0/5 if no separation of variables

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Question 6

For $0 \leq t \leq 12$, a particle moves along the x -axis. The velocity of the particle at time t is given by $v(t) = \cos\left(\frac{\pi}{6}t\right)$. The particle is at position $x = -2$ at time $t = 0$.

- (a) For $0 \leq t \leq 12$, when is the particle moving to the left?
- (b) Write, but do not evaluate, an integral expression that gives the total distance traveled by the particle from time $t = 0$ to time $t = 6$.
- (c) Find the acceleration of the particle at time t . Is the speed of the particle increasing, decreasing, or neither at time $t = 4$? Explain your reasoning.
- (d) Find the position of the particle at time $t = 4$.

(a) $v(t) = \cos\left(\frac{\pi}{6}t\right) = 0 \Rightarrow t = 3, 9$

The particle is moving to the left when $v(t) < 0$.
This occurs when $3 < t < 9$.

2 : $\begin{cases} 1 : \text{considers } v(t) = 0 \\ 1 : \text{interval} \end{cases}$

(b) $\int_0^6 |v(t)| dt$

1 : answer

(c) $a(t) = -\frac{\pi}{6} \sin\left(\frac{\pi}{6}t\right)$

$$a(4) = -\frac{\pi}{6} \sin\left(\frac{2\pi}{3}\right) = -\frac{\sqrt{3}\pi}{12} < 0$$

$$v(4) = \cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2} < 0$$

The speed is increasing at time $t = 4$, because velocity and acceleration have the same sign.

3 : $\begin{cases} 1 : a(t) \\ 2 : \text{conclusion with reason} \end{cases}$

(d) $x(4) = -2 + \int_0^4 \cos\left(\frac{\pi}{6}t\right) dt$

$$= -2 + \left[\frac{6}{\pi} \sin\left(\frac{\pi}{6}t\right) \right]_0^4$$

$$= -2 + \frac{6}{\pi} \left[\sin\left(\frac{2\pi}{3}\right) - 0 \right]$$

$$= -2 + \frac{6}{\pi} \cdot \frac{\sqrt{3}}{2} = -2 + \frac{3\sqrt{3}}{\pi}$$

3 : $\begin{cases} 1 : \text{antiderivative} \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$