



## AP<sup>®</sup> Calculus AB 2013 Free-Response Questions

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**CALCULUS AB**  
**SECTION II, Part A**  
**Time—30 minutes**  
**Number of problems—2**

**A graphing calculator is required for these problems.**

1. On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by  $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$ , where  $t$  is measured in hours and  $0 \leq t \leq 8$ . At the beginning of the workday ( $t = 0$ ), the plant has 500 tons of unprocessed gravel. During the hours of operation,  $0 \leq t \leq 8$ , the plant processes gravel at a constant rate of 100 tons per hour.
- (a) Find  $G'(5)$ . Using correct units, interpret your answer in the context of the problem.
- (b) Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.
- (c) Is the amount of unprocessed gravel at the plant increasing or decreasing at time  $t = 5$  hours? Show the work that leads to your answer.
- (d) What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.
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2. A particle moves along a straight line. For  $0 \leq t \leq 5$ , the velocity of the particle is given by  $v(t) = -2 + (t^2 + 3t)^{6/5} - t^3$ , and the position of the particle is given by  $s(t)$ . It is known that  $s(0) = 10$ .
- (a) Find all values of  $t$  in the interval  $2 \leq t \leq 4$  for which the speed of the particle is 2.
  - (b) Write an expression involving an integral that gives the position  $s(t)$ . Use this expression to find the position of the particle at time  $t = 5$ .
  - (c) Find all times  $t$  in the interval  $0 \leq t \leq 5$  at which the particle changes direction. Justify your answer.
  - (d) Is the speed of the particle increasing or decreasing at time  $t = 4$ ? Give a reason for your answer.
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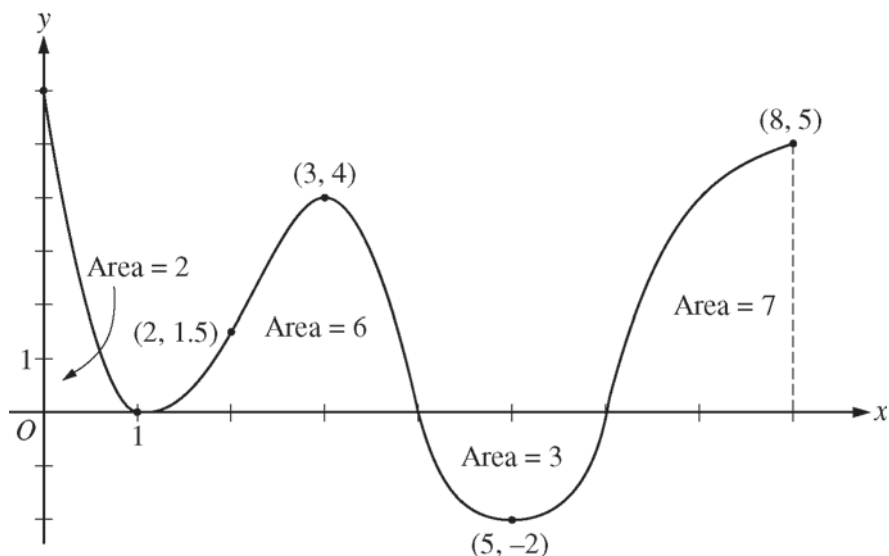
**END OF PART A OF SECTION II**

**CALCULUS AB**  
**SECTION II, Part B**  
**Time—60 minutes**  
**Number of problems—4**

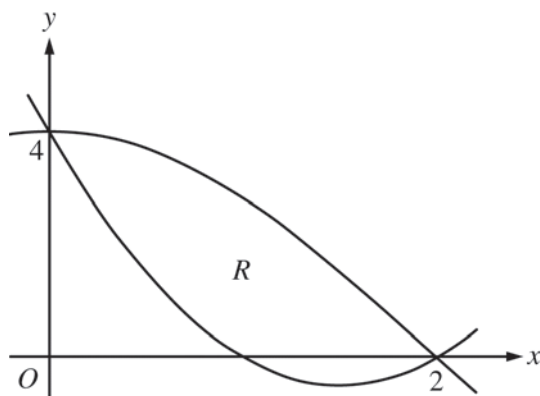
**No calculator is allowed for these problems.**

$t$ (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

3. Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time  $t$ ,  $0 \leq t \leq 6$ , is given by a differentiable function  $C$ , where  $t$  is measured in minutes. Selected values of  $C(t)$ , measured in ounces, are given in the table above.
- (a) Use the data in the table to approximate  $C'(3.5)$ . Show the computations that lead to your answer, and indicate units of measure.
- (b) Is there a time  $t$ ,  $2 \leq t \leq 4$ , at which  $C'(t) = 2$ ? Justify your answer.
- (c) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of  $\frac{1}{6} \int_0^6 C(t) dt$ . Using correct units, explain the meaning of  $\frac{1}{6} \int_0^6 C(t) dt$  in the context of the problem.
- (d) The amount of coffee in the cup, in ounces, is modeled by  $B(t) = 16 - 16e^{-0.4t}$ . Using this model, find the rate at which the amount of coffee in the cup is changing when  $t = 5$ .
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Graph of  $f'$ 

4. The figure above shows the graph of  $f'$ , the derivative of a twice-differentiable function  $f$ , on the closed interval  $0 \leq x \leq 8$ . The graph of  $f'$  has horizontal tangent lines at  $x = 1$ ,  $x = 3$ , and  $x = 5$ . The areas of the regions between the graph of  $f'$  and the  $x$ -axis are labeled in the figure. The function  $f$  is defined for all real numbers and satisfies  $f(8) = 4$ .
- Find all values of  $x$  on the open interval  $0 < x < 8$  for which the function  $f$  has a local minimum. Justify your answer.
  - Determine the absolute minimum value of  $f$  on the closed interval  $0 \leq x \leq 8$ . Justify your answer.
  - On what open intervals contained in  $0 < x < 8$  is the graph of  $f$  both concave down and increasing? Explain your reasoning.
  - The function  $g$  is defined by  $g(x) = (f(x))^3$ . If  $f(3) = -\frac{5}{2}$ , find the slope of the line tangent to the graph of  $g$  at  $x = 3$ .



5. Let  $f(x) = 2x^2 - 6x + 4$  and  $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$ . Let  $R$  be the region bounded by the graphs of  $f$  and  $g$ , as shown in the figure above.
- Find the area of  $R$ .
  - Write, but do not evaluate, an integral expression that gives the volume of the solid generated when  $R$  is rotated about the horizontal line  $y = 4$ .
  - The region  $R$  is the base of a solid. For this solid, each cross section perpendicular to the  $x$ -axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.
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6. Consider the differential equation  $\frac{dy}{dx} = e^y(3x^2 - 6x)$ . Let  $y = f(x)$  be the particular solution to the differential equation that passes through  $(1, 0)$ .
- (a) Write an equation for the line tangent to the graph of  $f$  at the point  $(1, 0)$ . Use the tangent line to approximate  $f(1.2)$ .
- (b) Find  $y = f(x)$ , the particular solution to the differential equation that passes through  $(1, 0)$ .
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**STOP**

**END OF EXAM**