2018

AP Calculus AB

Scoring Guidelines

Question 1

(a) $\int_0^{300} r(t)\,dt = 270$

According to the model, 270 people enter the line for the escalator during the time interval $0 \le t \le 300$.

2 : { 1 : integral; 1 : answer }

(b) $20 + \int_0^{300} (r(t) - 0.7)\,dt = 20 + \int_0^{300} r(t)\,dt - 0.7 \cdot 300 = 80$

According to the model, 80 people are in line at time $t = 300$.

2 : { 1 : considers rate out; 1 : answer }

(c) Based on part (b), the number of people in line at time $t = 300$ is 80.

The first time t that there are no people in line is

$300 + \frac{80}{0.7} = 414.286$ (or 414.285) seconds.

1 : answer

(d) The total number of people in line at time t, $0 \le t \le 300$, is modeled by $20 + \int_0^{t} r(x)\,dx - 0.7t$.

$r(t) - 0.7 = 0 \Rightarrow t_1 = 33.013298,\ t_2 = 166.574719$

t	People in line for escalator
0	20
t_1	3.803
t_2	158.070
300	80

The number of people in line is a minimum at time $t = 33.013$ seconds, when there are 4 people in line.

4 : { 1 : considers $r(t) - 0.7 = 0$; 1 : identifies $t = 33.013$; 1 : answers; 1 : justification }

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Question 2

(a) $v'(3) = -2.118$ The acceleration of the particle at time $t = 3$ is -2.118.	1 : answer
(b) $x(3) = x(0) + \int_0^3 v(t)\,dt = -5 + \int_0^3 v(t)\,dt = -1.760213$ The position of the particle at time $t = 3$ is -1.760.	3 : { 1 : $\int_0^3 v(t)\,dt$ 1 : uses initial condition 1 : answer
(c) $\int_0^{3.5} v(t)\,dt = 2.844$ (or 2.843) $\int_0^{3.5} \lvert v(t) \rvert\,dt = 3.737$ The integral $\int_0^{3.5} v(t)\,dt$ is the displacement of the particle over the time interval $0 \le t \le 3.5$. The integral $\int_0^{3.5} \lvert v(t) \rvert\,dt$ is the total distance traveled by the particle over the time interval $0 \le t \le 3.5$.	3 : { 1 : answers 2 : interpretations of $\int_0^{3.5} v(t)\,dt$ and $\int_0^{3.5} \lvert v(t) \rvert\,dt$
(d) $v(t) = x_2'(t)$ $v(t) = 2t - 1 \Rightarrow t = 1.57054$ The two particles are moving with the same velocity at time $t = 1.571$ (or 1.570).	2 : { 1 : sets $v(t) = x_2'(t)$ 1 : answer

Question 3

(a) $f(-5) = f(1) + \int_1^{-5} g(x)\,dx = f(1) - \int_{-5}^{1} g(x)\,dx$ $= 3 - \left(-9 - \frac{3}{2} + 1\right) = 3 - \left(-\frac{19}{2}\right) = \frac{25}{2}$	$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(b) $\int_1^6 g(x)\,dx = \int_1^3 g(x)\,dx + \int_3^6 g(x)\,dx$ $= \int_1^3 2\,dx + \int_3^6 2(x-4)^2\,dx$ $= 4 + \left[\frac{2}{3}(x-4)^3\right]_{x=3}^{x=6} = 4 + \frac{16}{3} - \left(-\frac{2}{3}\right) = 10$	$3 : \begin{cases} 1 : \text{split at } x = 3 \\ 1 : \text{antiderivative of } 2(x-4)^2 \\ 1 : \text{answer} \end{cases}$
(c) The graph of f is increasing and concave up on $0 < x < 1$ and $4 < x < 6$ because $f'(x) = g(x) > 0$ and $f'(x) = g(x)$ is increasing on those intervals.	$2 : \begin{cases} 1 : \text{intervals} \\ 1 : \text{reason} \end{cases}$
(d) The graph of f has a point of inflection at $x = 4$ because $f'(x) = g(x)$ changes from decreasing to increasing at $x = 4$.	$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$

Question 4

(a) $H'(6) \approx \frac{H(7) - H(5)}{7 - 5} = \frac{11 - 6}{2} = \frac{5}{2}$

$H'(6)$ is the rate at which the height of the tree is changing, in meters per year, at time $t = 6$ years.

$2 : \begin{cases} 1 : \text{estimate} \\ 1 : \text{interpretation with units} \end{cases}$

(b) $\frac{H(5) - H(3)}{5 - 3} = \frac{6 - 2}{2} = 2$

Because H is differentiable on $3 \le t \le 5$, H is continuous on $3 \le t \le 5$.

By the Mean Value Theorem, there exists a value c, $3 < c < 5$, such that $H'(c) = 2$.

$2 : \begin{cases} 1 : \frac{H(5) - H(3)}{5 - 3} \\ 1 : \text{conclusion using Mean Value Theorem} \end{cases}$

(c) The average height of the tree over the time interval $2 \le t \le 10$ is given by $\frac{1}{10 - 2}\int_2^{10} H(t)\,dt$.

$$\frac{1}{8}\int_2^{10} H(t)\,dt \approx \frac{1}{8}\left(\frac{1.5 + 2}{2}\cdot 1 + \frac{2 + 6}{2}\cdot 2 + \frac{6 + 11}{2}\cdot 2 + \frac{11 + 15}{2}\cdot 3\right)$$
$$= \frac{1}{8}(65.75) = \frac{263}{32}$$

The average height of the tree over the time interval $2 \le t \le 10$ is $\frac{263}{32}$ meters.

$2 : \begin{cases} 1 : \text{trapezoidal sum} \\ 1 : \text{approximation} \end{cases}$

(d) $G(x) = 50 \Rightarrow x = 1$

$$\frac{d}{dt}(G(x)) = \frac{d}{dx}(G(x))\cdot\frac{dx}{dt} = \frac{(1 + x)100 - 100x \cdot 1}{(1 + x)^2}\cdot\frac{dx}{dt} = \frac{100}{(1 + x)^2}\cdot\frac{dx}{dt}$$

$$\left.\frac{d}{dt}(G(x))\right|_{x=1} = \frac{100}{(1 + 1)^2}\cdot 0.03 = \frac{3}{4}$$

According to the model, the rate of change of the height of the tree with respect to time when the tree is 50 meters tall is $\frac{3}{4}$ meter per year.

$3 : \begin{cases} 2 : \frac{d}{dt}(G(x)) \\ 1 : \text{answer} \end{cases}$

Note: max $1/3$ [1-0] if no chain rule

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Question 5

(a) The average rate of change of f on the interval $0 \le x \le \pi$ is $\frac{f(\pi) - f(0)}{\pi - 0} = \frac{-e^{\pi} - 1}{\pi}$.

1 : answer

(b) $f'(x) = e^x \cos x - e^x \sin x$

$$f'\left(\frac{3\pi}{2}\right) = e^{3\pi/2} \cos\left(\frac{3\pi}{2}\right) - e^{3\pi/2} \sin\left(\frac{3\pi}{2}\right) = e^{3\pi/2}$$

The slope of the line tangent to the graph of f at $x = \frac{3\pi}{2}$ is $e^{3\pi/2}$.

2 : { 1 : $f'(x)$
1 : slope }

(c) $f'(x) = 0 \Rightarrow \cos x - \sin x = 0 \Rightarrow x = \frac{\pi}{4}, x = \frac{5\pi}{4}$

x	$f(x)$
0	1
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}} e^{\pi/4}$
$\frac{5\pi}{4}$	$-\frac{1}{\sqrt{2}} e^{5\pi/4}$
2π	$e^{2\pi}$

The absolute minimum value of f on $0 \le x \le 2\pi$ is $-\frac{1}{\sqrt{2}} e^{5\pi/4}$.

3 : { 1 : sets $f'(x) = 0$
1 : identifies $x = \frac{\pi}{4}, x = \frac{5\pi}{4}$ as candidates
1 : answer with justification }

(d) $\lim_{x \to \pi/2} f(x) = 0$

Because g is differentiable, g is continuous.

$\lim_{x \to \pi/2} g(x) = g\left(\frac{\pi}{2}\right) = 0$

By L'Hospital's Rule,

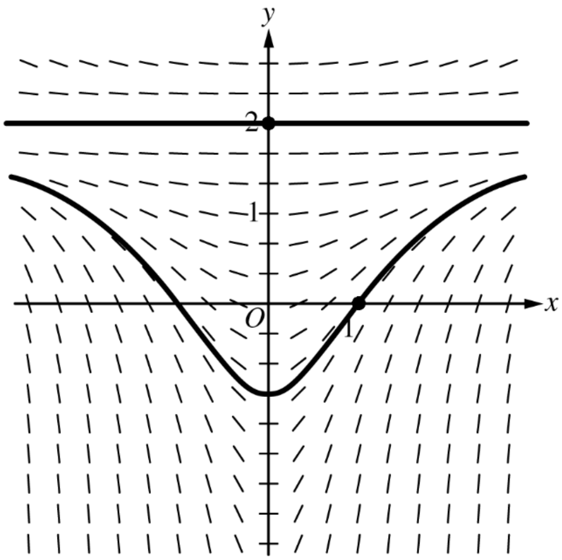
$$\lim_{x \to \pi/2} \frac{f(x)}{g(x)} = \lim_{x \to \pi/2} \frac{f'(x)}{g'(x)} = \frac{-e^{\pi/2}}{2}.$$

3 : { 1 : g is continuous at $x = \frac{\pi}{2}$ and limits equal 0
1 : applies L'Hospital's Rule
1 : answer }

Note: max $1/3$ [1-0-0] if no limit notation attached to a ratio of derivatives

Question 6

(a)



$2: \begin{cases} 1: \text{solution curve through } (0, 2) \\ 1: \text{solution curve through } (1, 0) \end{cases}$

Curves must go through the indicated points, follow the given slope lines, and extend to the boundary of the slope field.

(b) $\left.\dfrac{dy}{dx}\right|_{(x,\,y)=(1,\,0)} = \dfrac{4}{3}$

An equation for the line tangent to the graph of $y = f(x)$ at $x = 1$ is $y = \frac{4}{3}(x-1)$.

$f(0.7) \approx \frac{4}{3}(0.7 - 1) = -0.4$

$2: \begin{cases} 1: \text{equation of tangent line} \\ 1: \text{approximation} \end{cases}$

(c) $\dfrac{dy}{dx} = \dfrac{1}{3}x(y-2)^2$

$$\int \frac{dy}{(y-2)^2} = \int \frac{1}{3}x\,dx$$

$$\frac{-1}{y-2} = \frac{1}{6}x^2 + C$$

$$\frac{1}{2} = \frac{1}{6} + C \Rightarrow C = \frac{1}{3}$$

$$\frac{-1}{y-2} = \frac{1}{6}x^2 + \frac{1}{3} = \frac{x^2+2}{6}$$

$$y = 2 - \frac{6}{x^2+2}$$

Note: this solution is valid for $-\infty < x < \infty$.

$5: \begin{cases} 1: \text{separation of variables} \\ 2: \text{antiderivatives} \\ 1: \text{constant of integration and uses initial condition} \\ 1: \text{solves for } y \end{cases}$

Note: $0/5$ if no separation of variables

Note: max $3/5$ [1-2-0-0] if no constant of integration