

2019

AP[®]

 CollegeBoard

AP[®] Calculus AB

Free-Response Questions

CALCULUS AB

SECTION II, Part A

Time—30 minutes

Number of questions—2

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

1. Fish enter a lake at a rate modeled by the function E given by $E(t) = 20 + 15 \sin\left(\frac{\pi t}{6}\right)$. Fish leave the lake at a rate modeled by the function L given by $L(t) = 4 + 2^{0.1t^2}$. Both $E(t)$ and $L(t)$ are measured in fish per hour, and t is measured in hours since midnight ($t = 0$).
- (a) How many fish enter the lake over the 5-hour period from midnight ($t = 0$) to 5 A.M. ($t = 5$)? Give your answer to the nearest whole number.
- (b) What is the average number of fish that leave the lake per hour over the 5-hour period from midnight ($t = 0$) to 5 A.M. ($t = 5$)?
- (c) At what time t , for $0 \leq t \leq 8$, is the greatest number of fish in the lake? Justify your answer.
- (d) Is the rate of change in the number of fish in the lake increasing or decreasing at 5 A.M. ($t = 5$)? Explain your reasoning.
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t (hours)	0	0.3	1.7	2.8	4
$v_P(t)$ (meters per hour)	0	55	-29	55	48

2. The velocity of a particle, P , moving along the x -axis is given by the differentiable function v_P , where $v_P(t)$ is measured in meters per hour and t is measured in hours. Selected values of $v_P(t)$ are shown in the table above. Particle P is at the origin at time $t = 0$.
- (a) Justify why there must be at least one time t , for $0.3 \leq t \leq 2.8$, at which $v_P'(t)$, the acceleration of particle P , equals 0 meters per hour per hour.
- (b) Use a trapezoidal sum with the three subintervals $[0, 0.3]$, $[0.3, 1.7]$, and $[1.7, 2.8]$ to approximate the value of $\int_0^{2.8} v_P(t) dt$.
- (c) A second particle, Q , also moves along the x -axis so that its velocity for $0 \leq t \leq 4$ is given by $v_Q(t) = 45\sqrt{t} \cos(0.063t^2)$ meters per hour. Find the time interval during which the velocity of particle Q is at least 60 meters per hour. Find the distance traveled by particle Q during the interval when the velocity of particle Q is at least 60 meters per hour.
- (d) At time $t = 0$, particle Q is at position $x = -90$. Using the result from part (b) and the function v_Q from part (c), approximate the distance between particles P and Q at time $t = 2.8$.
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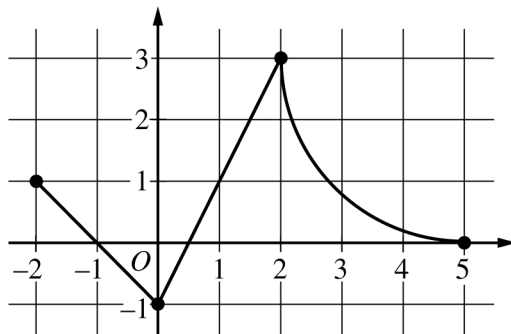
END OF PART A OF SECTION II

CALCULUS AB
SECTION II, Part B

Time—1 hour

Number of questions—4

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.



Graph of f

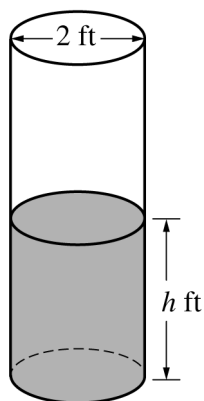
3. The continuous function f is defined on the closed interval $-6 \leq x \leq 5$. The figure above shows a portion of the graph of f , consisting of two line segments and a quarter of a circle centered at the point $(5, 3)$. It is known that the point $(3, 3 - \sqrt{5})$ is on the graph of f .

(a) If $\int_{-6}^5 f(x) dx = 7$, find the value of $\int_{-6}^{-2} f(x) dx$. Show the work that leads to your answer.

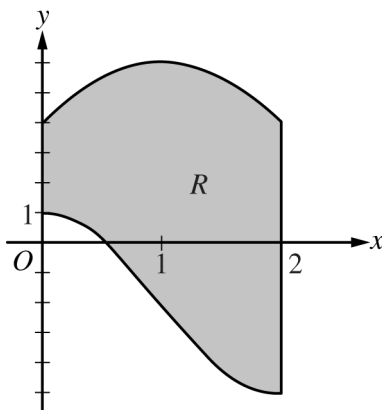
(b) Evaluate $\int_3^5 (2f'(x) + 4) dx$.

(c) The function g is given by $g(x) = \int_{-2}^x f(t) dt$. Find the absolute maximum value of g on the interval $-2 \leq x \leq 5$. Justify your answer.

(d) Find $\lim_{x \rightarrow 1} \frac{10^x - 3f'(x)}{f(x) - \arctan x}$.



4. A cylindrical barrel with a diameter of 2 feet contains collected rainwater, as shown in the figure above. The water drains out through a valve (not shown) at the bottom of the barrel. The rate of change of the height h of the water in the barrel with respect to time t is modeled by $\frac{dh}{dt} = -\frac{1}{10}\sqrt{h}$, where h is measured in feet and t is measured in seconds. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)
- (a) Find the rate of change of the volume of water in the barrel with respect to time when the height of the water is 4 feet. Indicate units of measure.
- (b) When the height of the water is 3 feet, is the rate of change of the height of the water with respect to time increasing or decreasing? Explain your reasoning.
- (c) At time $t = 0$ seconds, the height of the water is 5 feet. Use separation of variables to find an expression for h in terms of t .
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5. Let R be the region enclosed by the graphs of $g(x) = -2 + 3 \cos\left(\frac{\pi}{2}x\right)$ and $h(x) = 6 - 2(x - 1)^2$, the y -axis, and the vertical line $x = 2$, as shown in the figure above.

(a) Find the area of R .

(b) Region R is the base of a solid. For the solid, at each x the cross section perpendicular to the x -axis has

area $A(x) = \frac{1}{x+3}$. Find the volume of the solid.

(c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 6$.

6. Functions f , g , and h are twice-differentiable functions with $g(2) = h(2) = 4$. The line $y = 4 + \frac{2}{3}(x - 2)$ is tangent to both the graph of g at $x = 2$ and the graph of h at $x = 2$.
- (a) Find $h'(2)$.
- (b) Let a be the function given by $a(x) = 3x^3h(x)$. Write an expression for $a'(x)$. Find $a'(2)$.
- (c) The function h satisfies $h(x) = \frac{x^2 - 4}{1 - (f(x))^3}$ for $x \neq 2$. It is known that $\lim_{x \rightarrow 2} h(x)$ can be evaluated using L'Hospital's Rule. Use $\lim_{x \rightarrow 2} h(x)$ to find $f(2)$ and $f'(2)$. Show the work that leads to your answers.
- (d) It is known that $g(x) \leq h(x)$ for $1 < x < 3$. Let k be a function satisfying $g(x) \leq k(x) \leq h(x)$ for $1 < x < 3$. Is k continuous at $x = 2$? Justify your answer.
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STOP
END OF EXAM