



AP Calculus BC 2000 Scoring Guidelines

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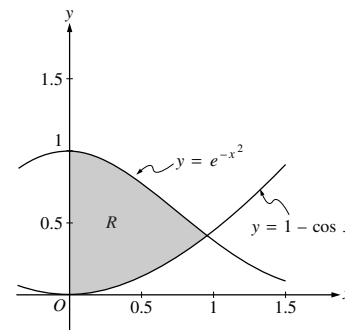
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AP Calculus AB-1 / BC-1

Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$, and the y -axis, as shown in the figure above.

- Find the area of the region R .
- Find the volume of the solid generated when the region R is revolved about the x -axis.
- The region R is the base of a solid. For this solid, each cross section perpendicular to the x -axis is a square. Find the volume of this solid.



Region R

$$e^{-x^2} = 1 - \cos x \text{ at } x = 0.941944 = A$$

$$\begin{aligned} \text{(a) Area} &= \int_0^A (e^{-x^2} - (1 - \cos x)) dx \\ &= 0.590 \text{ or } 0.591 \end{aligned}$$

$$\begin{aligned} \text{(b) Volume} &= \pi \int_0^A \left((e^{-x^2})^2 - (1 - \cos x)^2 \right) dx \\ &= 0.55596\pi = 1.746 \text{ or } 1.747 \end{aligned}$$

$$\begin{aligned} \text{(c) Volume} &= \int_0^A (e^{-x^2} - (1 - \cos x))^2 dx \\ &= 0.461 \end{aligned}$$

1 : Correct limits in an integral in (a), (b), or (c).

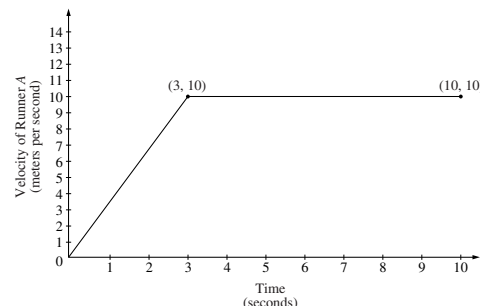
2 $\left\{ \begin{array}{l} 1 : \text{integrand} \\ 1 : \text{answer} \end{array} \right.$

3 $\left\{ \begin{array}{l} 2 : \text{integrand and constant} \\ < -1 > \text{ each error} \\ 1 : \text{answer} \end{array} \right.$

3 $\left\{ \begin{array}{l} 2 : \text{integrand} \\ < -1 > \text{ each error} \\ \text{Note: } 0/2 \text{ if not of the form} \\ \quad k \int_c^d (f(x) - g(x))^2 dx \\ 1 : \text{answer} \end{array} \right.$

AP Calculus AB–2 / BC–2

Two runners, A and B , run on a straight racetrack for $0 \leq t \leq 10$ seconds. The graph above, which consists of two line segments, shows the velocity, in meters per second, of Runner A . The velocity, in meters per second, of Runner B is given by the function v defined by $v(t) = \frac{24t}{2t + 3}$.



- (a) Find the velocity of Runner A and the velocity of Runner B at time $t = 2$ seconds. Indicate units of measure.
- (b) Find the acceleration of Runner A and the acceleration of Runner B at time $t = 2$ seconds. Indicate units of measure.
- (c) Find the total distance run by Runner A and the total distance run by Runner B over the time interval $0 \leq t \leq 10$ seconds. Indicate units of measure.

(a) Runner A : velocity $= \frac{10}{3} \cdot 2 = \frac{20}{3}$
 $= 6.666$ or 6.667 meters/sec

Runner B : $v(2) = \frac{48}{7} = 6.857$ meters/sec

(b) Runner A : acceleration $= \frac{10}{3} = 3.333$ meters/sec²

Runner B : $a(2) = v'(2) = \frac{72}{(2t + 3)^2} \Big|_{t=2}$
 $= \frac{72}{49} = 1.469$ meters/sec²

(c) Runner A : distance $= \frac{1}{2}(3)(10) + 7(10) = 85$ meters

Runner B : distance $= \int_0^{10} \frac{24t}{2t + 3} dt = 83.336$ meters

2 { 1 : velocity for Runner A
1 : velocity for Runner B

2 { 1 : acceleration for Runner A
1 : acceleration for Runner B

4 { 2 : distance for Runner A
1 : method
1 : answer
2 : distance for Runner B
1 : integral
1 : answer

(units) meters/sec in part (a), meters/sec² in part (b), and meters in part (c), or equivalent.

1: units

AP Calculus BC-3

The Taylor series about $x = 5$ for a certain function f converges to $f(x)$ for all x in the interval of convergence. The n th derivative of f at $x = 5$ is given by $f^{(n)}(5) = \frac{(-1)^n n!}{2^n (n + 2)}$, and $f(5) = \frac{1}{2}$.

- (a) Write the third-degree Taylor polynomial for f about $x = 5$.
- (b) Find the radius of convergence of the Taylor series for f about $x = 5$.
- (c) Show that the sixth-degree Taylor polynomial for f about $x = 5$ approximates $f(6)$ with error less than $\frac{1}{1000}$.

(a) $f'(5) = \frac{-1!}{2(3)}, f''(5) = \frac{2!}{4(4)}, f'''(5) = \frac{-3!}{8(5)}$

$$P_3(f, 5)(x) = \frac{1}{2} - \frac{1}{6}(x - 5) + \frac{1}{16}(x - 5)^2 - \frac{1}{40}(x - 5)^3$$

(b) $a_n = \frac{f^{(n)}(5)}{n!} = \frac{(-1)^n}{2^n (n + 2)}$

$$\lim_{n \rightarrow \infty} \left| \frac{\frac{(-1)^{n+1}(x - 5)^{n+1}}{2^{n+1}(n + 3)}}{\frac{(-1)^n(x - 5)^n}{2^n(n + 2)}} \right| = \lim_{n \rightarrow \infty} \frac{1}{2} \left(\frac{n + 2}{n + 3} \right) |x - 5|$$

$$= \frac{|x - 5|}{2} < 1$$

The radius of convergence is 2.

- (c) The Taylor series about $x = 5$ for the function f , when evaluated at $x = 6$, is an alternating series with absolute value of terms decreasing to 0. The error in approximating $f(6)$ with the 6th degree Taylor polynomial at $x = 6$ is less than the first omitted term in the series.

$$|f(6) - P_6(f, 5)(6)| \leq \frac{1}{2^7(9)} = \frac{1}{1152} < \frac{1}{1000}$$

3 : $P_3(f, 5)(x)$

<-1> each error or missing term

Note: <-1> max for improper use of extra terms, equality or +...

- 1 : general term
- 1 : sets up ratio test
- 4 { 1 : computes the limit
- 1 : applies ratio test to get radius of convergence

- 2 { 1 : error bound $< \frac{1}{1000}$
- 1 : refers to an alternating series and indicates the error bound is found from the next term

AP Calculus BC-4

A moving particle has position $(x(t), y(t))$ at time t . The position of the particle at time $t = 1$ is $(2, 6)$ and the velocity vector at any time $t > 0$ is given by $\left(1 - \frac{1}{t^2}, 2 + \frac{1}{t^2}\right)$.

- (a) Find the acceleration vector at time $t = 3$.
 (b) Find the position of the particle at time $t = 3$.
 (c) For what time $t > 0$ does the line tangent to the path of the particle at $(x(t), y(t))$ have a slope of 8?
 (d) The particle approaches a line as $t \rightarrow \infty$. Find the slope of this line. Show the work that leads to your conclusion.

(a) acceleration vector = $(x''(t), y''(t)) = \left(\frac{2}{t^3}, -\frac{2}{t^3}\right)$

$$(x''(3), y''(3)) = \left(\frac{2}{27}, -\frac{2}{27}\right)$$

$$2 \begin{cases} 1: \text{ components of acceleration} \\ \text{vector as a function of } t \\ 1: \text{ acceleration vector at } t = 3 \end{cases}$$

(b) $(x(t), y(t)) = \left(t + \frac{1}{t} + C_1, 2t - \frac{1}{t} + C_2\right)$

$$(2, 6) = (x(1), y(1)) = (2 + C_1, 1 + C_2)$$

$$C_1 = 0, C_2 = 5$$

$$(x(3), y(3)) = \left(3 + \frac{1}{3}, 6 - \frac{1}{3} + 5\right) = \left(\frac{10}{3}, \frac{32}{3}\right)$$

$$3 \begin{cases} 1: \text{ antidifferentiation} \\ 1: \text{ uses initial condition at } t = 1 \\ 1: \text{ position at } t = 3 \end{cases}$$

Note: max 1/3 [1-0-0] if no constants of integration

(c) $\frac{dy}{dx} = \frac{2 + \frac{1}{t^2}}{1 - \frac{1}{t^2}} = 8$

$$2 + \frac{1}{t^2} = 8\left(1 - \frac{1}{t^2}\right); \quad t^2 = \frac{9}{6}$$

$$t = \sqrt{\frac{3}{2}}$$

$$2 \begin{cases} 1: \frac{dy}{dx} = 8 \text{ as equation in } t \\ 1: \text{ solution for } t \end{cases}$$

(d) $\lim_{t \rightarrow \infty} \frac{dy}{dx} = \lim_{t \rightarrow \infty} \frac{2 + \frac{1}{t^2}}{1 - \frac{1}{t^2}} = 2$

– or –

Since $x(t) \rightarrow \infty$ as $t \rightarrow \infty$, the slope of the line is

$$\lim_{t \rightarrow \infty} \frac{y(t)}{x(t)} = \lim_{t \rightarrow \infty} \frac{2t - \frac{1}{t} + 5}{t + \frac{1}{t}} = 2$$

$$2 \begin{cases} 1: \text{ considers limit of } \frac{dy}{dx} \text{ or } \frac{y(t)}{x(t)} \\ 1: \text{ answer} \end{cases}$$

Note: 0/2 if no consideration of limit

AP Calculus AB–5 / BC–5

Consider the curve given by $xy^2 - x^3y = 6$.

- (a) Show that $\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$.
- (b) Find all points on the curve whose x -coordinate is 1, and write an equation for the tangent line at each of these points.
- (c) Find the x -coordinate of each point on the curve where the tangent line is vertical.

(a) $y^2 + 2xy \frac{dy}{dx} - 3x^2y - x^3 \frac{dy}{dx} = 0$

$$\frac{dy}{dx}(2xy - x^3) = 3x^2y - y^2$$

$$\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$$

(b) When $x = 1$, $y^2 - y = 6$
 $y^2 - y - 6 = 0$
 $(y - 3)(y + 2) = 0$
 $y = 3, y = -2$

At $(1, 3)$, $\frac{dy}{dx} = \frac{9 - 9}{6 - 1} = 0$

Tangent line equation is $y = 3$

At $(1, -2)$, $\frac{dy}{dx} = \frac{-6 - 4}{-4 - 1} = \frac{-10}{-5} = 2$

Tangent line equation is $y + 2 = 2(x - 1)$

(c) Tangent line is vertical when $2xy - x^3 = 0$

$$x(2y - x^2) = 0 \text{ gives } x = 0 \text{ or } y = \frac{1}{2}x^2$$

There is no point on the curve with x -coordinate 0.

When $y = \frac{1}{2}x^2$, $\frac{1}{4}x^5 - \frac{1}{2}x^5 = 6$

$$-\frac{1}{4}x^5 = 6$$

$$x = \sqrt[5]{-24}$$

$$2 \left\{ \begin{array}{l} 1 : \text{implicit differentiation} \\ 1 : \text{verifies expression for } \frac{dy}{dx} \end{array} \right.$$

$$4 \left\{ \begin{array}{l} 1 : y^2 - y = 6 \\ 1 : \text{solves for } y \\ 2 : \text{tangent lines} \end{array} \right.$$

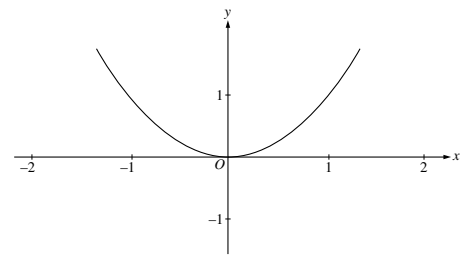
Note: 0/4 if not solving an equation of the form $y^2 - y = k$

$$3 \left\{ \begin{array}{l} 1 : \text{sets denominator of } \frac{dy}{dx} \text{ equal to 0} \\ 1 : \text{substitutes } y = \frac{1}{2}x^2 \text{ or } x = \pm\sqrt{2y} \\ \text{into the equation for the curve} \\ 1 : \text{solves for } x\text{-coordinate} \end{array} \right.$$

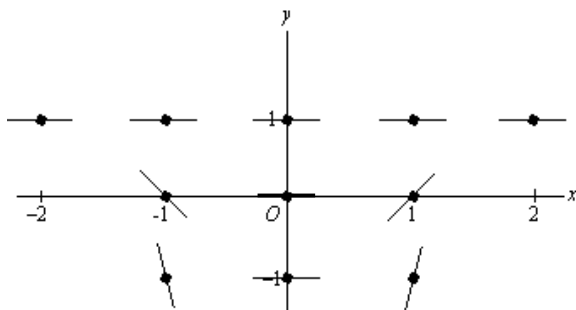
AP Calculus BC-6

Consider the differential equation given by $\frac{dy}{dx} = x(y - 1)^2$.

- (a) On the axes provided, sketch a slope field for the given differential equation at the eleven points indicated.
- (b) Use the slope field for the given differential equation to explain why a solution could not have the graph shown below.
- (c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = -1$.
- (d) Find the range of the solution found in part (c).



(a)



(b) The graph does not have slope 0 where $y = 1$.
- or -

The slope field shown suggests that solutions are asymptotic to $y = 1$ from below, but the graph does not exhibit this behavior.

(c) $\frac{1}{(y - 1)^2} dy = x dx$

$$-\frac{1}{y - 1} = \frac{1}{2}x^2 + C$$

$$\frac{1}{2} = 0 + C; \quad C = \frac{1}{2}$$

$$-\frac{1}{y - 1} = \frac{1}{2}(x^2 + 1); \quad y = 1 - \frac{2}{x^2 + 1}$$

(d) range is $-1 \leq y < 1$

- 1 : zero slope at 7 points with $y = 1$ and $x = 0$
- 2 { 1 : negative slope at $(-1, 0)$ and $(-1, -1)$
positive slope at $(1, 0)$ and $(1, -1)$
steeper slope at $y = -1$ than $y = 0$

1 : reason

- 5 { 1 : separates variables
1 : antiderivatives
1 : constant of integration
1 : uses initial condition $f(0) = -1$
1 : solves for y
0/1 if y is linear

Note: max 2/5 [1-1-0-0-0] if no constant of integration

Note: 0/5 if no separation of variables

1 : answer

0/1 if -1 not in range