



## AP Calculus BC 2001 Free-Response Questions

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**CALCULUS BC**  
**SECTION II, Part A**  
**Time—45 minutes**  
**Number of problems—3**

**A graphing calculator is required for some problems or parts of problems.**

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1. An object moving along a curve in the  $xy$ -plane has position  $(x(t), y(t))$  at time  $t$  with

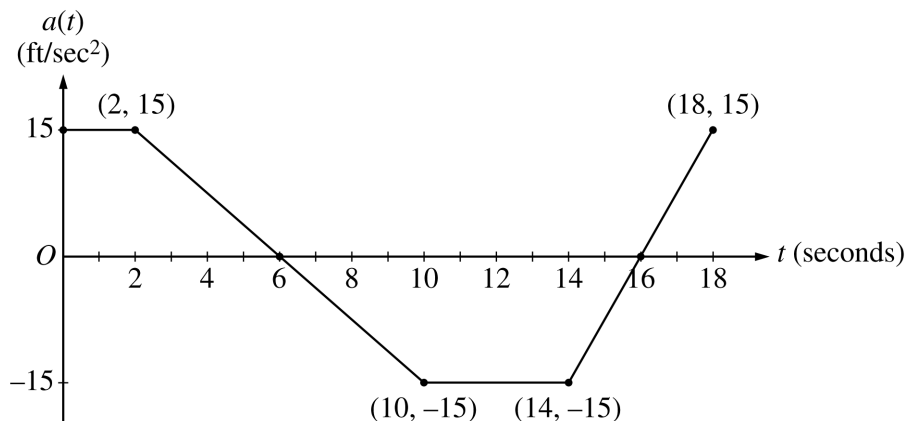
$$\frac{dx}{dt} = \cos(t^3) \text{ and } \frac{dy}{dt} = 3 \sin(t^2)$$

for  $0 \leq t \leq 3$ . At time  $t = 2$ , the object is at position  $(4, 5)$ .

- (a) Write an equation for the line tangent to the curve at  $(4, 5)$ .
  - (b) Find the speed of the object at time  $t = 2$ .
  - (c) Find the total distance traveled by the object over the time interval  $0 \leq t \leq 1$ .
  - (d) Find the position of the object at time  $t = 3$ .
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$t$ (days)	$W(t)$ (°C)
0	20
3	31
6	28
9	24
12	22
15	21

2. The temperature, in degrees Celsius (°C), of the water in a pond is a differentiable function  $W$  of time  $t$ . The table above shows the water temperature as recorded every 3 days over a 15-day period.
- (a) Use data from the table to find an approximation for  $W'(12)$ . Show the computations that lead to your answer. Indicate units of measure.
- (b) Approximate the average temperature, in degrees Celsius, of the water over the time interval  $0 \leq t \leq 15$  days by using a trapezoidal approximation with subintervals of length  $\Delta t = 3$  days.
- (c) A student proposes the function  $P$ , given by  $P(t) = 20 + 10te^{(-t/3)}$ , as a model for the temperature of the water in the pond at time  $t$ , where  $t$  is measured in days and  $P(t)$  is measured in degrees Celsius. Find  $P'(12)$ . Using appropriate units, explain the meaning of your answer in terms of water temperature.
- (d) Use the function  $P$  defined in part (c) to find the average value, in degrees Celsius, of  $P(t)$  over the time interval  $0 \leq t \leq 15$  days.
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3. A car is traveling on a straight road with velocity 55 ft/sec at time  $t = 0$ . For  $0 \leq t \leq 18$  seconds, the car's acceleration  $a(t)$ , in  $\text{ft/sec}^2$ , is the piecewise linear function defined by the graph above.
- Is the velocity of the car increasing at  $t = 2$  seconds? Why or why not?
  - At what time in the interval  $0 \leq t \leq 18$ , other than  $t = 0$ , is the velocity of the car 55 ft/sec? Why?
  - On the time interval  $0 \leq t \leq 18$ , what is the car's absolute maximum velocity, in ft/sec, and at what time does it occur? Justify your answer.
  - At what times in the interval  $0 \leq t \leq 18$ , if any, is the car's velocity equal to zero? Justify your answer.
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**END OF PART A OF SECTION II**

**CALCULUS BC**  
**SECTION II, Part B**  
**Time—45 minutes**  
**Number of problems—3**

**No calculator is allowed for these problems.**

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4. Let  $h$  be a function defined for all  $x \neq 0$  such that  $h(4) = -3$  and the derivative of  $h$  is given by

$$h'(x) = \frac{x^2 - 2}{x} \text{ for all } x \neq 0.$$

- (a) Find all values of  $x$  for which the graph of  $h$  has a horizontal tangent, and determine whether  $h$  has a local maximum, a local minimum, or neither at each of these values. Justify your answers.
- (b) On what intervals, if any, is the graph of  $h$  concave up? Justify your answer.
- (c) Write an equation for the line tangent to the graph of  $h$  at  $x = 4$ .
- (d) Does the line tangent to the graph of  $h$  at  $x = 4$  lie above or below the graph of  $h$  for  $x > 4$ ? Why?

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5. Let  $f$  be the function satisfying  $f'(x) = -3xf(x)$ , for all real numbers  $x$ , with  $f(1) = 4$  and  $\lim_{x \rightarrow \infty} f(x) = 0$ .

- (a) Evaluate  $\int_1^{\infty} -3xf(x)dx$ . Show the work that leads to your answer.
  - (b) Use Euler's method, starting at  $x = 1$  with a step size of 0.5, to approximate  $f(2)$ .
  - (c) Write an expression for  $y = f(x)$  by solving the differential equation  $\frac{dy}{dx} = -3xy$  with the initial condition  $f(1) = 4$ .
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6. A function  $f$  is defined by

$$f(x) = \frac{1}{3} + \frac{2}{3^2}x + \frac{3}{3^3}x^2 + \cdots + \frac{n+1}{3^{n+1}}x^n + \cdots$$

for all  $x$  in the interval of convergence of the given power series.

(a) Find the interval of convergence for this power series. Show the work that leads to your answer.

(b) Find  $\lim_{x \rightarrow 0} \frac{f(x) - \frac{1}{3}}{x}$ .

(c) Write the first three nonzero terms and the general term for an infinite series that represents  $\int_0^1 f(x) dx$ .

(d) Find the sum of the series determined in part (c).

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**END OF EXAMINATION**