



## AP<sup>®</sup> Calculus BC 2002 Free-Response Questions

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**CALCULUS BC**  
**SECTION II, Part A**  
**Time—45 minutes**  
**Number of problems—3**

**A graphing calculator is required for some problems or parts of problems.**

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1. Let  $f$  and  $g$  be the functions given by  $f(x) = e^{-x}$  and  $g(x) = \ln x$ .
- (a) Find the area of the region enclosed by the graphs of  $f$  and  $g$  between  $x = \frac{1}{2}$  and  $x = 1$ .
- (b) Find the volume of the solid generated when the region enclosed by the graphs of  $f$  and  $g$  between  $x = \frac{1}{2}$  and  $x = 1$  is revolved about the line  $y = 4$ .
- (c) Let  $h$  be the function given by  $h(x) = f(x) - g(x)$ . Find the absolute minimum value of  $h(x)$  on the closed interval  $\frac{1}{2} \leq x \leq 1$ , and find the absolute maximum value of  $h(x)$  on the closed interval  $\frac{1}{2} \leq x \leq 1$ . Show the analysis that leads to your answers.
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2. The rate at which people enter an amusement park on a given day is modeled by the function  $E$  defined by

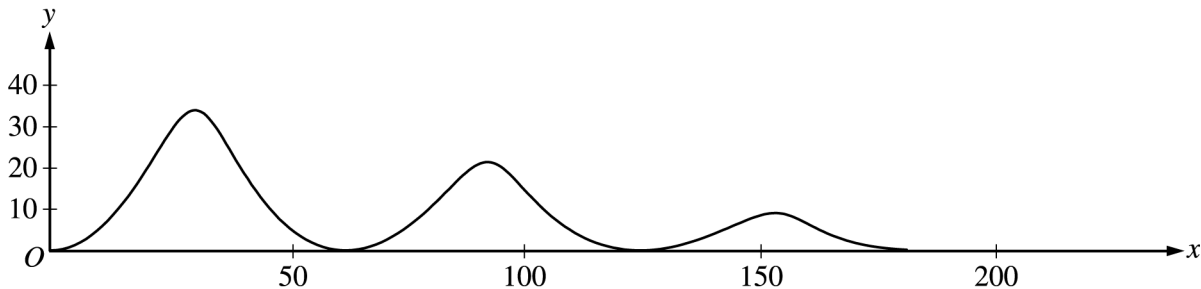
$$E(t) = \frac{15600}{(t^2 - 24t + 160)}.$$

The rate at which people leave the same amusement park on the same day is modeled by the function  $L$  defined by

$$L(t) = \frac{9890}{(t^2 - 38t + 370)}.$$

Both  $E(t)$  and  $L(t)$  are measured in people per hour and time  $t$  is measured in hours after midnight. These functions are valid for  $9 \leq t \leq 23$ , the hours during which the park is open. At time  $t = 9$ , there are no people in the park.

- (a) How many people have entered the park by 5:00 P.M. ( $t = 17$ )? Round your answer to the nearest whole number.
- (b) The price of admission to the park is \$15 until 5:00 P.M. ( $t = 17$ ). After 5:00 P.M., the price of admission to the park is \$11. How many dollars are collected from admissions to the park on the given day? Round your answer to the nearest whole number.
- (c) Let  $H(t) = \int_9^t (E(x) - L(x)) dx$  for  $9 \leq t \leq 23$ . The value of  $H(17)$  to the nearest whole number is 3725. Find the value of  $H'(17)$ , and explain the meaning of  $H(17)$  and  $H'(17)$  in the context of the amusement park.
- (d) At what time  $t$ , for  $9 \leq t \leq 23$ , does the model predict that the number of people in the park is a maximum?
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3. The figure above shows the path traveled by a roller coaster car over the time interval  $0 \leq t \leq 18$  seconds. The position of the car at time  $t$  seconds can be modeled parametrically by

$$x(t) = 10t + 4 \sin t$$

$$y(t) = (20 - t)(1 - \cos t),$$

where  $x$  and  $y$  are measured in meters. The derivatives of these functions are given by

$$x'(t) = 10 + 4 \cos t$$

$$y'(t) = (20 - t) \sin t + \cos t - 1.$$

- Find the slope of the path at time  $t = 2$ . Show the computations that lead to your answer.
- Find the acceleration vector of the car at the time when the car's horizontal position is  $x = 140$ .
- Find the time  $t$  at which the car is at its maximum height, and find the speed, in m/sec, of the car at this time.
- For  $0 < t < 18$ , there are two times at which the car is at ground level ( $y = 0$ ). Find these two times and write an expression that gives the average speed, in m/sec, of the car between these two times. Do not evaluate the expression.

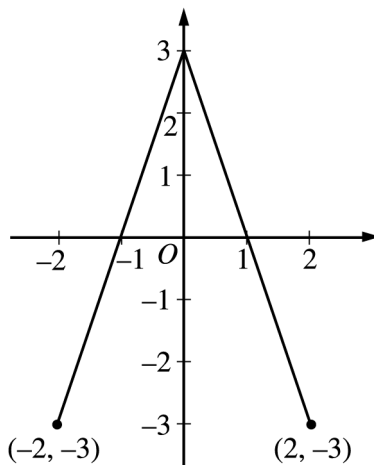
**END OF PART A OF SECTION II**

CALCULUS BC  
SECTION II, Part B

Time—45 minutes

Number of problems—3

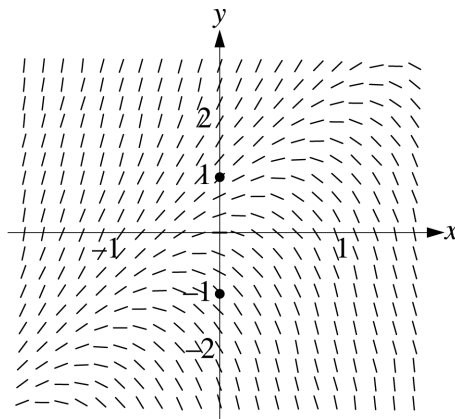
No calculator is allowed for these problems.

Graph of  $f$ 

4. The graph of the function  $f$  shown above consists of two line segments. Let  $g$  be the function given by  $g(x) = \int_0^x f(t) dt$ .
- (a) Find  $g(-1)$ ,  $g'(-1)$ , and  $g''(-1)$ .
- (b) For what values of  $x$  in the open interval  $(-2, 2)$  is  $g$  increasing? Explain your reasoning.
- (c) For what values of  $x$  in the open interval  $(-2, 2)$  is the graph of  $g$  concave down? Explain your reasoning.
- (d) On the axes provided, sketch the graph of  $g$  on the closed interval  $[-2, 2]$ .  
(Note: The axes are provided in the pink test booklet only.)
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5. Consider the differential equation  $\frac{dy}{dx} = 2y - 4x$ .

- (a) The slope field for the given differential equation is provided. Sketch the solution curve that passes through the point  $(0, 1)$  and sketch the solution curve that passes through the point  $(0, -1)$ .  
 (Note: Use the slope field provided in the pink test booklet.)



- (b) Let  $f$  be the function that satisfies the given differential equation with the initial condition  $f(0) = 1$ . Use Euler's method, starting at  $x = 0$  with a step size of 0.1, to approximate  $f(0.2)$ . Show the work that leads to your answer.
- (c) Find the value of  $b$  for which  $y = 2x + b$  is a solution to the given differential equation. Justify your answer.
- (d) Let  $g$  be the function that satisfies the given differential equation with the initial condition  $g(0) = 0$ . Does the graph of  $g$  have a local extremum at the point  $(0, 0)$ ? If so, is the point a local maximum or a local minimum? Justify your answer.

6. The Maclaurin series for the function  $f$  is given by

$$f(x) = \sum_{n=0}^{\infty} \frac{(2x)^{n+1}}{n+1} = 2x + \frac{4x^2}{2} + \frac{8x^3}{3} + \frac{16x^4}{4} + \cdots + \frac{(2x)^{n+1}}{n+1} + \cdots$$

on its interval of convergence.

- (a) Find the interval of convergence of the Maclaurin series for  $f$ . Justify your answer.
- (b) Find the first four terms and the general term for the Maclaurin series for  $f'(x)$ .
- (c) Use the Maclaurin series you found in part (b) to find the value of  $f'\left(-\frac{1}{3}\right)$ .

### END OF EXAMINATION