



AP[®] Calculus BC 2003 Free-Response Questions

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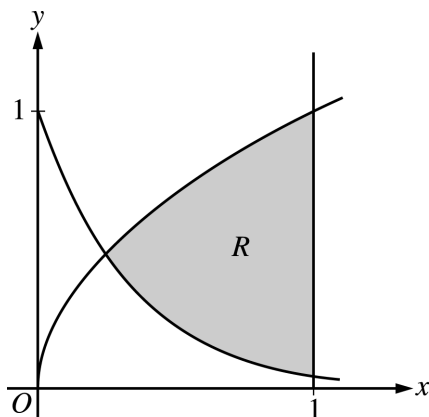
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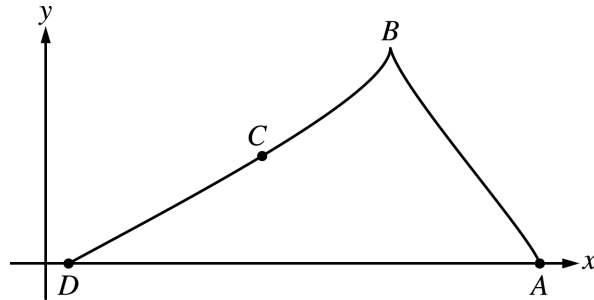
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CALCULUS BC
SECTION II, Part A
Time—45 minutes
Number of problems—3

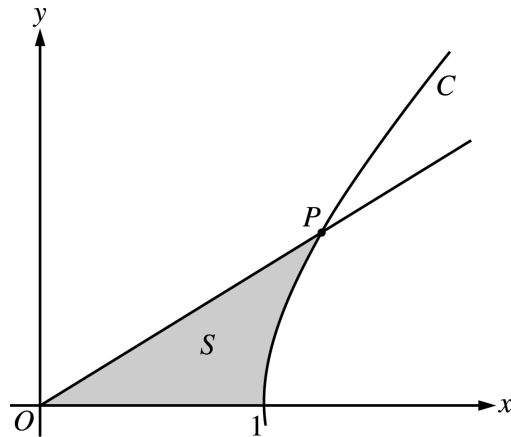
A graphing calculator is required for some problems or parts of problems.



1. Let R be the shaded region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-3x}$ and the vertical line $x = 1$, as shown in the figure above.
- Find the area of R .
 - Find the volume of the solid generated when R is revolved about the horizontal line $y = 1$.
 - The region R is the base of a solid. For this solid, each cross section perpendicular to the x -axis is a rectangle whose height is 5 times the length of its base in region R . Find the volume of this solid.
-



2. A particle starts at point A on the positive x -axis at time $t = 0$ and travels along the curve from A to B to C to D , as shown above. The coordinates of the particle's position $(x(t), y(t))$ are differentiable functions of t , where $x'(t) = \frac{dx}{dt} = -9\cos\left(\frac{\pi t}{6}\right)\sin\left(\frac{\pi\sqrt{t+1}}{2}\right)$ and $y'(t) = \frac{dy}{dt}$ is not explicitly given. At time $t = 9$, the particle reaches its final position at point D on the positive x -axis.
- At point C , is $\frac{dy}{dt}$ positive? At point C , is $\frac{dx}{dt}$ positive? Give a reason for each answer.
 - The slope of the curve is undefined at point B . At what time t is the particle at point B ?
 - The line tangent to the curve at the point $(x(8), y(8))$ has equation $y = \frac{5}{9}x - 2$. Find the velocity vector and the speed of the particle at this point.
 - How far apart are points A and D , the initial and final positions, respectively, of the particle?
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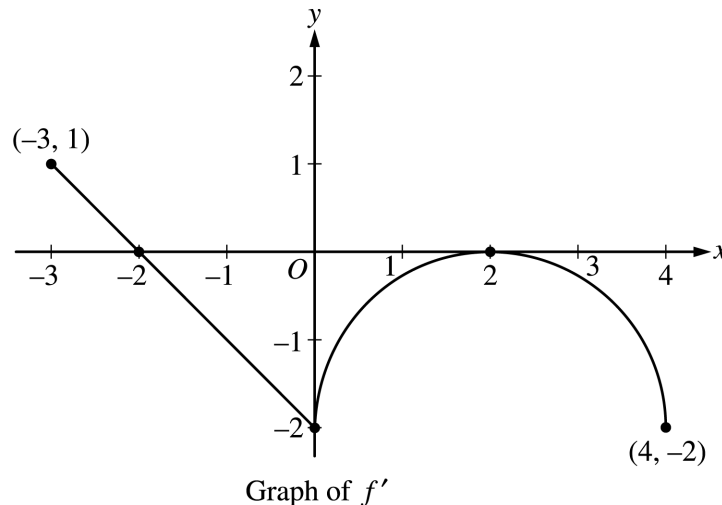


3. The figure above shows the graphs of the line $x = \frac{5}{3}y$ and the curve C given by $x = \sqrt{1 + y^2}$. Let S be the shaded region bounded by the two graphs and the x -axis. The line and the curve intersect at point P .
- Find the coordinates of point P and the value of $\frac{dx}{dy}$ for curve C at point P .
 - Set up and evaluate an integral expression with respect to y that gives the area of S .
 - Curve C is a part of the curve $x^2 - y^2 = 1$. Show that $x^2 - y^2 = 1$ can be written as the polar equation $r^2 = \frac{1}{\cos^2 \theta - \sin^2 \theta}$.
 - Use the polar equation given in part (c) to set up an integral expression with respect to the polar angle θ that represents the area of S .
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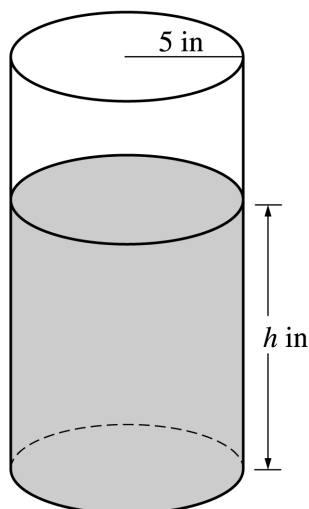
END OF PART A OF SECTION II

CALCULUS BC
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



4. Let f be a function defined on the closed interval $-3 \leq x \leq 4$ with $f(0) = 3$. The graph of f' , the derivative of f , consists of one line segment and a semicircle, as shown above.
- On what intervals, if any, is f increasing? Justify your answer.
 - Find the x -coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$. Justify your answer.
 - Find an equation for the line tangent to the graph of f at the point $(0, 3)$.
 - Find $f(-3)$ and $f(4)$. Show the work that leads to your answers.
-



5. A coffeepot has the shape of a cylinder with radius 5 inches, as shown in the figure above. Let h be the depth of the coffee in the pot, measured in inches, where h is a function of time t , measured in seconds. The volume V of coffee in the pot is changing at the rate of $-5\pi\sqrt{h}$ cubic inches per second. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)
- (a) Show that $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$.
- (b) Given that $h = 17$ at time $t = 0$, solve the differential equation $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$ for h as a function of t .
- (c) At what time t is the coffeepot empty?

6. The function f is defined by the power series

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n+1)!} = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \cdots + \frac{(-1)^n x^{2n}}{(2n+1)!} + \cdots$$

for all real numbers x .

- (a) Find $f'(0)$ and $f''(0)$. Determine whether f has a local maximum, a local minimum, or neither at $x = 0$. Give a reason for your answer.
- (b) Show that $1 - \frac{1}{3!}$ approximates $f(1)$ with error less than $\frac{1}{100}$.
- (c) Show that $y = f(x)$ is a solution to the differential equation $xy' + y = \cos x$.

END OF EXAMINATION