



AP[®] Calculus BC 2005 Free-Response Questions

The College Board: Connecting Students to College Success

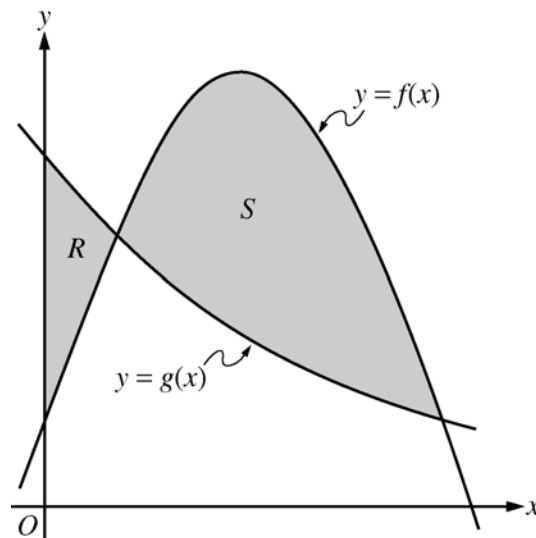
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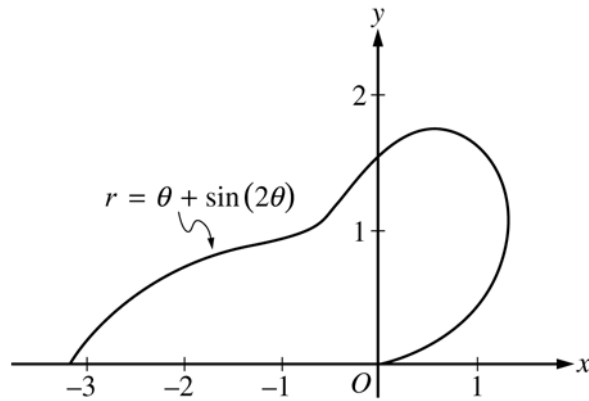
CALCULUS BC
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let f and g be the functions given by $f(x) = \frac{1}{4} + \sin(\pi x)$ and $g(x) = 4^{-x}$. Let R be the shaded region in the first quadrant enclosed by the y -axis and the graphs of f and g , and let S be the shaded region in the first quadrant enclosed by the graphs of f and g , as shown in the figure above.
- Find the area of R .
 - Find the area of S .
 - Find the volume of the solid generated when S is revolved about the horizontal line $y = -1$.
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WRITE ALL WORK IN THE TEST BOOKLET.



2. The curve above is drawn in the xy -plane and is described by the equation in polar coordinates $r = \theta + \sin(2\theta)$ for $0 \leq \theta \leq \pi$, where r is measured in meters and θ is measured in radians. The derivative of r with respect to θ is given by $\frac{dr}{d\theta} = 1 + 2\cos(2\theta)$.
- (a) Find the area bounded by the curve and the x -axis.
- (b) Find the angle θ that corresponds to the point on the curve with x -coordinate -2 .
- (c) For $\frac{\pi}{3} < \theta < \frac{2\pi}{3}$, $\frac{dr}{d\theta}$ is negative. What does this fact say about r ? What does this fact say about the curve?
- (d) Find the value of θ in the interval $0 \leq \theta \leq \frac{\pi}{2}$ that corresponds to the point on the curve in the first quadrant with greatest distance from the origin. Justify your answer.
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2005 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS

Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ ($^{\circ}\text{C}$)	100	93	70	62	55

3. A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature $T(x)$, in degrees Celsius ($^{\circ}\text{C}$), of the wire x cm from the heated end. The function T is decreasing and twice differentiable.
- (a) Estimate $T'(7)$. Show the work that leads to your answer. Indicate units of measure.
- (b) Write an integral expression in terms of $T(x)$ for the average temperature of the wire. Estimate the average temperature of the wire using a trapezoidal sum with the four subintervals indicated by the data in the table. Indicate units of measure.
- (c) Find $\int_0^8 T'(x) dx$, and indicate units of measure. Explain the meaning of $\int_0^8 T'(x) dx$ in terms of the temperature of the wire.
- (d) Are the data in the table consistent with the assertion that $T''(x) > 0$ for every x in the interval $0 < x < 8$? Explain your answer.
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END OF PART A OF SECTION II

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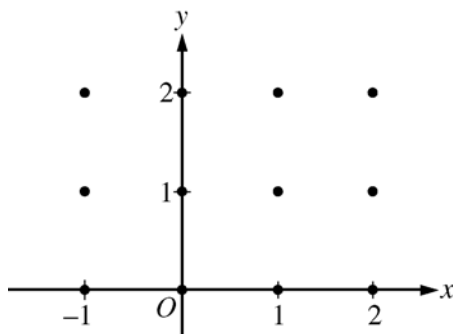
**CALCULUS BC
SECTION II, Part B
Time—45 minutes
Number of problems—3**

No calculator is allowed for these problems.

4. Consider the differential equation $\frac{dy}{dx} = 2x - y$.

- (a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated, and sketch the solution curve that passes through the point $(0, 1)$.

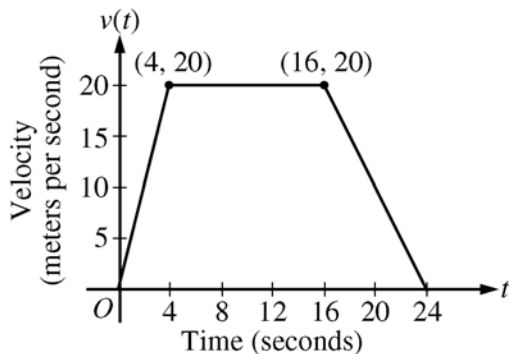
(Note: Use the axes provided in the pink test booklet.)



- (b) The solution curve that passes through the point $(0, 1)$ has a local minimum at $x = \ln\left(\frac{3}{2}\right)$. What is the y -coordinate of this local minimum?
- (c) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(0) = 1$. Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $f(-0.4)$. Show the work that leads to your answer.
- (d) Find $\frac{d^2y}{dx^2}$ in terms of x and y . Determine whether the approximation found in part (c) is less than or greater than $f(-0.4)$. Explain your reasoning.

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5. A car is traveling on a straight road. For $0 \leq t \leq 24$ seconds, the car's velocity $v(t)$, in meters per second, is modeled by the piecewise-linear function defined by the graph above.
- Find $\int_0^{24} v(t) dt$. Using correct units, explain the meaning of $\int_0^{24} v(t) dt$.
 - For each of $v'(4)$ and $v'(20)$, find the value or explain why it does not exist. Indicate units of measure.
 - Let $a(t)$ be the car's acceleration at time t , in meters per second per second. For $0 < t < 24$, write a piecewise-defined function for $a(t)$.
 - Find the average rate of change of v over the interval $8 \leq t \leq 20$. Does the Mean Value Theorem guarantee a value of c , for $8 < c < 20$, such that $v'(c)$ is equal to this average rate of change? Why or why not?
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6. Let f be a function with derivatives of all orders and for which $f(2) = 7$. When n is odd, the n th derivative of f at $x = 2$ is 0. When n is even and $n \geq 2$, the n th derivative of f at $x = 2$ is given by $f^{(n)}(2) = \frac{(n-1)!}{3^n}$.
- Write the sixth-degree Taylor polynomial for f about $x = 2$.
 - In the Taylor series for f about $x = 2$, what is the coefficient of $(x - 2)^{2n}$ for $n \geq 1$?
 - Find the interval of convergence of the Taylor series for f about $x = 2$. Show the work that leads to your answer.
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END OF EXAM