



**AP<sup>®</sup> Calculus BC  
2006 Scoring Guidelines  
Form B**

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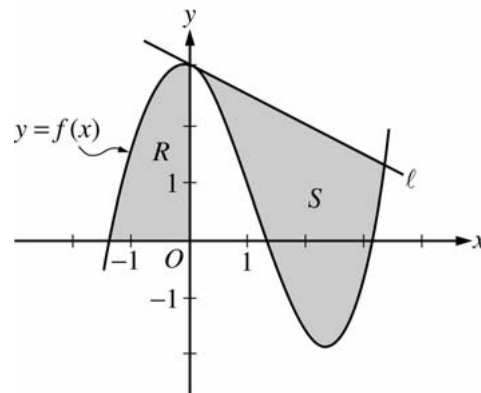
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**Question 1**

Let  $f$  be the function given by  $f(x) = \frac{x^3}{4} - \frac{x^2}{3} - \frac{x}{2} + 3\cos x$ . Let  $R$  be the shaded region in the second quadrant bounded by the graph of  $f$ , and let  $S$  be the shaded region bounded by the graph of  $f$  and line  $\ell$ , the line tangent to the graph of  $f$  at  $x = 0$ , as shown above.



- (a) Find the area of  $R$ .
- (b) Find the volume of the solid generated when  $R$  is rotated about the horizontal line  $y = -2$ .
- (c) Write, but do not evaluate, an integral expression that can be used to find the area of  $S$ .

For  $x < 0$ ,  $f(x) = 0$  when  $x = -1.37312$ .  
Let  $P = -1.37312$ .

(a) Area of  $R = \int_P^0 f(x) dx = 2.903$

2 :  $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) Volume =  $\pi \int_P^0 ((f(x) + 2)^2 - 4) dx = 59.361$

4 :  $\begin{cases} 1 : \text{limits and constant} \\ 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(c) The equation of the tangent line  $\ell$  is  $y = 3 - \frac{1}{2}x$ .

The graph of  $f$  and line  $\ell$  intersect at  $A = 3.38987$ .

Area of  $S = \int_0^A \left( \left( 3 - \frac{1}{2}x \right) - f(x) \right) dx$

3 :  $\begin{cases} 1 : \text{tangent line} \\ 1 : \text{integrand} \\ 1 : \text{limits} \end{cases}$

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**Question 2**

An object moving along a curve in the  $xy$ -plane is at position  $(x(t), y(t))$  at time  $t$ , where

$$\frac{dx}{dt} = \tan(e^{-t}) \text{ and } \frac{dy}{dt} = \sec(e^{-t})$$

for  $t \geq 0$ . At time  $t = 1$ , the object is at position  $(2, -3)$ .

- (a) Write an equation for the line tangent to the curve at position  $(2, -3)$ .  
 (b) Find the acceleration vector and the speed of the object at time  $t = 1$ .  
 (c) Find the total distance traveled by the object over the time interval  $1 \leq t \leq 2$ .  
 (d) Is there a time  $t \geq 0$  at which the object is on the  $y$ -axis? Explain why or why not.

(a) 
$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\sec(e^{-t})}{\tan(e^{-t})} = \frac{1}{\sin(e^{-t})}$$

$$\left. \frac{dy}{dx} \right|_{(2, -3)} = \frac{1}{\sin(e^{-1})} = 2.780 \text{ or } 2.781$$

$$y + 3 = \frac{1}{\sin(e^{-1})}(x - 2)$$

$$2 : \begin{cases} 1 : \left. \frac{dy}{dx} \right|_{(2, -3)} \\ 1 : \text{equation of tangent line} \end{cases}$$

(b)  $x''(1) = -0.42253, y''(1) = -0.15196$

$$a(1) = \langle -0.423, -0.152 \rangle \text{ or } \langle -0.422, -0.151 \rangle.$$

$$2 : \begin{cases} 1 : \text{acceleration vector} \\ 1 : \text{speed} \end{cases}$$

speed  $\sqrt{(\sec(e^{-1}))^2 + (\tan(e^{-1}))^2} = 1.138 \text{ or } 1.139$

(c)  $\int_1^2 \sqrt{(x'(t))^2 + (y'(t))^2} dt = 1.059$

$$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$$

(d)  $x(0) = x(1) - \int_0^1 x'(t) dt = 2 - 0.775553 > 0$

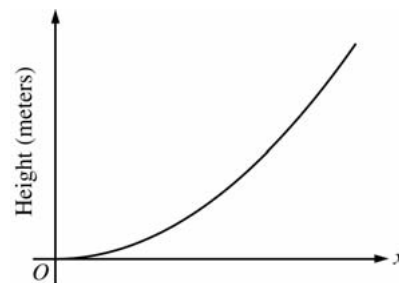
$$3 : \begin{cases} 1 : x(0) \text{ expression} \\ 1 : x'(t) > 0 \\ 1 : \text{conclusion and reason} \end{cases}$$

The particle starts to the right of the  $y$ -axis.  
 Since  $x'(t) > 0$  for all  $t \geq 0$ , the object is always moving to the right and thus is never on the  $y$ -axis.

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**Question 3**

The figure above is the graph of a function of  $x$ , which models the height of a skateboard ramp. The function meets the following requirements.



- (i) At  $x = 0$ , the value of the function is 0, and the slope of the graph of the function is 0.
  - (ii) At  $x = 4$ , the value of the function is 1, and the slope of the graph of the function is 1.
  - (iii) Between  $x = 0$  and  $x = 4$ , the function is increasing.
- (a) Let  $f(x) = ax^2$ , where  $a$  is a nonzero constant. Show that it is not possible to find a value for  $a$  so that  $f$  meets requirement (ii) above.
- (b) Let  $g(x) = cx^3 - \frac{x^2}{16}$ , where  $c$  is a nonzero constant. Find the value of  $c$  so that  $g$  meets requirement (ii) above. Show the work that leads to your answer.
- (c) Using the function  $g$  and your value of  $c$  from part (b), show that  $g$  does not meet requirement (iii) above.
- (d) Let  $h(x) = \frac{x^n}{k}$ , where  $k$  is a nonzero constant and  $n$  is a positive integer. Find the values of  $k$  and  $n$  so that  $h$  meets requirement (ii) above. Show that  $h$  also meets requirements (i) and (iii) above.

(a)  $f(4) = 1$  implies that  $a = \frac{1}{16}$  and  $f'(4) = 2a(4) = 1$   
implies that  $a = \frac{1}{8}$ . Thus,  $f$  cannot satisfy (ii).

2 :  $\begin{cases} 1 : a = \frac{1}{16} \text{ or } a = \frac{1}{8} \\ 1 : \text{shows } a \text{ does not work} \end{cases}$

(b)  $g(4) = 64c - 1 = 1$  implies that  $c = \frac{1}{32}$ .  
When  $c = \frac{1}{32}$ ,  $g'(4) = 3c(4)^2 - \frac{2(4)}{16} = 3\left(\frac{1}{32}\right)(16) - \frac{1}{2} = 1$

1 : value of  $c$

(c)  $g'(x) = \frac{3}{32}x^2 - \frac{x}{8} = \frac{1}{32}x(3x - 4)$   
 $g'(x) < 0$  for  $0 < x < \frac{4}{3}$ , so  $g$  does not satisfy (iii).

2 :  $\begin{cases} 1 : g'(x) \\ 1 : \text{explanation} \end{cases}$

(d)  $h(4) = \frac{4^n}{k} = 1$  implies that  $4^n = k$ .  
 $h'(4) = \frac{n4^{n-1}}{k} = \frac{n4^{n-1}}{4^n} = \frac{n}{4} = 1$  gives  $n = 4$  and  $k = 4^4 = 256$ .

4 :  $\begin{cases} 1 : \frac{4^n}{k} = 1 \\ 1 : \frac{n4^{n-1}}{k} = 1 \\ 1 : \text{values for } k \text{ and } n \\ 1 : \text{verifications} \end{cases}$

$$h(x) = \frac{x^4}{256} \Rightarrow h(0) = 0.$$

$$h'(x) = \frac{4x^3}{256} \Rightarrow h'(0) = 0 \text{ and } h'(x) > 0 \text{ for } 0 < x < 4.$$

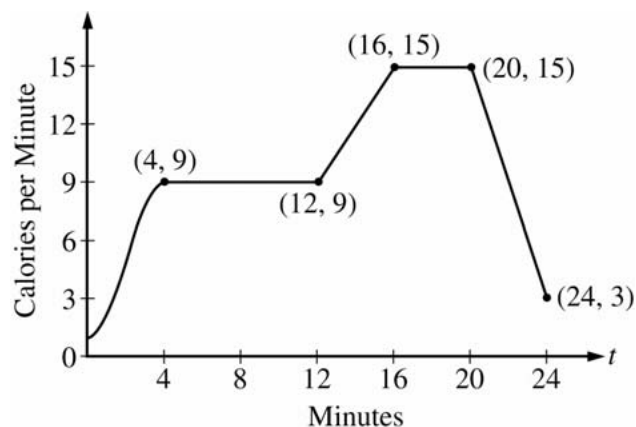
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**Question 4**

The rate, in calories per minute, at which a person using an exercise machine burns calories is modeled by the function  $f$ . In the figure above,  $f(t) = -\frac{1}{4}t^3 + \frac{3}{2}t^2 + 1$  for

$0 \leq t \leq 4$  and  $f$  is piecewise linear for  $4 \leq t \leq 24$ .

- (a) Find  $f'(22)$ . Indicate units of measure.
- (b) For the time interval  $0 \leq t \leq 24$ , at what time  $t$  is  $f$  increasing at its greatest rate? Show the reasoning that supports your answer.
- (c) Find the total number of calories burned over the time interval  $6 \leq t \leq 18$  minutes.
- (d) The setting on the machine is now changed so that the person burns  $f(t) + c$  calories per minute. For this setting, find  $c$  so that an average of 15 calories per minute is burned during the time interval  $6 \leq t \leq 18$ .



(a)  $f'(22) = \frac{15 - 3}{20 - 24} = -3$  calories/min/min

(b)  $f$  is increasing on  $[0, 4]$  and on  $[12, 16]$ .

On  $(12, 16)$ ,  $f'(t) = \frac{15 - 9}{16 - 12} = \frac{3}{2}$  since  $f$  has constant slope on this interval.

On  $(0, 4)$ ,  $f'(t) = -\frac{3}{4}t^2 + 3t$  and

$f''(t) = -\frac{3}{2}t + 3 = 0$  when  $t = 2$ . This is where  $f'$  has a maximum on  $[0, 4]$  since  $f'' > 0$  on  $(0, 2)$  and  $f'' < 0$  on  $(2, 4)$ .

On  $[0, 24]$ ,  $f$  is increasing at its greatest rate when  $t = 2$  because  $f'(2) = 3 > \frac{3}{2}$ .

(c)  $\int_6^{18} f(t) dt = 6(9) + \frac{1}{2}(4)(9 + 15) + 2(15)$   
 $= 132$  calories

(d) We want  $\frac{1}{12} \int_6^{18} (f(t) + c) dt = 15$ .

This means  $132 + 12c = 15(12)$ . So,  $c = 4$ .

OR

Currently, the average is  $\frac{132}{12} = 11$  calories/min.

Adding  $c$  to  $f(t)$  will shift the average by  $c$ .

So  $c = 4$  to get an average of 15 calories/min.

1 :  $f'(22)$  and units

4 :  $\begin{cases} 1 : f' \text{ on } (0, 4) \\ 1 : \text{shows } f' \text{ has a max at } t = 2 \text{ on } (0, 4) \\ 1 : \text{shows for } 12 < t < 16, f'(t) < f'(2) \\ 1 : \text{answer} \end{cases}$

2 :  $\begin{cases} 1 : \text{method} \\ 1 : \text{answer} \end{cases}$

2 :  $\begin{cases} 1 : \text{setup} \\ 1 : \text{value of } c \end{cases}$

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**Question 5**

Let  $f$  be a function with  $f(4) = 1$  such that all points  $(x, y)$  on the graph of  $f$  satisfy the differential equation

$$\frac{dy}{dx} = 2y(3 - x).$$

Let  $g$  be a function with  $g(4) = 1$  such that all points  $(x, y)$  on the graph of  $g$  satisfy the logistic differential equation

$$\frac{dy}{dx} = 2y(3 - y).$$

- (a) Find  $y = f(x)$ .
- (b) Given that  $g(4) = 1$ , find  $\lim_{x \rightarrow \infty} g(x)$  and  $\lim_{x \rightarrow \infty} g'(x)$ . (It is not necessary to solve for  $g(x)$  or to show how you arrived at your answers.)
- (c) For what value of  $y$  does the graph of  $g$  have a point of inflection? Find the slope of the graph of  $g$  at the point of inflection. (It is not necessary to solve for  $g(x)$ .)

(a)  $\frac{dy}{dx} = 2y(3 - x)$

$$\frac{1}{y} dy = 2(3 - x) dx$$

$$\ln|y| = 6x - x^2 + C$$

$$0 = 24 - 16 + C$$

$$C = -8$$

$$\ln|y| = 6x - x^2 - 8$$

$$y = e^{6x - x^2 - 8} \text{ for } -\infty < x < \infty$$

$$5 : \begin{cases} 1 : \text{separates variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solution} \end{cases}$$

Note: max 2/5 [1-1-0-0-0] if no constant of integration

Note: 0/5 if no separation of variables

(b)  $\lim_{x \rightarrow \infty} g(x) = 3$

$$\lim_{x \rightarrow \infty} g'(x) = 0$$

$$2 : \begin{cases} 1 : \lim_{x \rightarrow \infty} g(x) = 3 \\ 1 : \lim_{x \rightarrow \infty} g'(x) = 0 \end{cases}$$

(c)  $\frac{d^2y}{dx^2} = (6 - 4y)\frac{dy}{dx}$

Because  $\frac{dy}{dx} \neq 0$  at any point on the graph of  $g$ , the

concavity only changes sign at  $y = \frac{3}{2}$ , half the carrying capacity.

$$\left. \frac{dy}{dx} \right|_{y=3/2} = 2\left(\frac{3}{2}\right)\left(3 - \frac{3}{2}\right) = \frac{9}{2}$$

$$2 : \begin{cases} 1 : y = \frac{3}{2} \\ 1 : \left. \frac{dy}{dx} \right|_{y=3/2} \end{cases}$$

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**Question 6**

The function  $f$  is defined by  $f(x) = \frac{1}{1+x^3}$ . The Maclaurin series for  $f$  is given by

$$1 - x^3 + x^6 - x^9 + \dots + (-1)^n x^{3n} + \dots,$$

which converges to  $f(x)$  for  $-1 < x < 1$ .

- (a) Find the first three nonzero terms and the general term for the Maclaurin series for  $f'(x)$ .
- (b) Use your results from part (a) to find the sum of the infinite series  $-\frac{3}{2^2} + \frac{6}{2^5} - \frac{9}{2^8} + \dots + (-1)^n \frac{3n}{2^{3n-1}} + \dots$ .
- (c) Find the first four nonzero terms and the general term for the Maclaurin series representing  $\int_0^x f(t) dt$ .
- (d) Use the first three nonzero terms of the infinite series found in part (c) to approximate  $\int_0^{1/2} f(t) dt$ . What are the properties of the terms of the series representing  $\int_0^{1/2} f(t) dt$  that guarantee that this approximation is within  $\frac{1}{10,000}$  of the exact value of the integral?

(a)  $f'(x) = -3x^2 + 6x^5 - 9x^8 + \dots + 3n(-1)^n x^{3n-1} + \dots$

2 :  $\begin{cases} 1 : \text{first three terms} \\ 1 : \text{general term} \end{cases}$

(b) The given series is the Maclaurin series for  $f'(x)$  with  $x = \frac{1}{2}$ .

$$f'(x) = -(1+x^3)^{-2}(3x^2)$$

Thus, the sum of the series is  $f'\left(\frac{1}{2}\right) = -\frac{3\left(\frac{1}{4}\right)}{\left(1+\frac{1}{8}\right)^2} = -\frac{16}{27}$ .

2 :  $\begin{cases} 1 : f'(x) \\ 1 : f'\left(\frac{1}{2}\right) \end{cases}$

(c)  $\int_0^x \frac{1}{1+t^3} dt = x - \frac{x^4}{4} + \frac{x^7}{7} - \frac{x^{10}}{10} + \dots + \frac{(-1)^n x^{3n+1}}{3n+1} + \dots$

2 :  $\begin{cases} 1 : \text{first four terms} \\ 1 : \text{general term} \end{cases}$

(d)  $\int_0^{1/2} \frac{1}{1+t^3} dt \approx \frac{1}{2} - \frac{\left(\frac{1}{2}\right)^4}{4} + \frac{\left(\frac{1}{2}\right)^7}{7}$ .

The series in part (c) with  $x = \frac{1}{2}$  has terms that alternate, decrease in absolute value, and have limit 0. Hence the error is bounded by the absolute value of the next term.

$$\left| \int_0^{1/2} \frac{1}{1+t^3} dt - \left( \frac{1}{2} - \frac{\left(\frac{1}{2}\right)^4}{4} + \frac{\left(\frac{1}{2}\right)^7}{7} \right) \right| < \frac{\left(\frac{1}{2}\right)^{10}}{10} = \frac{1}{10240} < 0.0001$$

3 :  $\begin{cases} 1 : \text{approximation} \\ 1 : \text{properties of terms} \\ 1 : \text{absolute value of fourth term} < 0.0001 \end{cases}$