



AP[®] Calculus BC 2007 Scoring Guidelines

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Question 1

Let R be the region in the first and second quadrants bounded above by the graph of $y = \frac{20}{1+x^2}$ and below by the horizontal line $y = 2$.

- (a) Find the area of R .
 (b) Find the volume of the solid generated when R is rotated about the x -axis.
 (c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x -axis are semicircles. Find the volume of this solid.

$$\frac{20}{1+x^2} = 2 \text{ when } x = \pm 3$$

(a) Area = $\int_{-3}^3 \left(\frac{20}{1+x^2} - 2 \right) dx = 37.961$ or 37.962

(b) Volume = $\pi \int_{-3}^3 \left(\left(\frac{20}{1+x^2} \right)^2 - 2^2 \right) dx = 1871.190$

(c) Volume = $\frac{\pi}{2} \int_{-3}^3 \left(\frac{1}{2} \left(\frac{20}{1+x^2} - 2 \right) \right)^2 dx$
 $= \frac{\pi}{8} \int_{-3}^3 \left(\frac{20}{1+x^2} - 2 \right)^2 dx = 174.268$

1 : correct limits in an integral in (a), (b), or (c)

2 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

3 : $\begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

3 : $\begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

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Question 2

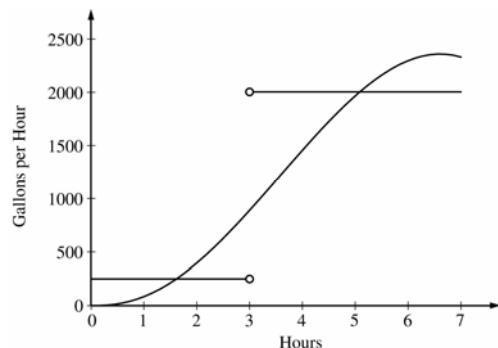
The amount of water in a storage tank, in gallons, is modeled by a continuous function on the time interval $0 \leq t \leq 7$, where t is measured in hours. In this model, rates are given as follows:

(i) The rate at which water enters the tank is

$$f(t) = 100t^2 \sin(\sqrt{t}) \text{ gallons per hour for } 0 \leq t \leq 7.$$

(ii) The rate at which water leaves the tank is

$$g(t) = \begin{cases} 250 & \text{for } 0 \leq t < 3 \\ 2000 & \text{for } 3 < t \leq 7 \end{cases} \text{ gallons per hour.}$$



The graphs of f and g , which intersect at $t = 1.617$ and $t = 5.076$, are shown in the figure above. At time $t = 0$, the amount of water in the tank is 5000 gallons.

- (a) How many gallons of water enter the tank during the time interval $0 \leq t \leq 7$? Round your answer to the nearest gallon.
- (b) For $0 \leq t \leq 7$, find the time intervals during which the amount of water in the tank is decreasing. Give a reason for each answer.
- (c) For $0 \leq t \leq 7$, at what time t is the amount of water in the tank greatest? To the nearest gallon, compute the amount of water at this time. Justify your answer.

(a) $\int_0^7 f(t) dt \approx 8264$ gallons

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) The amount of water in the tank is decreasing on the intervals $0 \leq t \leq 1.617$ and $3 \leq t \leq 5.076$ because $f(t) < g(t)$ for $0 \leq t < 1.617$ and $3 < t < 5.076$.

2 : $\begin{cases} 1 : \text{intervals} \\ 1 : \text{reason} \end{cases}$

(c) Since $f(t) - g(t)$ changes sign from positive to negative only at $t = 3$, the candidates for the absolute maximum are at $t = 0, 3$, and 7 .

5 : $\begin{cases} 1 : \text{identifies } t = 3 \text{ as a candidate} \\ 1 : \text{integrand} \\ 1 : \text{amount of water at } t = 3 \\ 1 : \text{amount of water at } t = 7 \\ 1 : \text{conclusion} \end{cases}$

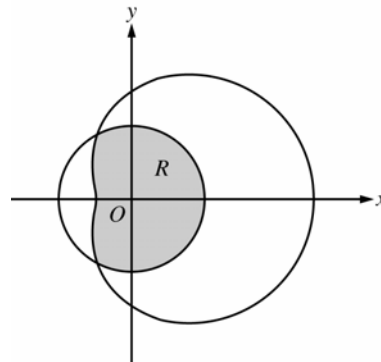
t (hours)	gallons of water
0	5000
3	$5000 + \int_0^3 f(t) dt - 250(3) = 5126.591$
7	$5126.591 + \int_3^7 f(t) dt - 2000(4) = 4513.807$

The amount of water in the tank is greatest at 3 hours. At that time, the amount of water in the tank, rounded to the nearest gallon, is 5127 gallons.

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Question 3

The graphs of the polar curves $r = 2$ and $r = 3 + 2\cos \theta$ are shown in the figure above. The curves intersect when $\theta = \frac{2\pi}{3}$ and $\theta = \frac{4\pi}{3}$.



- (a) Let R be the region that is inside the graph of $r = 2$ and also inside the graph of $r = 3 + 2\cos \theta$, as shaded in the figure above. Find the area of R .
- (b) A particle moving with nonzero velocity along the polar curve given by $r = 3 + 2\cos \theta$ has position $(x(t), y(t))$ at time t , with $\theta = 0$ when $t = 0$. This particle moves along the curve so that $\frac{dr}{dt} = \frac{dr}{d\theta}$.

Find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{3}$ and interpret your answer in terms of the motion of the particle.

- (c) For the particle described in part (b), $\frac{dy}{dt} = \frac{dy}{d\theta}$. Find the value of $\frac{dy}{dt}$ at $\theta = \frac{\pi}{3}$ and interpret your answer in terms of the motion of the particle.

(a)
$$\text{Area} = \frac{2}{3}\pi(2)^2 + \frac{1}{2}\int_{2\pi/3}^{4\pi/3}(3 + 2\cos \theta)^2 d\theta$$

$$= 10.370$$

4 : $\left\{ \begin{array}{l} 1 : \text{area of circular sector} \\ 2 : \text{integral for section of limaçon} \\ \quad 1 : \text{integrand} \\ \quad 1 : \text{limits and constant} \\ 1 : \text{answer} \end{array} \right.$

(b)
$$\left. \frac{dr}{dt} \right|_{\theta=\pi/3} = \left. \frac{dr}{d\theta} \right|_{\theta=\pi/3} = -1.732$$

2 : $\left\{ \begin{array}{l} 1 : \left. \frac{dr}{dt} \right|_{\theta=\pi/3} \\ 1 : \text{interpretation} \end{array} \right.$

The particle is moving closer to the origin, since $\frac{dr}{dt} < 0$

and $r > 0$ when $\theta = \frac{\pi}{3}$.

(c)
$$y = r \sin \theta = (3 + 2\cos \theta) \sin \theta$$

$$\left. \frac{dy}{dt} \right|_{\theta=\pi/3} = \left. \frac{dy}{d\theta} \right|_{\theta=\pi/3} = 0.5$$

3 : $\left\{ \begin{array}{l} 1 : \text{expression for } y \text{ in terms of } \theta \\ 1 : \left. \frac{dy}{dt} \right|_{\theta=\pi/3} \\ 1 : \text{interpretation} \end{array} \right.$

The particle is moving away from the x -axis, since

$\frac{dy}{dt} > 0$ and $y > 0$ when $\theta = \frac{\pi}{3}$.

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Question 4

Let f be the function defined for $x > 0$, with $f(e) = 2$ and f' , the first derivative of f , given by $f'(x) = x^2 \ln x$.

- (a) Write an equation for the line tangent to the graph of f at the point $(e, 2)$.
 (b) Is the graph of f concave up or concave down on the interval $1 < x < 3$? Give a reason for your answer.
 (c) Use antidifferentiation to find $f(x)$.

(a) $f'(e) = e^2$

An equation for the line tangent to the graph of f at the point $(e, 2)$ is $y - 2 = e^2(x - e)$.

$$2: \begin{cases} 1: f'(e) \\ 1: \text{equation of tangent line} \end{cases}$$

(b) $f''(x) = x + 2x \ln x$.

For $1 < x < 3$, $x > 0$ and $\ln x > 0$, so $f''(x) > 0$. Thus, the graph of f is concave up on $(1, 3)$.

$$3: \begin{cases} 2: f''(x) \\ 1: \text{answer with reason} \end{cases}$$

- (c) Since $f(x) = \int (x^2 \ln x) dx$, we consider integration by parts.

$$\begin{aligned} u &= \ln x & dv &= x^2 dx \\ du &= \frac{1}{x} dx & v &= \int (x^2) dx = \frac{1}{3}x^3 \end{aligned}$$

Therefore,

$$\begin{aligned} f(x) &= \int (x^2 \ln x) dx \\ &= \frac{1}{3}x^3 \ln x - \int \left(\frac{1}{3}x^3 \cdot \frac{1}{x} \right) dx \\ &= \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C. \end{aligned}$$

Since $f(e) = 2$, $2 = \frac{e^3}{3} - \frac{e^3}{9} + C$ and $C = 2 - \frac{2}{9}e^3$.

Thus, $f(x) = \frac{x^3}{3} \ln x - \frac{1}{9}x^3 + 2 - \frac{2}{9}e^3$.

$$4: \begin{cases} 2: \text{antiderivative} \\ 1: \text{uses } f(e) = 2 \\ 1: \text{answer} \end{cases}$$

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Question 5

t (minutes)	0	2	5	7	11	12
$r'(t)$ (feet per minute)	5.7	4.0	2.0	1.2	0.6	0.5

The volume of a spherical hot air balloon expands as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by a twice-differentiable function r of time t , where t is measured in minutes. For $0 < t < 12$, the graph of r is concave down. The table above gives selected values of the rate of change, $r'(t)$, of the radius of the balloon over the time interval $0 \leq t \leq 12$. The radius of the balloon is 30 feet when $t = 5$. (Note: The volume of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$.)

- (a) Estimate the radius of the balloon when $t = 5.4$ using the tangent line approximation at $t = 5$. Is your estimate greater than or less than the true value? Give a reason for your answer.
- (b) Find the rate of change of the volume of the balloon with respect to time when $t = 5$. Indicate units of measure.
- (c) Use a right Riemann sum with the five subintervals indicated by the data in the table to approximate $\int_0^{12} r'(t) dt$. Using correct units, explain the meaning of $\int_0^{12} r'(t) dt$ in terms of the radius of the balloon.
- (d) Is your approximation in part (c) greater than or less than $\int_0^{12} r'(t) dt$? Give a reason for your answer.

(a) $r(5.4) \approx r(5) + r'(5)\Delta t = 30 + 2(0.4) = 30.8$ ft
 Since the graph of r is concave down on the interval $5 < t < 5.4$, this estimate is greater than $r(5.4)$.

2 : $\begin{cases} 1 : \text{estimate} \\ 1 : \text{conclusion with reason} \end{cases}$

(b) $\frac{dV}{dt} = 3\left(\frac{4}{3}\right)\pi r^2 \frac{dr}{dt}$
 $\left. \frac{dV}{dt} \right|_{t=5} = 4\pi(30)^2 2 = 7200\pi \text{ ft}^3/\text{min}$

3 : $\begin{cases} 2 : \frac{dV}{dt} \\ 1 : \text{answer} \end{cases}$

(c) $\int_0^{12} r'(t) dt \approx 2(4.0) + 3(2.0) + 2(1.2) + 4(0.6) + 1(0.5)$
 $= 19.3$ ft
 $\int_0^{12} r'(t) dt$ is the change in the radius, in feet, from $t = 0$ to $t = 12$ minutes.

2 : $\begin{cases} 1 : \text{approximation} \\ 1 : \text{explanation} \end{cases}$

(d) Since r is concave down, r' is decreasing on $0 < t < 12$. Therefore, this approximation, 19.3 ft, is less than $\int_0^{12} r'(t) dt$.

1 : conclusion with reason

Units of ft^3/min in part (b) and ft in part (c)

1 : units in (b) and (c)

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Question 6

Let f be the function given by $f(x) = e^{-x^2}$.

- (a) Write the first four nonzero terms and the general term of the Taylor series for f about $x = 0$.
- (b) Use your answer to part (a) to find $\lim_{x \rightarrow 0} \frac{1 - x^2 - f(x)}{x^4}$.
- (c) Write the first four nonzero terms of the Taylor series for $\int_0^x e^{-t^2} dt$ about $x = 0$. Use the first two terms of your answer to estimate $\int_0^{1/2} e^{-t^2} dt$.
- (d) Explain why the estimate found in part (c) differs from the actual value of $\int_0^{1/2} e^{-t^2} dt$ by less than $\frac{1}{200}$.

$$\begin{aligned} \text{(a)} \quad e^{-x^2} &= 1 + \frac{(-x^2)}{1!} + \frac{(-x^2)^2}{2!} + \frac{(-x^2)^3}{3!} + \cdots + \frac{(-x^2)^n}{n!} + \cdots \\ &= 1 - x^2 + \frac{x^4}{2} - \frac{x^6}{6} + \cdots + \frac{(-1)^n x^{2n}}{n!} + \cdots \end{aligned}$$

$$3 : \begin{cases} 1 : \text{two of } 1, -x^2, \frac{x^4}{2}, -\frac{x^6}{6} \\ 1 : \text{remaining terms} \\ 1 : \text{general term} \end{cases}$$

$$\text{(b)} \quad \frac{1 - x^2 - f(x)}{x^4} = -\frac{1}{2} + \frac{x^2}{6} + \sum_{n=4}^{\infty} \frac{(-1)^{n+1} x^{2n-4}}{n!}$$

$$\text{Thus, } \lim_{x \rightarrow 0} \left(\frac{1 - x^2 - f(x)}{x^4} \right) = -\frac{1}{2}.$$

1 : answer

$$\begin{aligned} \text{(c)} \quad \int_0^x e^{-t^2} dt &= \int_0^x \left(1 - t^2 + \frac{t^4}{2} - \frac{t^6}{6} + \cdots + \frac{(-1)^n t^{2n}}{n!} + \cdots \right) dt \\ &= x - \frac{x^3}{3} + \frac{x^5}{10} - \frac{x^7}{42} + \cdots \end{aligned}$$

$$3 : \begin{cases} 1 : \text{two terms} \\ 1 : \text{remaining terms} \\ 1 : \text{estimate} \end{cases}$$

Using the first two terms of this series, we estimate that

$$\int_0^{1/2} e^{-t^2} dt \approx \frac{1}{2} - \left(\frac{1}{3}\right)\left(\frac{1}{8}\right) = \frac{11}{24}.$$

$$\text{(d)} \quad \left| \int_0^{1/2} e^{-t^2} dt - \frac{11}{24} \right| < \left(\frac{1}{2}\right)^5 \cdot \frac{1}{10} = \frac{1}{320} < \frac{1}{200}, \text{ since}$$

$$\int_0^{1/2} e^{-t^2} dt = \sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{1}{2}\right)^{2n+1}}{n!(2n+1)}, \text{ which is an alternating}$$

series with individual terms that decrease in absolute value to 0.

$$2 : \begin{cases} 1 : \text{uses the third term as} \\ \quad \text{the error bound} \\ 1 : \text{explanation} \end{cases}$$