



AP[®] Calculus BC
2009 Free-Response Questions
Form B

The College Board

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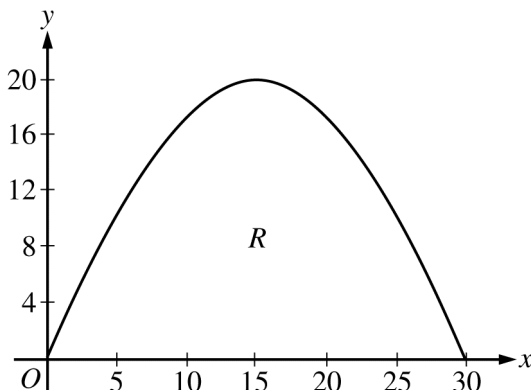
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CALCULUS BC
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

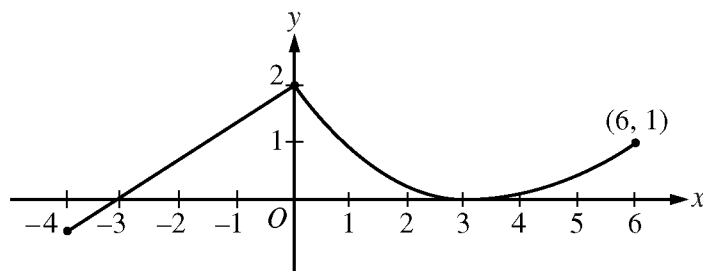


1. A baker is creating a birthday cake. The base of the cake is the region R in the first quadrant under the graph of $y = f(x)$ for $0 \leq x \leq 30$, where $f(x) = 20\sin\left(\frac{\pi x}{30}\right)$. Both x and y are measured in centimeters. The region R is shown in the figure above. The derivative of f is $f'(x) = \frac{2\pi}{3}\cos\left(\frac{\pi x}{30}\right)$.
- The region R is cut out of a 30-centimeter-by-20-centimeter rectangular sheet of cardboard, and the remaining cardboard is discarded. Find the area of the discarded cardboard.
 - The cake is a solid with base R . Cross sections of the cake perpendicular to the x -axis are semicircles. If the baker uses 0.05 gram of unsweetened chocolate for each cubic centimeter of cake, how many grams of unsweetened chocolate will be in the cake?
 - Find the perimeter of the base of the cake.

WRITE ALL WORK IN THE EXAM BOOKLET.

2. A storm washed away sand from a beach, causing the edge of the water to get closer to a nearby road. The rate at which the distance between the road and the edge of the water was changing during the storm is modeled by $f(t) = \sqrt{t} + \cos t - 3$ meters per hour, t hours after the storm began. The edge of the water was 35 meters from the road when the storm began, and the storm lasted 5 hours. The derivative of $f(t)$ is $f'(t) = \frac{1}{2\sqrt{t}} - \sin t$.
- (a) What was the distance between the road and the edge of the water at the end of the storm?
- (b) Using correct units, interpret the value $f'(4) = 1.007$ in terms of the distance between the road and the edge of the water.
- (c) At what time during the 5 hours of the storm was the distance between the road and the edge of the water decreasing most rapidly? Justify your answer.
- (d) After the storm, a machine pumped sand back onto the beach so that the distance between the road and the edge of the water was growing at a rate of $g(p)$ meters per day, where p is the number of days since pumping began. Write an equation involving an integral expression whose solution would give the number of days that sand must be pumped to restore the original distance between the road and the edge of the water.
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Graph of f

3. A continuous function f is defined on the closed interval $-4 \leq x \leq 6$. The graph of f consists of a line segment and a curve that is tangent to the x -axis at $x = 3$, as shown in the figure above. On the interval $0 < x < 6$, the function f is twice differentiable, with $f''(x) > 0$.
- (a) Is f differentiable at $x = 0$? Use the definition of the derivative with one-sided limits to justify your answer.
- (b) For how many values of a , $-4 \leq a < 6$, is the average rate of change of f on the interval $[a, 6]$ equal to 0? Give a reason for your answer.
- (c) Is there a value of a , $-4 \leq a < 6$, for which the Mean Value Theorem, applied to the interval $[a, 6]$, guarantees a value c , $a < c < 6$, at which $f'(c) = \frac{1}{3}$? Justify your answer.
- (d) The function g is defined by $g(x) = \int_0^x f(t) dt$ for $-4 \leq x \leq 6$. On what intervals contained in $[-4, 6]$ is the graph of g concave up? Explain your reasoning.

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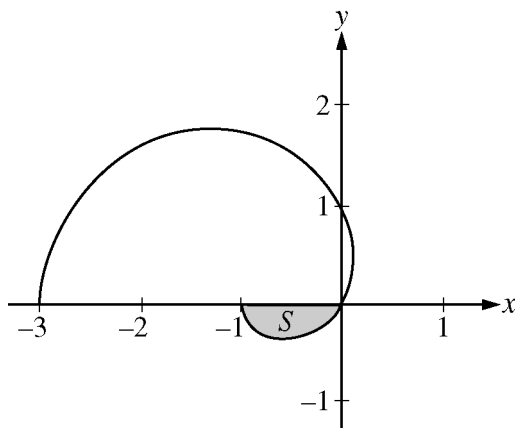
END OF PART A OF SECTION II

CALCULUS BC
SECTION II, Part B

Time—45 minutes

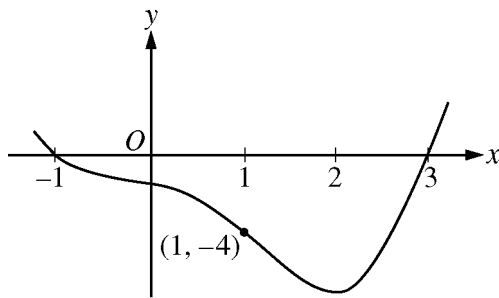
Number of problems—3

No calculator is allowed for these problems.



4. The graph of the polar curve $r = 1 - 2\cos\theta$ for $0 \leq \theta \leq \pi$ is shown above. Let S be the shaded region in the third quadrant bounded by the curve and the x -axis.
- (a) Write an integral expression for the area of S .
- (b) Write expressions for $\frac{dx}{d\theta}$ and $\frac{dy}{d\theta}$ in terms of θ .
- (c) Write an equation in terms of x and y for the line tangent to the graph of the polar curve at the point where $\theta = \frac{\pi}{2}$. Show the computations that lead to your answer.

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Graph of f'

5. Let f be a twice-differentiable function defined on the interval $-1.2 < x < 3.2$ with $f(1) = 2$. The graph of f' , the derivative of f , is shown above. The graph of f' crosses the x -axis at $x = -1$ and $x = 3$ and has a horizontal tangent at $x = 2$. Let g be the function given by $g(x) = e^{f(x)}$.
- Write an equation for the line tangent to the graph of g at $x = 1$.
 - For $-1.2 < x < 3.2$, find all values of x at which g has a local maximum. Justify your answer.
 - The second derivative of g is $g''(x) = e^{f(x)}[(f'(x))^2 + f''(x)]$. Is $g''(-1)$ positive, negative, or zero? Justify your answer.
 - Find the average rate of change of g' , the derivative of g , over the interval $[1, 3]$.

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6. The function f is defined by the power series

$$f(x) = 1 + (x + 1) + (x + 1)^2 + \cdots + (x + 1)^n + \cdots = \sum_{n=0}^{\infty} (x + 1)^n$$

for all real numbers x for which the series converges.

- (a) Find the interval of convergence of the power series for f . Justify your answer.
- (b) The power series above is the Taylor series for f about $x = -1$. Find the sum of the series for f .
- (c) Let g be the function defined by $g(x) = \int_{-1}^x f(t) dt$. Find the value of $g\left(-\frac{1}{2}\right)$, if it exists, or explain why $g\left(-\frac{1}{2}\right)$ cannot be determined.
- (d) Let h be the function defined by $h(x) = f(x^2 - 1)$. Find the first three nonzero terms and the general term of the Taylor series for h about $x = 0$, and find the value of $h\left(\frac{1}{2}\right)$.

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END OF EXAM