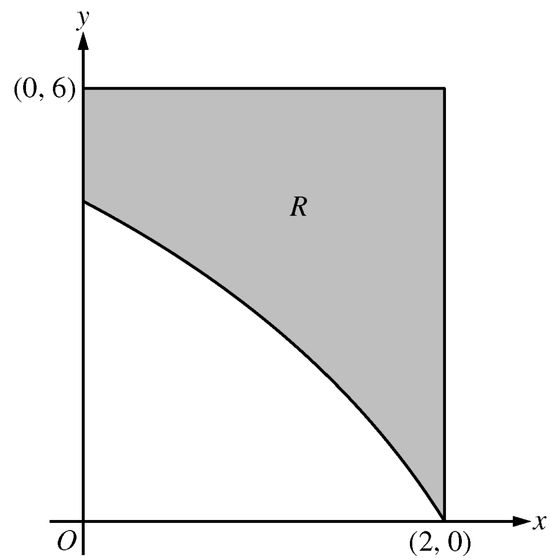
CollegeBoard
Advanced Placement Program

AP® Calculus BC
2010 Free-Response Questions
Form B

The College Board

CALCULUS BC
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. In the figure above, R is the shaded region in the first quadrant bounded by the graph of $y = 4\ln(3 - x)$, the horizontal line $y = 6$, and the vertical line $x = 2$.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 8$.

   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of the solid.

WRITE ALL WORK IN THE EXAM BOOKLET.

2. The velocity vector of a particle moving in the xy-plane has components given by

$$\frac{dx}{dt} = 14\cos\left(t^2\right)\sin\left(e^t\right) \text{ and } \frac{dy}{dt} = 1 + 2\sin\left(t^2\right), \text{ for } 0 \le t \le 1.5.$$

At time $t = 0,$ the position of the particle is $(-2, 3).$

(a) For $0 < t < 1.5,$ find all values of t at which the line tangent to the path of the particle is vertical.

(b) Write an equation for the line tangent to the path of the particle at $t = 1.$

(c) Find the speed of the particle at $t = 1.$

(d) Find the acceleration vector of the particle at $t = 1.$

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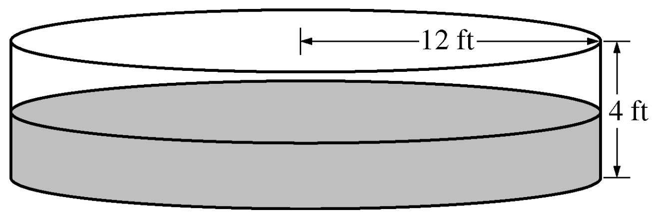
t	0	2	4	6	8	10	12
$P(t)$	0	46	53	57	60	62	63



3. The figure above shows an aboveground swimming pool in the shape of a cylinder with a radius of 12 feet and a height of 4 feet. The pool contains 1000 cubic feet of water at time $t = 0.$ During the time interval $0 \le t \le 12$ hours, water is pumped into the pool at the rate $P(t)$ cubic feet per hour. The table above gives values of $P(t)$ for selected values of $t.$ During the same time interval, water is leaking from the pool at the rate $R(t)$ cubic feet per hour, where $R(t) = 25e^{-0.05t}.$ (Note: The volume V of a cylinder with radius r and height h is given by $V = \pi r^2 h.$)

(a) Use a midpoint Riemann sum with three subintervals of equal length to approximate the total amount of water that was pumped into the pool during the time interval $0 \le t \le 12$ hours. Show the computations that lead to your answer.

(b) Calculate the total amount of water that leaked out of the pool during the time interval $0 \le t \le 12$ hours.

(c) Use the results from parts (a) and (b) to approximate the volume of water in the pool at time $t = 12$ hours. Round your answer to the nearest cubic foot.

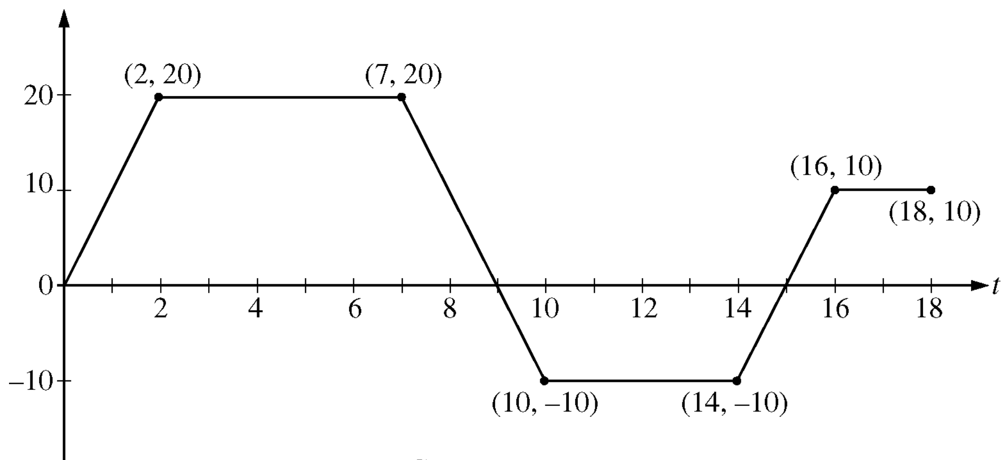
(d) Find the rate at which the volume of water in the pool is increasing at time $t = 8$ hours. How fast is the water level in the pool rising at $t = 8$ hours? Indicate units of measure in both answers.

WRITE ALL WORK IN THE EXAM BOOKLET.

END OF PART A OF SECTION II

CALCULUS BC
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



Graph of v

4. A squirrel starts at building A at time $t = 0$ and travels along a straight, horizontal wire connected to building B. For $0 \le t \le 18$, the squirrel's velocity is modeled by the piecewise-linear function defined by the graph above.
   (a) At what times in the interval $0 < t < 18$, if any, does the squirrel change direction? Give a reason for your answer.
   (b) At what time in the interval $0 \le t \le 18$ is the squirrel farthest from building A? How far from building A is the squirrel at that time?
   (c) Find the total distance the squirrel travels during the time interval $0 \le t \le 18$.
   (d) Write expressions for the squirrel's acceleration $a(t)$, velocity $v(t)$, and distance $x(t)$ from building A that are valid for the time interval $7 < t < 10$.

WRITE ALL WORK IN THE EXAM BOOKLET.

5. Let f and g be the functions defined by $f(x) = \frac{1}{x}$ and $g(x) = \frac{4x}{1 + 4x^2}$, for all $x > 0$.

   (a) Find the absolute maximum value of g on the open interval $(0, \infty)$ if the maximum exists. Find the absolute minimum value of g on the open interval $(0, \infty)$ if the minimum exists. Justify your answers.

   (b) Find the area of the unbounded region in the first quadrant to the right of the vertical line $x = 1$, below the graph of f, and above the graph of g.

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6. The Maclaurin series for the function f is given by $f(x) = \sum_{n=2}^{\infty} \frac{(-1)^n (2x)^n}{n - 1}$ on its interval of convergence.

   (a) Find the interval of convergence for the Maclaurin series of f. Justify your answer.

   (b) Show that $y = f(x)$ is a solution to the differential equation $xy' - y = \frac{4x^2}{1 + 2x}$ for $|x| < R$, where R is the radius of convergence from part (a).

WRITE ALL WORK IN THE EXAM BOOKLET.

END OF EXAM