



AP[®] Calculus BC 2011 Scoring Guidelines Form B

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2011 SCORING GUIDELINES (Form B)

Question 1

A cylindrical can of radius 10 millimeters is used to measure rainfall in Stormville. The can is initially empty, and rain enters the can during a 60-day period. The height of water in the can is modeled by the function S , where $S(t)$ is measured in millimeters and t is measured in days for $0 \leq t \leq 60$. The rate at which the height of the water is rising in the can is given by $S'(t) = 2\sin(0.03t) + 1.5$.

- (a) According to the model, what is the height of the water in the can at the end of the 60-day period?
- (b) According to the model, what is the average rate of change in the height of water in the can over the 60-day period? Show the computations that lead to your answer. Indicate units of measure.
- (c) Assuming no evaporation occurs, at what rate is the volume of water in the can changing at time $t = 7$? Indicate units of measure.
- (d) During the same 60-day period, rain on Monsoon Mountain accumulates in a can identical to the one in Stormville. The height of the water in the can on Monsoon Mountain is modeled by the function M , where $M(t) = \frac{1}{400}(3t^3 - 30t^2 + 330t)$. The height $M(t)$ is measured in millimeters, and t is measured in days for $0 \leq t \leq 60$. Let $D(t) = M'(t) - S'(t)$. Apply the Intermediate Value Theorem to the function D on the interval $0 \leq t \leq 60$ to justify that there exists a time t , $0 < t < 60$, at which the heights of water in the two cans are changing at the same rate.

(a) $S(60) = \int_0^{60} S'(t) dt = 171.813 \text{ mm}$

3 : $\begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(b) $\frac{S(60) - S(0)}{60} = 2.863 \text{ or } 2.864 \text{ mm/day}$

1 : answer

(c) $V(t) = 100\pi S(t)$
 $V'(7) = 100\pi S'(7) = 602.218$

2 : $\begin{cases} 1 : \text{relationship between } V \text{ and } S \\ 1 : \text{answer} \end{cases}$

The volume of water in the can is increasing at a rate of $602.218 \text{ mm}^3/\text{day}$.

(d) $D(0) = -0.675 < 0$ and $D(60) = 69.37730 > 0$

2 : $\begin{cases} 1 : \text{considers } D(0) \text{ and } D(60) \\ 1 : \text{justification} \end{cases}$

Because D is continuous, the Intermediate Value Theorem implies that there is a time t , $0 < t < 60$, at which $D(t) = 0$. At this time, the heights of water in the two cans are changing at the same rate.

1 : units in (b) or (c)

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Question 2

The polar curve r is given by $r(\theta) = 3\theta + \sin \theta$, where $0 \leq \theta \leq 2\pi$.

- (a) Find the area in the second quadrant enclosed by the coordinate axes and the graph of r .
- (b) For $\frac{\pi}{2} \leq \theta \leq \pi$, there is one point P on the polar curve r with x -coordinate -3 . Find the angle θ that corresponds to point P . Find the y -coordinate of point P . Show the work that leads to your answers.
- (c) A particle is traveling along the polar curve r so that its position at time t is $(x(t), y(t))$ and such that $\frac{d\theta}{dt} = 2$. Find $\frac{dy}{dt}$ at the instant that $\theta = \frac{2\pi}{3}$, and interpret the meaning of your answer in the context of the problem.

(a) $\text{Area} = \frac{1}{2} \int_{\pi/2}^{\pi} (r(\theta))^2 d\theta = 47.513$

$$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and constant} \\ 1 : \text{answer} \end{cases}$$

(b) $-3 = r(\theta)\cos \theta = (3\theta + \sin \theta)\cos \theta$
 $\theta = 2.01692$
 $y = r(\theta)\sin(\theta) = 6.272$

$$3 : \begin{cases} 1 : \text{equation} \\ 1 : \text{value of } \theta \\ 1 : \text{y-coordinate} \end{cases}$$

(c) $y = r(\theta)\sin \theta = (3\theta + \sin \theta)\sin \theta$
 $\frac{dy}{dt} \Big|_{\theta=2\pi/3} = \left[\frac{dy}{d\theta} \cdot \frac{d\theta}{dt} \right]_{\theta=2\pi/3} = -2.819$

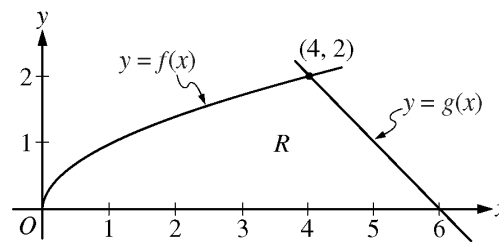
$$3 : \begin{cases} 1 : \text{uses chain rule} \\ 1 : \text{answer} \\ 1 : \text{interpretation} \end{cases}$$

The y -coordinate of the particle is decreasing at a rate of 2.819.

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Question 3

The functions f and g are given by $f(x) = \sqrt{x}$ and $g(x) = 6 - x$. Let R be the region bounded by the x -axis and the graphs of f and g , as shown in the figure above.



- (a) Find the area of R .
- (b) The region R is the base of a solid. For each y , where $0 \leq y \leq 2$, the cross section of the solid taken perpendicular to the y -axis is a rectangle whose base lies in R and whose height is $2y$. Write, but do not evaluate, an integral expression that gives the volume of the solid.
- (c) There is a point P on the graph of f at which the line tangent to the graph of f is perpendicular to the graph of g . Find the coordinates of point P .

(a) Area = $\int_0^4 \sqrt{x} \, dx + \frac{1}{2} \cdot 2 \cdot 2 = \frac{2}{3}x^{3/2} \Big|_{x=0}^{x=4} + 2 = \frac{22}{3}$

3 : { 1 : integral
1 : antiderivative
1 : answer

(b) $y = \sqrt{x} \Rightarrow x = y^2$
 $y = 6 - x \Rightarrow x = 6 - y$

Width = $(6 - y) - y^2$

Volume = $\int_0^2 2y(6 - y - y^2) \, dy$

3 : { 2 : integrand
1 : answer

(c) $g'(x) = -1$

Thus a line perpendicular to the graph of g has slope 1.

$f'(x) = \frac{1}{2\sqrt{x}}$

$\frac{1}{2\sqrt{x}} = 1 \Rightarrow x = \frac{1}{4}$

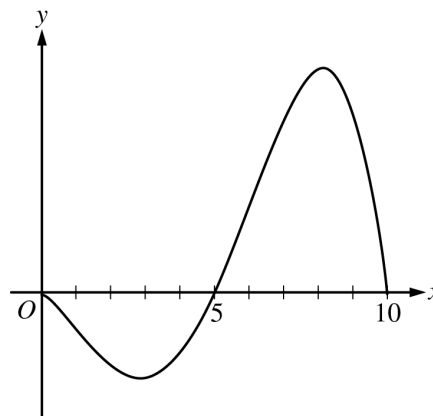
The point P has coordinates $\left(\frac{1}{4}, \frac{1}{2}\right)$.

3 : { 1 : $f'(x)$
1 : equation
1 : answer

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Question 4

The graph of the differentiable function $y = f(x)$ with domain $0 \leq x \leq 10$ is shown in the figure above. The area of the region enclosed between the graph of f and the x -axis for $0 \leq x \leq 5$ is 10, and the area of the region enclosed between the graph of f and the x -axis for $5 \leq x \leq 10$ is 27. The arc length for the portion of the graph of f between $x = 0$ and $x = 5$ is 11, and the arc length for the portion of the graph of f between $x = 5$ and $x = 10$ is 18. The function f has exactly two critical points that are located at $x = 3$ and $x = 8$.



Graph of f

- (a) Find the average value of f on the interval $0 \leq x \leq 5$.
- (b) Evaluate $\int_0^{10} (3f(x) + 2) dx$. Show the computations that lead to your answer.
- (c) Let $g(x) = \int_5^x f(t) dt$. On what intervals, if any, is the graph of g both concave up and decreasing? Explain your reasoning.
- (d) The function h is defined by $h(x) = 2f\left(\frac{x}{2}\right)$. The derivative of h is $h'(x) = f'\left(\frac{x}{2}\right)$. Find the arc length of the graph of $y = h(x)$ from $x = 0$ to $x = 20$.

(a) Average value = $\frac{1}{5} \int_0^5 f(x) dx = \frac{-10}{5} = -2$

1 : answer

(b) $\int_0^{10} (3f(x) + 2) dx = 3\left(\int_0^5 f(x) dx + \int_5^{10} f(x) dx\right) + 20$
 $= 3(-10 + 27) + 20 = 71$

2 : answer

- (c) $g'(x) = f(x)$
 $g'(x) < 0$ on $0 < x < 5$
 $g'(x)$ is increasing on $3 < x < 8$.
 The graph of g is concave up and decreasing on $3 < x < 5$.

3 : $\begin{cases} 1 : g'(x) = f(x) \\ 1 : \text{analysis} \\ 1 : \text{answer and reason} \end{cases}$

(d) Arc length = $\int_0^{20} \sqrt{1 + (h'(x))^2} dx = \int_0^{20} \sqrt{1 + \left(f'\left(\frac{x}{2}\right)\right)^2} dx$

Let $u = \frac{x}{2}$. Then $du = \frac{1}{2} dx$ and

$$\int_0^{20} \sqrt{1 + \left(f'\left(\frac{x}{2}\right)\right)^2} dx = 2 \int_0^{10} \sqrt{1 + (f'(u))^2} du = 2(11 + 18) = 58$$

3 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{substitution} \\ 1 : \text{answer} \end{cases}$

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Question 5

t (seconds)	0	10	40	60
$B(t)$ (meters)	100	136	9	49
$v(t)$ (meters per second)	2.0	2.3	2.5	4.6

Ben rides a unicycle back and forth along a straight east-west track. The twice-differentiable function B models Ben's position on the track, measured in meters from the western end of the track, at time t , measured in seconds from the start of the ride. The table above gives values for $B(t)$ and Ben's velocity, $v(t)$, measured in meters per second, at selected times t .

- (a) Use the data in the table to approximate Ben's acceleration at time $t = 5$ seconds. Indicate units of measure.
- (b) Using correct units, interpret the meaning of $\int_0^{60} |v(t)| dt$ in the context of this problem. Approximate $\int_0^{60} |v(t)| dt$ using a left Riemann sum with the subintervals indicated by the data in the table.
- (c) For $40 \leq t \leq 60$, must there be a time t when Ben's velocity is 2 meters per second? Justify your answer.
- (d) A light is directly above the western end of the track. Ben rides so that at time t , the distance $L(t)$ between Ben and the light satisfies $(L(t))^2 = 12^2 + (B(t))^2$. At what rate is the distance between Ben and the light changing at time $t = 40$?

(a) $a(5) \approx \frac{v(10) - v(0)}{10 - 0} = \frac{0.3}{10} = 0.03 \text{ meters/sec}^2$

1 : answer

- (b) $\int_0^{60} |v(t)| dt$ is the total distance, in meters, Ben rides over the 60-second interval $t = 0$ to $t = 60$.

2 : $\left\{ \begin{array}{l} 1 : \text{meaning of integral} \\ 1 : \text{approximation} \end{array} \right.$

$$\int_0^{60} |v(t)| dt \approx 2.0 \cdot 10 + 2.3(40 - 10) + 2.5(60 - 40) = 139 \text{ meters}$$

- (c) Because $\frac{B(60) - B(40)}{60 - 40} = \frac{49 - 9}{20} = 2$, the Mean Value Theorem implies there is a time t , $40 < t < 60$, such that $v(t) = 2$.

2 : $\left\{ \begin{array}{l} 1 : \text{difference quotient} \\ 1 : \text{conclusion with justification} \end{array} \right.$

(d) $2L(t)L'(t) = 2B(t)B'(t)$

$$L'(40) = \frac{B(40)v(40)}{L(40)} = \frac{9 \cdot 2.5}{\sqrt{144 + 81}} = \frac{3}{2} \text{ meters/sec}$$

3 : $\left\{ \begin{array}{l} 1 : \text{derivatives} \\ 1 : \text{uses } B'(t) = v(t) \\ 1 : \text{answer} \end{array} \right.$

1 : units in (a) or (b)

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Question 6

Let $f(x) = \ln(1 + x^3)$.

- (a) The Maclaurin series for $\ln(1 + x)$ is $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots + (-1)^{n+1} \cdot \frac{x^n}{n} + \dots$. Use the series to write the first four nonzero terms and the general term of the Maclaurin series for f .
- (b) The radius of convergence of the Maclaurin series for f is 1. Determine the interval of convergence. Show the work that leads to your answer.
- (c) Write the first four nonzero terms of the Maclaurin series for $f'(t^2)$. If $g(x) = \int_0^x f'(t^2) dt$, use the first two nonzero terms of the Maclaurin series for g to approximate $g(1)$.
- (d) The Maclaurin series for g , evaluated at $x = 1$, is a convergent alternating series with individual terms that decrease in absolute value to 0. Show that your approximation in part (c) must differ from $g(1)$ by less than $\frac{1}{5}$.

(a) $x^3 - \frac{x^6}{2} + \frac{x^9}{3} - \frac{x^{12}}{4} + \dots + (-1)^{n+1} \cdot \frac{x^{3n}}{n} + \dots$

2 : $\begin{cases} 1 : \text{first four terms} \\ 1 : \text{general term} \end{cases}$

- (b) The interval of convergence is centered at $x = 0$.

At $x = -1$, the series is $-1 - \frac{1}{2} - \frac{1}{3} - \frac{1}{4} - \dots - \frac{1}{n} - \dots$, which diverges because the harmonic series diverges.

At $x = 1$, the series is $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots + (-1)^{n+1} \cdot \frac{1}{n} + \dots$, the alternating harmonic series, which converges.

Therefore the interval of convergence is $-1 < x \leq 1$.

2 : answer with analysis

- (c) The Maclaurin series for $f'(x)$, $f'(t^2)$, and $g(x)$ are

$$f'(x) : \sum_{n=1}^{\infty} (-1)^{n+1} \cdot 3x^{3n-1} = 3x^2 - 3x^5 + 3x^8 - 3x^{11} + \dots$$

$$f'(t^2) : \sum_{n=1}^{\infty} (-1)^{n+1} \cdot 3t^{6n-2} = 3t^4 - 3t^{10} + 3t^{16} - 3t^{22} + \dots$$

$$g(x) : \sum_{n=1}^{\infty} (-1)^{n+1} \cdot \frac{3x^{6n-1}}{6n-1} = \frac{3x^5}{5} - \frac{3x^{11}}{11} + \frac{3x^{17}}{17} - \frac{3x^{23}}{23} + \dots$$

$$\text{Thus } g(1) \approx \frac{3}{5} - \frac{3}{11} = \frac{18}{55}.$$

4 : $\begin{cases} 1 : \text{two terms for } f'(t^2) \\ 1 : \text{other terms for } f'(t^2) \\ 1 : \text{first two terms for } g(x) \\ 1 : \text{approximation} \end{cases}$

- (d) The Maclaurin series for g evaluated at $x = 1$ is alternating, and the terms decrease in absolute value to 0.

$$\text{Thus } \left| g(1) - \frac{18}{55} \right| < \frac{3 \cdot 1^{17}}{17} = \frac{3}{17} < \frac{1}{5}.$$

1 : analysis