



AP[®] Calculus BC 2011 Scoring Guidelines

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Question 1

At time t , a particle moving in the xy -plane is at position $(x(t), y(t))$, where $x(t)$ and $y(t)$ are not explicitly given. For $t \geq 0$, $\frac{dx}{dt} = 4t + 1$ and $\frac{dy}{dt} = \sin(t^2)$. At time $t = 0$, $x(0) = 0$ and $y(0) = -4$.

- (a) Find the speed of the particle at time $t = 3$, and find the acceleration vector of the particle at time $t = 3$.
 (b) Find the slope of the line tangent to the path of the particle at time $t = 3$.
 (c) Find the position of the particle at time $t = 3$.
 (d) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 3$.

(a) Speed $\sqrt{(x'(3))^2 + (y'(3))^2}$ 13.006 or 13.007

Acceleration = $\langle x''(3), y''(3) \rangle$
 $= \langle 4, -5.466 \rangle$ or $\langle 4, -5.467 \rangle$

2 : { 1 : speed
1 : acceleration

(b) Slope = $\frac{y'(3)}{x'(3)} = 0.031$ or 0.032

1 : answer

(c) $x(3) = 0 + \int_0^3 \frac{dx}{dt} dt = 21$

$y(3) = -4 + \int_0^3 \frac{dy}{dt} dt = -3.226$

At time $t = 3$, the particle is at position $(21, -3.226)$.

4 : { 2 : x -coordinate
1 : integral
1 : answer
2 : y -coordinate
1 : integral
1 : answer

(d) Distance = $\int_0^3 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt = 21.091$

2 : { 1 : integral
1 : answer

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Question 2

t (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

As a pot of tea cools, the temperature of the tea is modeled by a differentiable function H for $0 \leq t \leq 10$, where time t is measured in minutes and temperature $H(t)$ is measured in degrees Celsius. Values of $H(t)$ at selected values of time t are shown in the table above.

- (a) Use the data in the table to approximate the rate at which the temperature of the tea is changing at time $t = 3.5$. Show the computations that lead to your answer.
- (b) Using correct units, explain the meaning of $\frac{1}{10} \int_0^{10} H(t) dt$ in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate $\frac{1}{10} \int_0^{10} H(t) dt$.
- (c) Evaluate $\int_0^{10} H'(t) dt$. Using correct units, explain the meaning of the expression in the context of this problem.
- (d) At time $t = 0$, biscuits with temperature 100°C were removed from an oven. The temperature of the biscuits at time t is modeled by a differentiable function B for which it is known that $B'(t) = -13.84e^{-0.173t}$. Using the given models, at time $t = 10$, how much cooler are the biscuits than the tea?

(a)
$$H'(3.5) \approx \frac{H(5) - H(2)}{5 - 2}$$

$$= \frac{52 - 60}{3} = -2.666 \text{ or } -2.667 \text{ degrees Celsius per minute}$$

(b) $\frac{1}{10} \int_0^{10} H(t) dt$ is the average temperature of the tea, in degrees Celsius, over the 10 minutes.

$$\frac{1}{10} \int_0^{10} H(t) dt \approx \frac{1}{10} \left(2 \cdot \frac{66 + 60}{2} + 3 \cdot \frac{60 + 52}{2} + 4 \cdot \frac{52 + 44}{2} + 1 \cdot \frac{44 + 43}{2} \right)$$

$$= 52.95$$

(c) $\int_0^{10} H'(t) dt = H(10) - H(0) = 43 - 66 = -23$
 The temperature of the tea drops 23 degrees Celsius from time $t = 0$ to time $t = 10$ minutes.

(d) $B(10) = 100 + \int_0^{10} B'(t) dt = 34.18275$; $H(10) - B(10) = 8.817$
 The biscuits are 8.817 degrees Celsius cooler than the tea.

1 : answer

3 : $\begin{cases} 1 : \text{meaning of expression} \\ 1 : \text{trapezoidal sum} \\ 1 : \text{estimate} \end{cases}$

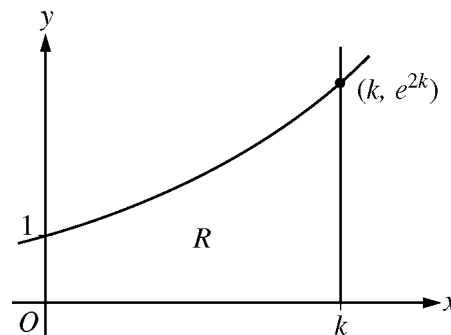
2 : $\begin{cases} 1 : \text{value of integral} \\ 1 : \text{meaning of expression} \end{cases}$

3 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{uses } B(0) = 100 \\ 1 : \text{answer} \end{cases}$

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Question 3

Let $f(x) = e^{2x}$. Let R be the region in the first quadrant bounded by the graph of f , the coordinate axes, and the vertical line $x = k$, where $k > 0$. The region R is shown in the figure above.



- (a) Write, but do not evaluate, an expression involving an integral that gives the perimeter of R in terms of k .
- (b) The region R is rotated about the x -axis to form a solid. Find the volume, V , of the solid in terms of k .
- (c) The volume V , found in part (b), changes as k changes. If $\frac{dk}{dt} = \frac{1}{3}$, determine $\frac{dV}{dt}$ when $k = \frac{1}{2}$.

(a) $f'(x) = 2e^{2x}$

$$\text{Perimeter} = 1 + k + e^{2k} + \int_0^k \sqrt{1 + (2e^{2x})^2} dx$$

3 : $\left\{ \begin{array}{l} 1 : f'(x) \\ 1 : \text{integral} \\ 1 : \text{answer} \end{array} \right.$

(b) $\text{Volume} = \pi \int_0^k (e^{2x})^2 dx = \pi \int_0^k e^{4x} dx = \frac{\pi}{4} e^{4x} \Big|_{x=0}^{x=k} = \frac{\pi}{4} e^{4k} - \frac{\pi}{4}$

4 : $\left\{ \begin{array}{l} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{array} \right.$

(c) $\frac{dV}{dt} = \pi e^{4k} \frac{dk}{dt}$

When $k = \frac{1}{2}$, $\frac{dV}{dt} = \pi e^2 \cdot \frac{1}{3}$.

2 : $\left\{ \begin{array}{l} 1 : \text{applies chain rule} \\ 1 : \text{answer} \end{array} \right.$

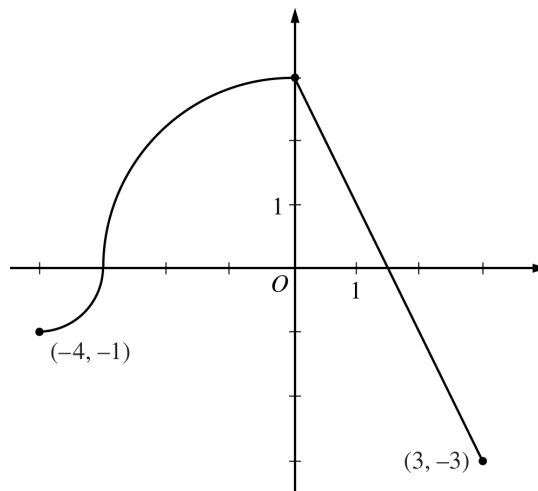
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Question 4

The continuous function f is defined on the interval $-4 \leq x \leq 3$. The graph of f consists of two quarter circles and one line segment, as shown in the figure above.

Let $g(x) = 2x + \int_0^x f(t) dt$.

- (a) Find $g(-3)$. Find $g'(x)$ and evaluate $g'(-3)$.
- (b) Determine the x -coordinate of the point at which g has an absolute maximum on the interval $-4 \leq x \leq 3$. Justify your answer.
- (c) Find all values of x on the interval $-4 < x < 3$ for which the graph of g has a point of inflection. Give a reason for your answer.
- (d) Find the average rate of change of f on the interval $-4 \leq x \leq 3$. There is no point c , $-4 < c < 3$, for which $f'(c)$ is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem.



Graph of f

(a) $g(-3) = 2(-3) + \int_0^{-3} f(t) dt = -6 - \frac{9\pi}{4}$
 $g'(x) = 2 + f(x)$
 $g'(-3) = 2 + f(-3) = 2$

$$3 : \begin{cases} 1 : g(-3) \\ 1 : g'(x) \\ 1 : g'(-3) \end{cases}$$

(b) $g'(x) = 0$ when $f(x) = -2$. This occurs at $x = \frac{5}{2}$.
 $g'(x) > 0$ for $-4 < x < \frac{5}{2}$ and $g'(x) < 0$ for $\frac{5}{2} < x < 3$.
 Therefore g has an absolute maximum at $x = \frac{5}{2}$.

$$3 : \begin{cases} 1 : \text{considers } g'(x) = 0 \\ 1 : \text{identifies interior candidate} \\ 1 : \text{answer with justification} \end{cases}$$

(c) $g''(x) = f'(x)$ changes sign only at $x = 0$. Thus the graph of g has a point of inflection at $x = 0$.

1 : answer with reason

(d) The average rate of change of f on the interval $-4 \leq x \leq 3$ is
 $\frac{f(3) - f(-4)}{3 - (-4)} = -\frac{2}{7}$.

$$2 : \begin{cases} 1 : \text{average rate of change} \\ 1 : \text{explanation} \end{cases}$$

To apply the Mean Value Theorem, f must be differentiable at each point in the interval $-4 < x < 3$. However, f is not differentiable at $x = -3$ and $x = 0$.

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Question 5

At the beginning of 2010, a landfill contained 1400 tons of solid waste. The increasing function W models the total amount of solid waste stored at the landfill. Planners estimate that W will satisfy the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ for the next 20 years. W is measured in tons, and t is measured in years from the start of 2010.

- (a) Use the line tangent to the graph of W at $t = 0$ to approximate the amount of solid waste that the landfill contains at the end of the first 3 months of 2010 (time $t = \frac{1}{4}$).
- (b) Find $\frac{d^2W}{dt^2}$ in terms of W . Use $\frac{d^2W}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the amount of solid waste that the landfill contains at time $t = \frac{1}{4}$.
- (c) Find the particular solution $W = W(t)$ to the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ with initial condition $W(0) = 1400$.

(a) $\left. \frac{dW}{dt} \right|_{t=0} = \frac{1}{25}(W(0) - 300) = \frac{1}{25}(1400 - 300) = 44$

The tangent line is $y = 1400 + 44t$.

$$W\left(\frac{1}{4}\right) \approx 1400 + 44\left(\frac{1}{4}\right) = 1411 \text{ tons}$$

$$2 : \begin{cases} 1 : \frac{dW}{dt} \text{ at } t = 0 \\ 1 : \text{answer} \end{cases}$$

(b) $\frac{d^2W}{dt^2} = \frac{1}{25} \frac{dW}{dt} = \frac{1}{625}(W - 300)$ and $W \geq 1400$

Therefore $\frac{d^2W}{dt^2} > 0$ on the interval $0 \leq t \leq \frac{1}{4}$.

The answer in part (a) is an underestimate.

$$2 : \begin{cases} 1 : \frac{d^2W}{dt^2} \\ 1 : \text{answer with reason} \end{cases}$$

(c) $\frac{dW}{dt} = \frac{1}{25}(W - 300)$

$$\int \frac{1}{W - 300} dW = \int \frac{1}{25} dt$$

$$\ln|W - 300| = \frac{1}{25}t + C$$

$$\ln(1400 - 300) = \frac{1}{25}(0) + C \Rightarrow \ln(1100) = C$$

$$W - 300 = 1100e^{\frac{1}{25}t}$$

$$W(t) = 300 + 1100e^{\frac{1}{25}t}, \quad 0 \leq t \leq 20$$

$$5 : \begin{cases} 1 : \text{separation of variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } W \end{cases}$$

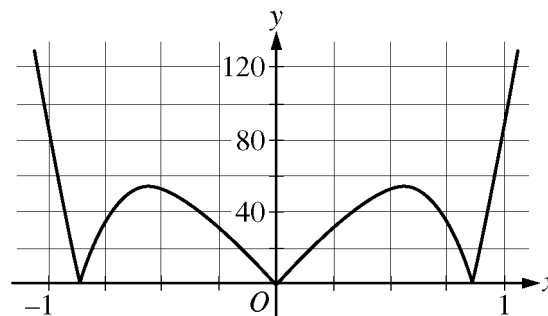
Note: max 2/5 [1-1-0-0-0] if no constant of integration

Note: 0/5 if no separation of variables

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Question 6

Let $f(x) = \sin(x^2) + \cos x$. The graph of $y = |f^{(5)}(x)|$ is shown above.



Graph of $y = |f^{(5)}(x)|$

- (a) Write the first four nonzero terms of the Taylor series for $\sin x$ about $x = 0$, and write the first four nonzero terms of the Taylor series for $\sin(x^2)$ about $x = 0$.
- (b) Write the first four nonzero terms of the Taylor series for $\cos x$ about $x = 0$. Use this series and the series for $\sin(x^2)$, found in part (a), to write the first four nonzero terms of the Taylor series for f about $x = 0$.
- (c) Find the value of $f^{(6)}(0)$.
- (d) Let $P_4(x)$ be the fourth-degree Taylor polynomial for f about $x = 0$. Using information from the graph of $y = |f^{(5)}(x)|$ shown above, show that $\left|P_4\left(\frac{1}{4}\right) - f\left(\frac{1}{4}\right)\right| < \frac{1}{3000}$.

(a) $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$
 $\sin(x^2) = x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \frac{x^{14}}{7!} + \dots$

3 : $\begin{cases} 1 : \text{series for } \sin x \\ 2 : \text{series for } \sin(x^2) \end{cases}$

(b) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$
 $f(x) = 1 + \frac{x^2}{2} + \frac{x^4}{4!} - \frac{121x^6}{6!} + \dots$

3 : $\begin{cases} 1 : \text{series for } \cos x \\ 2 : \text{series for } f(x) \end{cases}$

(c) $\frac{f^{(6)}(0)}{6!}$ is the coefficient of x^6 in the Taylor series for f about $x = 0$. Therefore $f^{(6)}(0) = -121$.

1 : answer

(d) The graph of $y = |f^{(5)}(x)|$ indicates that $\max_{0 \leq x \leq \frac{1}{4}} |f^{(5)}(x)| < 40$.

2 : $\begin{cases} 1 : \text{form of the error bound} \\ 1 : \text{analysis} \end{cases}$

Therefore

$$\left|P_4\left(\frac{1}{4}\right) - f\left(\frac{1}{4}\right)\right| \leq \frac{\max_{0 \leq x \leq \frac{1}{4}} |f^{(5)}(x)|}{5!} \cdot \left(\frac{1}{4}\right)^5 < \frac{40}{120 \cdot 4^5} = \frac{1}{3072} < \frac{1}{3000}.$$