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# AP<sup>®</sup> Calculus BC

## 2014 Scoring Guidelines

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**Question 1**

Grass clippings are placed in a bin, where they decompose. For  $0 \leq t \leq 30$ , the amount of grass clippings remaining in the bin is modeled by  $A(t) = 6.687(0.931)^t$ , where  $A(t)$  is measured in pounds and  $t$  is measured in days.

- (a) Find the average rate of change of  $A(t)$  over the interval  $0 \leq t \leq 30$ . Indicate units of measure.
- (b) Find the value of  $A'(15)$ . Using correct units, interpret the meaning of the value in the context of the problem.
- (c) Find the time  $t$  for which the amount of grass clippings in the bin is equal to the average amount of grass clippings in the bin over the interval  $0 \leq t \leq 30$ .
- (d) For  $t > 30$ ,  $L(t)$ , the linear approximation to  $A$  at  $t = 30$ , is a better model for the amount of grass clippings remaining in the bin. Use  $L(t)$  to predict the time at which there will be 0.5 pound of grass clippings remaining in the bin. Show the work that leads to your answer.

(a)  $\frac{A(30) - A(0)}{30 - 0} = -0.197$  (or  $-0.196$ ) lbs/day

1 : answer with units

(b)  $A'(15) = -0.164$  (or  $-0.163$ )

The amount of grass clippings in the bin is decreasing at a rate of 0.164 (or 0.163) lbs/day at time  $t = 15$  days.

2 :  $\begin{cases} 1 : A'(15) \\ 1 : \text{interpretation} \end{cases}$

(c)  $A(t) = \frac{1}{30} \int_0^{30} A(t) dt \Rightarrow t = 12.415$  (or 12.414)

2 :  $\begin{cases} 1 : \frac{1}{30} \int_0^{30} A(t) dt \\ 1 : \text{answer} \end{cases}$

(d)  $L(t) = A(30) + A'(30) \cdot (t - 30)$

$A'(30) = -0.055976$

$A(30) = 0.782928$

$L(t) = 0.5 \Rightarrow t = 35.054$

4 :  $\begin{cases} 2 : \text{expression for } L(t) \\ 1 : L(t) = 0.5 \\ 1 : \text{answer} \end{cases}$

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**Question 2**

The graphs of the polar curves  $r = 3$  and  $r = 3 - 2\sin(2\theta)$  are shown in the figure above for  $0 \leq \theta \leq \pi$ .

(a) Let  $R$  be the shaded region that is inside the graph of  $r = 3$  and inside the graph of  $r = 3 - 2\sin(2\theta)$ . Find the area of  $R$ .

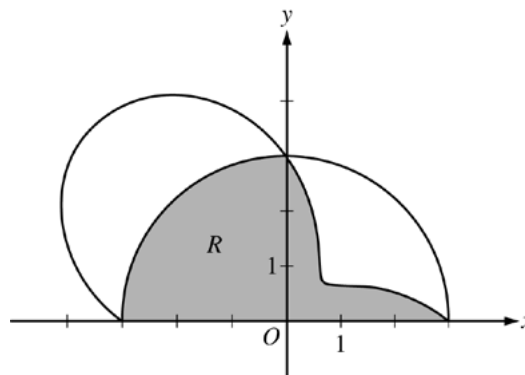
(b) For the curve  $r = 3 - 2\sin(2\theta)$ , find the value of  $\frac{dx}{d\theta}$  at

$$\theta = \frac{\pi}{6}.$$

(c) The distance between the two curves changes for  $0 < \theta < \frac{\pi}{2}$ .

Find the rate at which the distance between the two curves is changing with respect to  $\theta$  when  $\theta = \frac{\pi}{3}$ .

(d) A particle is moving along the curve  $r = 3 - 2\sin(2\theta)$  so that  $\frac{d\theta}{dt} = 3$  for all times  $t \geq 0$ . Find the value of  $\frac{dr}{dt}$  at  $\theta = \frac{\pi}{6}$ .



(a) 
$$\text{Area} = \frac{9\pi}{4} + \frac{1}{2} \int_0^{\pi/2} (3 - 2\sin(2\theta))^2 d\theta$$
  

$$= 9.708 \text{ (or } 9.707)$$

3 :  $\begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$

(b) 
$$x = (3 - 2\sin(2\theta))\cos\theta$$
  

$$\left. \frac{dx}{d\theta} \right|_{\theta=\pi/6} = -2.366$$

2 :  $\begin{cases} 1 : \text{expression for } x \\ 1 : \text{answer} \end{cases}$

(c) The distance between the two curves is  

$$D = 3 - (3 - 2\sin(2\theta)) = 2\sin(2\theta).$$

2 :  $\begin{cases} 1 : \text{expression for distance} \\ 1 : \text{answer} \end{cases}$

$$\left. \frac{dD}{d\theta} \right|_{\theta=\pi/3} = -2$$

(d) 
$$\frac{dr}{dt} = \frac{dr}{d\theta} \cdot \frac{d\theta}{dt} = \frac{dr}{d\theta} \cdot 3$$
  

$$\left. \frac{dr}{dt} \right|_{\theta=\pi/6} = (-2)(3) = -6$$

2 :  $\begin{cases} 1 : \text{chain rule with respect to } t \\ 1 : \text{answer} \end{cases}$

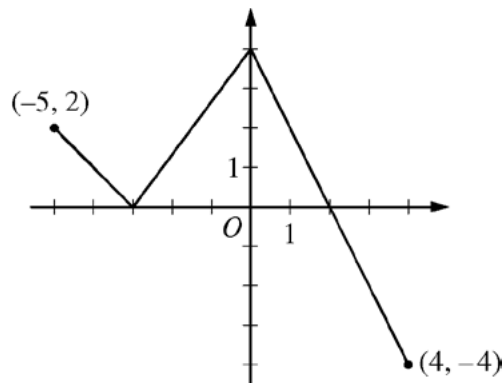
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**Question 3**

The function  $f$  is defined on the closed interval  $[-5, 4]$ . The graph of  $f$  consists of three line segments and is shown in the figure above.

Let  $g$  be the function defined by  $g(x) = \int_{-3}^x f(t) dt$ .

- (a) Find  $g(3)$ .
- (b) On what open intervals contained in  $-5 < x < 4$  is the graph of  $g$  both increasing and concave down? Give a reason for your answer.
- (c) The function  $h$  is defined by  $h(x) = \frac{g(x)}{5x}$ . Find  $h'(3)$ .
- (d) The function  $p$  is defined by  $p(x) = f(x^2 - x)$ . Find the slope of the line tangent to the graph of  $p$  at the point where  $x = -1$ .



Graph of  $f$

(a)  $g(3) = \int_{-3}^3 f(t) dt = 6 + 4 - 1 = 9$

1 : answer

(b)  $g'(x) = f(x)$

The graph of  $g$  is increasing and concave down on the intervals  $-5 < x < -3$  and  $0 < x < 2$  because  $g' = f$  is positive and decreasing on these intervals.

2 :  $\begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$

(c)  $h'(x) = \frac{5xg'(x) - g(x)5}{(5x)^2} = \frac{5xg'(x) - 5g(x)}{25x^2}$

3 :  $\begin{cases} 2 : h'(x) \\ 1 : \text{answer} \end{cases}$

$$h'(3) = \frac{(5)(3)g'(3) - 5g(3)}{25 \cdot 3^2}$$

$$= \frac{15(-2) - 5(9)}{225} = \frac{-75}{225} = -\frac{1}{3}$$

(d)  $p'(x) = f'(x^2 - x)(2x - 1)$

3 :  $\begin{cases} 2 : p'(x) \\ 1 : \text{answer} \end{cases}$

$$p'(-1) = f'(2)(-3) = (-2)(-3) = 6$$

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**Question 4**

Train  $A$  runs back and forth on an east-west section of railroad track. Train  $A$ 's velocity, measured in meters per minute, is given by a differentiable function  $v_A(t)$ , where time  $t$  is measured in minutes. Selected values for  $v_A(t)$  are given in the table above.

$t$ (minutes)	0	2	5	8	12
$v_A(t)$ (meters/minute)	0	100	40	-120	-150

- (a) Find the average acceleration of train  $A$  over the interval  $2 \leq t \leq 8$ .
- (b) Do the data in the table support the conclusion that train  $A$ 's velocity is  $-100$  meters per minute at some time  $t$  with  $5 < t < 8$ ? Give a reason for your answer.
- (c) At time  $t = 2$ , train  $A$ 's position is 300 meters east of the Origin Station, and the train is moving to the east. Write an expression involving an integral that gives the position of train  $A$ , in meters from the Origin Station, at time  $t = 12$ . Use a trapezoidal sum with three subintervals indicated by the table to approximate the position of the train at time  $t = 12$ .
- (d) A second train, train  $B$ , travels north from the Origin Station. At time  $t$  the velocity of train  $B$  is given by  $v_B(t) = -5t^2 + 60t + 25$ , and at time  $t = 2$  the train is 400 meters north of the station. Find the rate, in meters per minute, at which the distance between train  $A$  and train  $B$  is changing at time  $t = 2$ .

(a) average accel =  $\frac{v_A(8) - v_A(2)}{8 - 2} = \frac{-120 - 100}{6} = -\frac{110}{3}$  m/min<sup>2</sup>

(b)  $v_A$  is differentiable  $\Rightarrow v_A$  is continuous  
 $v_A(8) = -120 < -100 < 40 = v_A(5)$

Therefore, by the Intermediate Value Theorem, there is a time  $t$ ,  $5 < t < 8$ , such that  $v_A(t) = -100$ .

(c)  $s_A(12) = s_A(2) + \int_2^{12} v_A(t) dt = 300 + \int_2^{12} v_A(t) dt$   
 $\int_2^{12} v_A(t) dt \approx 3 \cdot \frac{100 + 40}{2} + 3 \cdot \frac{40 - 120}{2} + 4 \cdot \frac{-120 - 150}{2}$   
 $= -450$

$s_A(12) \approx 300 - 450 = -150$

The position of Train  $A$  at time  $t = 12$  minutes is approximately 150 meters west of Origin Station.

- (d) Let  $x$  be train  $A$ 's position,  $y$  train  $B$ 's position, and  $z$  the distance between train  $A$  and train  $B$ .

$z^2 = x^2 + y^2 \Rightarrow 2z \frac{dz}{dt} = 2x \frac{dx}{dt} + 2y \frac{dy}{dt}$

$x = 300, y = 400 \Rightarrow z = 500$

$v_B(2) = -20 + 120 + 25 = 125$

$500 \frac{dz}{dt} = (300)(100) + (400)(125)$

$\frac{dz}{dt} = \frac{80000}{500} = 160$  meters per minute

1 : average acceleration

2 :  $\begin{cases} 1 : v_A(8) < -100 < v_A(5) \\ 1 : \text{conclusion, using IVT} \end{cases}$

3 :  $\begin{cases} 1 : \text{position expression} \\ 1 : \text{trapezoidal sum} \\ 1 : \text{position at time } t = 12 \end{cases}$

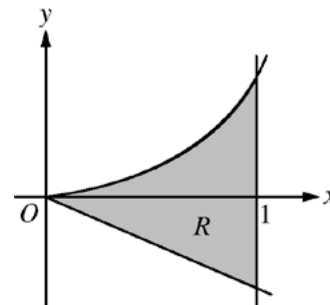
3 :  $\begin{cases} 2 : \text{implicit differentiation of} \\ \quad \text{distance relationship} \\ 1 : \text{answer} \end{cases}$

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**Question 5**

Let  $R$  be the shaded region bounded by the graph of  $y = xe^{x^2}$ , the line  $y = -2x$ , and the vertical line  $x = 1$ , as shown in the figure above.

- (a) Find the area of  $R$ .
- (b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when  $R$  is rotated about the horizontal line  $y = -2$ .
- (c) Write, but do not evaluate, an expression involving one or more integrals that gives the perimeter of  $R$ .



(a) Area =  $\int_0^1 (xe^{x^2} - (-2x)) dx$   
 $= \left[ \frac{1}{2}e^{x^2} + x^2 \right]_{x=0}^{x=1}$   
 $= \left( \frac{1}{2}e + 1 \right) - \frac{1}{2} = \frac{e + 1}{2}$

3 :  $\begin{cases} 1 : \text{integrand} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(b) Volume =  $\pi \int_0^1 \left[ (xe^{x^2} + 2)^2 - (-2x + 2)^2 \right] dx$

3 :  $\begin{cases} 2 : \text{integrand} \\ 1 : \text{limits and constant} \end{cases}$

(c)  $y' = \frac{d}{dx}(xe^{x^2}) = e^{x^2} + 2x^2e^{x^2} = e^{x^2}(1 + 2x^2)$

3 :  $\begin{cases} 1 : y' = e^{x^2}(1 + 2x^2) \\ 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

Perimeter =  $\sqrt{5} + 2 + e + \int_0^1 \sqrt{1 + [e^{x^2}(1 + 2x^2)]^2} dx$

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**Question 6**

The Taylor series for a function  $f$  about  $x = 1$  is given by  $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2^n}{n} (x-1)^n$  and converges to  $f(x)$  for  $|x-1| < R$ , where  $R$  is the radius of convergence of the Taylor series.

- (a) Find the value of  $R$ .
- (b) Find the first three nonzero terms and the general term of the Taylor series for  $f'$ , the derivative of  $f$ , about  $x = 1$ .
- (c) The Taylor series for  $f'$  about  $x = 1$ , found in part (b), is a geometric series. Find the function  $f'$  to which the series converges for  $|x-1| < R$ . Use this function to determine  $f$  for  $|x-1| < R$ .

- (a) Let  $a_n$  be the  $n$ th term of the Taylor series.

$$\begin{aligned} \frac{a_{n+1}}{a_n} &= \frac{(-1)^{n+2} 2^{n+1} (x-1)^{n+1}}{n+1} \cdot \frac{n}{(-1)^{n+1} 2^n (x-1)^n} \\ &= \frac{-2n(x-1)}{n+1} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left| \frac{-2n(x-1)}{n+1} \right| = 2|x-1|$$

$$2|x-1| < 1 \Rightarrow |x-1| < \frac{1}{2}$$

The radius of convergence is  $R = \frac{1}{2}$ .

- (b) The first three nonzero terms are

$$2 - 4(x-1) + 8(x-1)^2.$$

The general term is  $(-1)^{n+1} 2^n (x-1)^{n-1}$  for  $n \geq 1$ .

- (c) The common ratio is  $-2(x-1)$ .

$$f'(x) = \frac{2}{1 - (-2(x-1))} = \frac{2}{2x-1} \text{ for } |x-1| < \frac{1}{2}$$

$$f(x) = \int \frac{2}{2x-1} dx = \ln|2x-1| + C$$

$$f(1) = 0$$

$$\ln|1| + C = 0 \Rightarrow C = 0$$

$$f(x) = \ln|2x-1| \text{ for } |x-1| < \frac{1}{2}$$

3 :  $\begin{cases} 1 : \text{sets up ratio} \\ 1 : \text{computes limit of ratio} \\ 1 : \text{determines radius of convergence} \end{cases}$

3 :  $\begin{cases} 2 : \text{first three nonzero terms} \\ 1 : \text{general term} \end{cases}$

3 :  $\begin{cases} 1 : f'(x) \\ 1 : \text{antiderivative} \\ 1 : f(x) \end{cases}$