

2017

AP<sup>®</sup>

CollegeBoard

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# AP Calculus BC

## Scoring Guidelines

**AP<sup>®</sup> CALCULUS AB/CALCULUS BC  
2017 SCORING GUIDELINES**

**Question 1**

<p>(a) Volume = <math>\int_0^{10} A(h) dh</math>  <math>\approx (2 - 0) \cdot A(0) + (5 - 2) \cdot A(2) + (10 - 5) \cdot A(5)</math>  <math>= 2 \cdot 50.3 + 3 \cdot 14.4 + 5 \cdot 6.5</math>  <math>= 176.3</math> cubic feet</p> <p>(b) The approximation in part (a) is an overestimate because a left Riemann sum is used and <math>A</math> is decreasing.</p> <p>(c) <math>\int_0^{10} f(h) dh = 101.325338</math>   The volume is 101.325 cubic feet.</p> <p>(d) Using the model, <math>V(h) = \int_0^h f(x) dx</math>.</p> $\left. \frac{dV}{dt} \right _{h=5} = \left[ \frac{dV}{dh} \cdot \frac{dh}{dt} \right]_{h=5}$ $= \left[ f(h) \cdot \frac{dh}{dt} \right]_{h=5}$ $= f(5) \cdot 0.26 = 1.694419$ <p>When <math>h = 5</math>, the volume of water is changing at a rate of 1.694 cubic feet per minute.</p>	<p>1 : units in parts (a), (c), and (d)</p> <p>2 : <math>\left\{ \begin{array}{l} 1 : \text{left Riemann sum} \\ 1 : \text{approximation} \end{array} \right.</math></p> <p>1 : overestimate with reason</p> <p>2 : <math>\left\{ \begin{array}{l} 1 : \text{integral} \\ 1 : \text{answer} \end{array} \right.</math></p> <p>3 : <math>\left\{ \begin{array}{l} 2 : \frac{dV}{dt} \\ 1 : \text{answer} \end{array} \right.</math></p>
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**Question 2**

(a)  $\frac{1}{2} \int_0^{\pi/2} (f(\theta))^2 d\theta = 0.648414$

The area of  $R$  is 0.648.

2 :  $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b)  $\int_0^k ((g(\theta))^2 - (f(\theta))^2) d\theta = \frac{1}{2} \int_0^{\pi/2} ((g(\theta))^2 - (f(\theta))^2) d\theta$

— OR —

$$\int_0^k ((g(\theta))^2 - (f(\theta))^2) d\theta = \int_k^{\pi/2} ((g(\theta))^2 - (f(\theta))^2) d\theta$$

2 :  $\begin{cases} 1 : \text{integral expression} \\ \quad \text{for one region} \\ 1 : \text{equation} \end{cases}$

(c)  $w(\theta) = g(\theta) - f(\theta)$

$$w_A = \frac{\int_0^{\pi/2} w(\theta) d\theta}{\frac{\pi}{2} - 0} = 0.485446$$

The average value of  $w(\theta)$  on the interval  $\left[0, \frac{\pi}{2}\right]$  is 0.485.

3 :  $\begin{cases} 1 : w(\theta) \\ 1 : \text{integral} \\ 1 : \text{average value} \end{cases}$

(d)  $w(\theta) = w_A$  for  $0 \leq \theta \leq \frac{\pi}{2} \Rightarrow \theta = 0.517688$

$w(\theta) = w_A$  at  $\theta = 0.518$  (or 0.517).

$w'(0.518) < 0 \Rightarrow w(\theta)$  is decreasing at  $\theta = 0.518$ .

2 :  $\begin{cases} 1 : \text{solves } w(\theta) = w_A \\ 1 : \text{answer with reason} \end{cases}$

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**Question 3**

(a)  $f(-6) = f(-2) + \int_{-2}^{-6} f'(x) dx = 7 - \int_{-6}^{-2} f'(x) dx = 7 - 4 = 3$

$$f(5) = f(-2) + \int_{-2}^5 f'(x) dx = 7 - 2\pi + 3 = 10 - 2\pi$$

(b)  $f'(x) > 0$  on the intervals  $[-6, -2]$  and  $(2, 5]$ .

Therefore,  $f$  is increasing on the intervals  $[-6, -2]$  and  $[2, 5]$ .

(c) The absolute minimum will occur at a critical point where  $f'(x) = 0$  or at an endpoint.

$$f'(x) = 0 \Rightarrow x = -2, x = 2$$

$x$	$f(x)$
-6	3
-2	7
2	$7 - 2\pi$
5	$10 - 2\pi$

The absolute minimum value is  $f(2) = 7 - 2\pi$ .

(d)  $f''(-5) = \frac{2 - 0}{-6 - (-2)} = -\frac{1}{2}$

$$\lim_{x \rightarrow 3^-} \frac{f'(x) - f'(3)}{x - 3} = 2 \text{ and } \lim_{x \rightarrow 3^+} \frac{f'(x) - f'(3)}{x - 3} = -1$$

$f''(3)$  does not exist because

$$\lim_{x \rightarrow 3^-} \frac{f'(x) - f'(3)}{x - 3} \neq \lim_{x \rightarrow 3^+} \frac{f'(x) - f'(3)}{x - 3}.$$

3 :  $\begin{cases} 1 : \text{uses initial condition} \\ 1 : f(-6) \\ 1 : f(5) \end{cases}$

2 : answer with justification

2 :  $\begin{cases} 1 : \text{considers } x = 2 \\ 1 : \text{answer with justification} \end{cases}$

2 :  $\begin{cases} 1 : f''(-5) \\ 1 : f''(3) \text{ does not exist,} \\ \text{with explanation} \end{cases}$

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**Question 4**

(a)  $H'(0) = -\frac{1}{4}(91 - 27) = -16$   
 $H(0) = 91$

An equation for the tangent line is  $y = 91 - 16t$ .

The internal temperature of the potato at time  $t = 3$  minutes is approximately  $91 - 16 \cdot 3 = 43$  degrees Celsius.

(b)  $\frac{d^2H}{dt^2} = -\frac{1}{4} \frac{dH}{dt} = \left(-\frac{1}{4}\right)\left(-\frac{1}{4}\right)(H - 27) = \frac{1}{16}(H - 27)$

$$H > 27 \text{ for } t > 0 \Rightarrow \frac{d^2H}{dt^2} = \frac{1}{16}(H - 27) > 0 \text{ for } t > 0$$

Therefore, the graph of  $H$  is concave up for  $t > 0$ . Thus, the answer in part (a) is an underestimate.

(c)  $\frac{dG}{(G - 27)^{2/3}} = -dt$

$$\int \frac{dG}{(G - 27)^{2/3}} = \int (-1) dt$$

$$3(G - 27)^{1/3} = -t + C$$

$$3(91 - 27)^{1/3} = 0 + C \Rightarrow C = 12$$

$$3(G - 27)^{1/3} = 12 - t$$

$$G(t) = 27 + \left(\frac{12 - t}{3}\right)^3 \text{ for } 0 \leq t < 10$$

The internal temperature of the potato at time  $t = 3$  minutes is

$$27 + \left(\frac{12 - 3}{3}\right)^3 = 54 \text{ degrees Celsius.}$$

3 :  $\left\{ \begin{array}{l} 1 : \text{slope} \\ 1 : \text{tangent line} \\ 1 : \text{approximation} \end{array} \right.$

1 : underestimate with reason

5 :  $\left\{ \begin{array}{l} 1 : \text{separation of variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration and} \\ \quad \text{uses initial condition} \\ 1 : \text{equation involving } G \text{ and } t \\ 1 : G(t) \text{ and } G(3) \end{array} \right.$

Note: max 2/5 [1-1-0-0-0] if no constant of integration

Note: 0/5 if no separation of variables

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**Question 5**

(a)  $f'(x) = \frac{-3(4x - 7)}{(2x^2 - 7x + 5)^2}$

$$f'(3) = \frac{(-3)(5)}{(18 - 21 + 5)^2} = -\frac{15}{4}$$

(b)  $f'(x) = \frac{-3(4x - 7)}{(2x^2 - 7x + 5)^2} = 0 \Rightarrow x = \frac{7}{4}$

The only critical point in the interval  $1 < x < 2.5$  has  $x$ -coordinate  $\frac{7}{4}$ .

$f'$  changes sign from positive to negative at  $x = \frac{7}{4}$ .

Therefore,  $f$  has a relative maximum at  $x = \frac{7}{4}$ .

(c) 
$$\int_5^{\infty} f(x) dx = \lim_{b \rightarrow \infty} \int_5^b \frac{3}{2x^2 - 7x + 5} dx = \lim_{b \rightarrow \infty} \int_5^b \left( \frac{2}{2x - 5} - \frac{1}{x - 1} \right) dx$$

$$= \lim_{b \rightarrow \infty} \left[ \ln(2x - 5) - \ln(x - 1) \right]_5^b = \lim_{b \rightarrow \infty} \left[ \ln \left( \frac{2x - 5}{x - 1} \right) \right]_5^b$$

$$= \lim_{b \rightarrow \infty} \left[ \ln \left( \frac{2b - 5}{b - 1} \right) - \ln \left( \frac{5}{4} \right) \right] = \ln 2 - \ln \left( \frac{5}{4} \right) = \ln \left( \frac{8}{5} \right)$$

(d)  $f$  is continuous, positive, and decreasing on  $[5, \infty)$ .

The series converges by the integral test since  $\int_5^{\infty} \frac{3}{2x^2 - 7x + 5} dx$  converges.

— OR —

$$\frac{3}{2n^2 - 7n + 5} > 0 \text{ and } \frac{1}{n^2} > 0 \text{ for } n \geq 5.$$

Since  $\lim_{n \rightarrow \infty} \frac{\frac{3}{2n^2 - 7n + 5}}{\frac{1}{n^2}} = \frac{3}{2}$  and the series  $\sum_{n=5}^{\infty} \frac{1}{n^2}$  converges,

the series  $\sum_{n=5}^{\infty} \frac{3}{2n^2 - 7n + 5}$  converges by the limit comparison test.

2 :  $f'(3)$

2 :  $\left\{ \begin{array}{l} 1 : x\text{-coordinate} \\ 1 : \text{relative maximum} \\ \quad \text{with justification} \end{array} \right.$

3 :  $\left\{ \begin{array}{l} 1 : \text{antiderivative} \\ 1 : \text{limit expression} \\ 1 : \text{answer} \end{array} \right.$

2 : answer with conditions

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**Question 6**

(a)  $f(0) = 0$

$$f'(0) = 1$$

$$f''(0) = -1(1) = -1$$

$$f'''(0) = -2(-1) = 2$$

$$f^{(4)}(0) = -3(2) = -6$$

The first four nonzero terms are

$$0 + 1x + \frac{-1}{2!}x^2 + \frac{2}{3!}x^3 + \frac{-6}{4!}x^4 = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}.$$

The general term is  $\frac{(-1)^{n+1}x^n}{n}$ .

(b) For  $x = 1$ , the Maclaurin series becomes  $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ .

The series does not converge absolutely because the harmonic series diverges.

The series alternates with terms that decrease in magnitude to 0, and therefore the series converges conditionally.

(c) 
$$\int_0^x f(t) dt = \int_0^x \left( t - \frac{t^2}{2} + \frac{t^3}{3} - \frac{t^4}{4} + \dots + \frac{(-1)^{n+1}t^n}{n} + \dots \right) dt$$

$$= \left[ \frac{t^2}{2} - \frac{t^3}{3 \cdot 2} + \frac{t^4}{4 \cdot 3} - \frac{t^5}{5 \cdot 4} + \dots + \frac{(-1)^{n+1}t^{n+1}}{(n+1)n} + \dots \right]_{t=0}^{t=x}$$

$$= \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{12} - \frac{x^5}{20} + \dots + \frac{(-1)^{n+1}x^{n+1}}{(n+1)n} + \dots$$

(d) The terms alternate in sign and decrease in magnitude to 0. By the alternating series error bound, the error  $\left| P_4\left(\frac{1}{2}\right) - g\left(\frac{1}{2}\right) \right|$  is bounded by the magnitude of the first unused term,  $\left| -\frac{(1/2)^5}{20} \right|$ .

Thus,  $\left| P_4\left(\frac{1}{2}\right) - g\left(\frac{1}{2}\right) \right| \leq \left| -\frac{(1/2)^5}{20} \right| = \frac{1}{32 \cdot 20} < \frac{1}{500}$ .

3 :  $\begin{cases} 1 : f''(0), f'''(0), \text{ and } f^{(4)}(0) \\ 1 : \text{verify terms} \\ 1 : \text{general term} \end{cases}$

2 : converges conditionally with reason

3 :  $\begin{cases} 1 : \text{two terms} \\ 1 : \text{remaining terms} \\ 1 : \text{general term} \end{cases}$

1 : error bound