1997 AP Calculus AB: Section I, Part B

40 Minutes—Graphing Calculator Required

Notes: (1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

76. If $f(x) = \dfrac{e^{2x}}{2x}$, then $f'(x) =$

(A) 1

(B) $\dfrac{e^{2x}(1-2x)}{2x^2}$

(C) e^{2x}

(D) $\dfrac{e^{2x}(2x+1)}{x^2}$

(E) $\dfrac{e^{2x}(2x-1)}{2x^2}$

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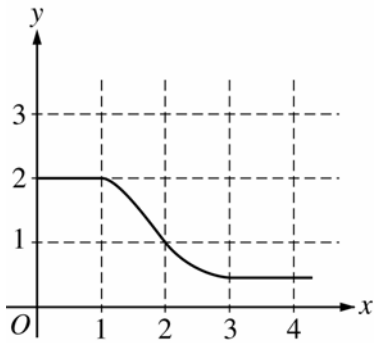
77. The graph of the function $y = x^3 + 6x^2 + 7x - 2\cos x$ changes concavity at $x =$

(A) –1.58 (B) –1.63 (C) –1.67 (D) –1.89 (E) –2.33



78. The graph of f is shown in the figure above. If $\int_1^3 f(x)\,dx = 2.3$ and $F'(x) = f(x)$, then $F(3) - F(0) =$

(A) 0.3 (B) 1.3 (C) 3.3 (D) 4.3 (E) 5.3

79. Let f be a function such that $\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}=5$. Which of the following must be true?

I. f is continuous at $x = 2$.

II. f is differentiable at $x = 2$.

III. The derivative of f is continuous at $x = 2$.

(A) I only (B) II only (C) I and II only (D) I and III only (E) II and III only

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80. Let f be the function given by $f(x)=2e^{4x^2}$. For what value of x is the slope of the line tangent to the graph of f at $\big(x, f(x)\big)$ equal to 3?

(A) 0.168 (B) 0.276 (C) 0.318 (D) 0.342 (E) 0.551

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81. A railroad track and a road cross at right angles. An observer stands on the road 70 meters south of the crossing and watches an eastbound train traveling at 60 meters per second. At how many meters per second is the train moving away from the observer 4 seconds after it passes through the intersection?

(A) 57.60 (B) 57.88 (C) 59.20 (D) 60.00 (E) 67.40

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82. If $y = 2x - 8$, what is the minimum value of the product xy ?

(A) –16 (B) –8 (C) –4 (D) 0 (E) 2

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83. What is the area of the region in the first quadrant enclosed by the graphs of $y=\cos x$, $y=x$, and the y-axis?

(A) 0.127 (B) 0.385 (C) 0.400 (D) 0.600 (E) 0.947

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84. The base of a solid S is the region enclosed by the graph of $y=\sqrt{\ln x}$, the line $x=e$, and the x-axis. If the cross sections of S perpendicular to the x-axis are squares, then the volume of S is

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) 1 (D) 2 (E) $\frac{1}{3}(e^3-1)$

85. If the derivative of f is given by $f'(x) = e^x - 3x^2$, at which of the following values of x does f have a relative maximum value?

(A) –0.46 (B) 0.20 (C) 0.91 (D) 0.95 (E) 3.73

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86. Let $f(x) = \sqrt{x}$. If the rate of change of f at $x = c$ is twice its rate of change at $x = 1$, then $c =$

(A) $\frac{1}{4}$ (B) 1 (C) 4 (D) $\frac{1}{\sqrt{2}}$ (E) $\frac{1}{2\sqrt{2}}$

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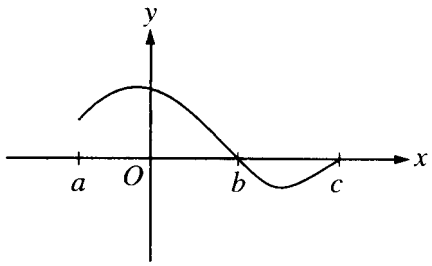
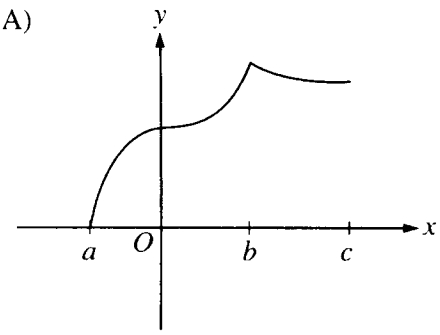
87. At time $t \geq 0$, the acceleration of a particle moving on the x-axis is $a(t) = t + \sin t$. At $t = 0$, the velocity of the particle is –2. For what value t will the velocity of the particle be zero?

(A) 1.02 (B) 1.48 (C) 1.85 (D) 2.81 (E) 3.14



88. Let $f(x) = \int_a^x h(t)\,dt$, where h has the graph shown above. Which of the following could be the graph of f?

(A)



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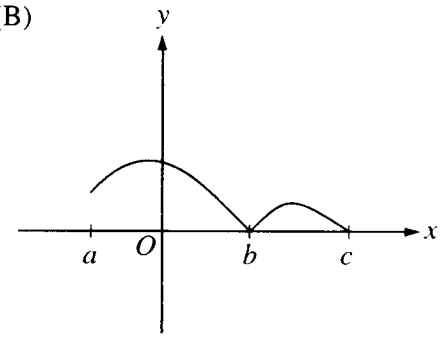

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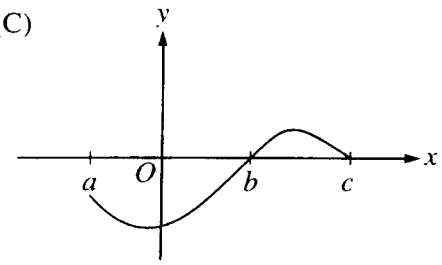

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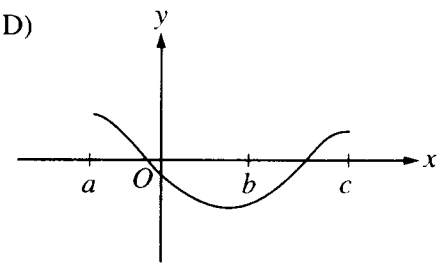
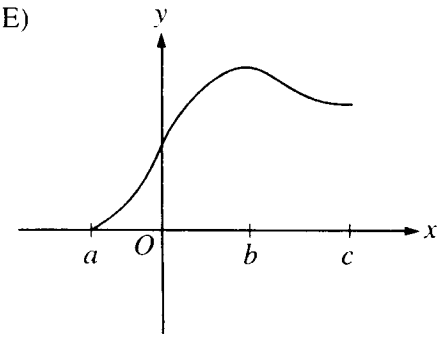


(E)



x	0	0.5	1.0	1.5	2.0
$f(x)$	3	3	5	8	13

89. A table of values for a continuous function f is shown above. If four equal subintervals of $[0,2]$ are used, which of the following is the trapezoidal approximation of $\int_0^2 f(x)\,dx$?

(A) 8 (B) 12 (C) 16 (D) 24 (E) 32

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90. Which of the following are antiderivatives of $f(x) = \sin x \cos x$?

I. $F(x) = \dfrac{\sin^2 x}{2}$

II. $F(x) = \dfrac{\cos^2 x}{2}$

III. $F(x) = \dfrac{-\cos(2x)}{4}$

(A) I only

(B) II only

(C) III only

(D) I and III only

(E) II and III only

76. E $f(x)=\dfrac{e^{2x}}{2x};\ f'(x)=\dfrac{2e^{2x}\cdot 2x-2e^{2x}}{4x^2}=\dfrac{e^{2x}(2x-1)}{2x^2}$

77. D $y=x^3+6x^2+7x-2\cos x$. Look at the graph of $y''=6x+12+2\cos x$ in the window [–3,–1] since that domain contains all the option values. y'' changes sign at $x=-1.89$.

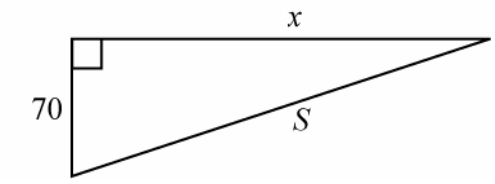
78. D $F(3)-F(0)=\int_0^3 f(x)dx=\int_0^1 f(x)dx+\int_1^3 f(x)dx=2+2.3=4.3$

(Count squares for $\int_0^1 f(x)dx$)

79. C The stem of the questions means $f'(2)=5$. Thus f is differentiable at $x=2$ and therefore continuous at $x=2$. We know nothing of the continuity of f'. I and II only.

80. A $f(x)=2e^{4x^2};\ f'(x)=16xe^{4x^2};$ We want $16xe^{4x^2}=3.$ Graph the derivative function and the function $y=3$, then find the intersection to get $x=0.168$.

81. A Let x be the distance of the train from the crossing. Then $\dfrac{dx}{dt}=60$.



$S^2=x^2+70^2\Rightarrow 2S\dfrac{dS}{dt}=2x\dfrac{dx}{dt}\Rightarrow\dfrac{dS}{dt}=\dfrac{x}{S}\dfrac{dx}{dt}.$

After 4 seconds, $x=240$ and so $S=250$.

Therefore $\dfrac{dS}{dt}=\dfrac{240}{250}(60)=57.6$

82. B $P(x)=2x^2-8x;\ P'(x)=4x-8;\ P'$ changes from negative to positive at $x=2$. $P(2)=-8$

83. C $\cos x=x$ at $x=0.739085$. Store this in A. $\int_0^A(\cos x-x)\,dx=0.400$

84. C Cross sections are squares with sides of length y.

Volume $=\int_1^e y^2dx=\int_1^e \ln x\,dx=(x\ln x-x)\big|_1^e=(e\ln e-e)-(0-1)=1$

85. C Look at the graph of f' and locate where the y changes from positive to negative. $x=0.91$

86. A $f(x)=\sqrt{x};\ f'(x)=\dfrac{1}{2\sqrt{x}};\ \dfrac{1}{2\sqrt{c}}=2\cdot\dfrac{1}{2\sqrt{1}}\ \Rightarrow\ c=\dfrac{1}{4}$

87. B $a(t) = t + \sin t$ and $v(0) = -2 \Rightarrow v(t) = \frac{1}{2}t^2 - \cos t - 1;\ v(t) = 0$ at $t = 1.48$

88. E $f(x) = \int_a^x h(x)dx \Rightarrow f(a) = 0,$ therefore only (A) or (E) are possible. But $f'(x) = h(x)$ and therefore f is differentiable at $x = b$. This is true for the graph in option (E) but not in option (A) where there appears to be a corner in the graph at $x = b$. Also, Since h is increasing at first, the graph of f must start out concave up. This is also true in (E) but not (A).

89. B $T = \frac{1}{2} \cdot \frac{1}{2}(3 + 2 \cdot 3 + 2 \cdot 5 + 2 \cdot 8 + 13) = 12$

90. D

$F(x) = \frac{1}{2}\sin^2 x$	$F'(x) = \sin x \cos x$	Yes
$F(x) = \frac{1}{2}\cos^2 x$	$F'(x) = -\cos x \sin x$	No
$F(x) = -\frac{1}{4}\cos(2x)$	$F'(x) = \frac{1}{2}\sin(2x) = \sin x \cos x$	Yes