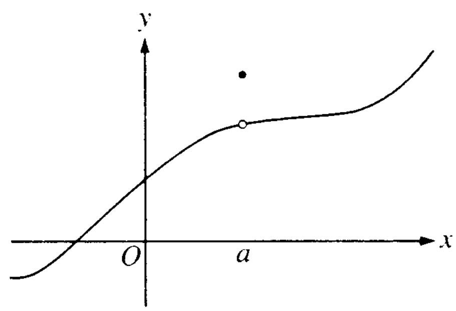
50 Minutes—Graphing Calculator Required

Notes: (1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.



76. The graph of a function f is shown above. Which of the following statements about f is false?

(A) f is continuous at $x = a$.

(B) f has a relative maximum at $x = a$.

(C) $x = a$ is in the domain of f.

(D) $\lim_{x\to a^+} f(x)$ is equal to $\lim_{x\to a^-} f(x)$.

(E) $\lim_{x\to a} f(x)$ exists.

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77. Let f be the function given by $f(x) = 3e^{2x}$ and let g be the function given by $g(x) = 6x^3$. At what value of x do the graphs of f and g have parallel tangent lines?

(A) -0.701
(B) -0.567
(C) -0.391
(D) -0.302
(E) -0.258

78. The radius of a circle is decreasing at a constant rate of 0.1 centimeter per second. In terms of the circumference C, what is the rate of change of the area of the circle, in square centimeters per second?

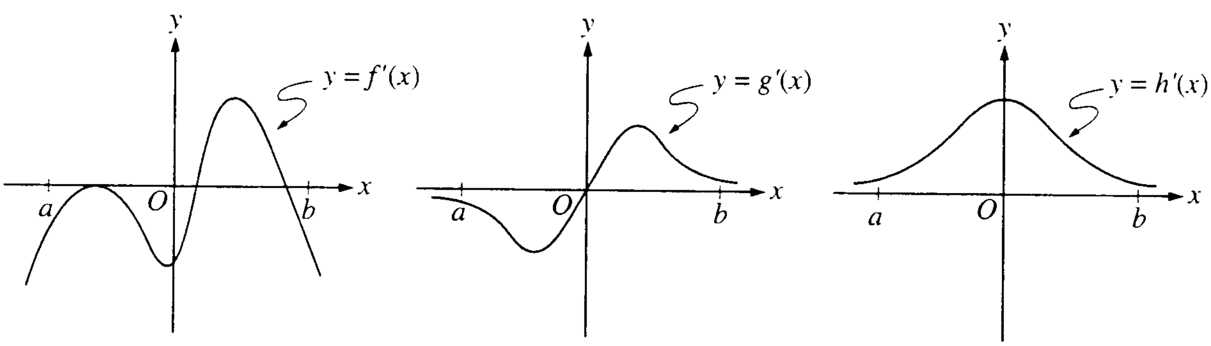
(A) $-(0.2)\pi C$

(B) $-(0.1)C$

(C) $-\dfrac{(0.1)C}{2\pi}$

(D) $(0.1)^2 C$

(E) $(0.1)^2 \pi C$



79. The graphs of the derivatives of the functions f, g, and h are shown above. Which of the functions f, g, or h have a relative maximum on the open interval $a < x < b$?

(A) f only
(B) g only
(C) h only
(D) f and g only
(E) f, g, and h

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80. The first derivative of the function f is given by $f'(x) = \dfrac{\cos^2 x}{x} - \dfrac{1}{5}$. How many critical values does f have on the open interval $(0,10)$?

(A) One
(B) Three
(C) Four
(D) Five
(E) Seven

81. Let f be the function given by $f(x) = |x|$. Which of the following statements about f are true?

I. f is continuous at $x = 0$.
II. f is differentiable at $x = 0$.
III. f has an absolute minimum at $x = 0$.

(A) I only (B) II only (C) III only (D) I and III only (E) II and III only

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82. If f is a continuous function and if $F'(x) = f(x)$ for all real numbers x, then $\int_1^3 f(2x)\,dx =$

(A) $2F(3) - 2F(1)$

(B) $\frac{1}{2}F(3) - \frac{1}{2}F(1)$

(C) $2F(6) - 2F(2)$

(D) $F(6) - F(2)$

(E) $\frac{1}{2}F(6) - \frac{1}{2}F(2)$

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83. If $a \neq 0$, then $\lim_{x \to a} \frac{x^2 - a^2}{x^4 - a^4}$ is

(A) $\frac{1}{a^2}$ (B) $\frac{1}{2a^2}$ (C) $\frac{1}{6a^2}$ (D) 0 (E) nonexistent

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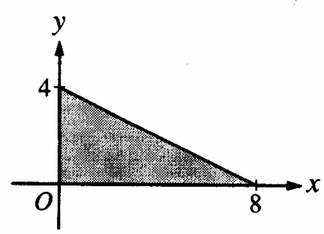
84. Population y grows according to the equation $\frac{dy}{dt} = ky$, where k is a constant and t is measured in years. If the population doubles every 10 years, then the value of k is

(A) 0.069 (B) 0.200 (C) 0.301 (D) 3.322 (E) 5.000

x	2	5	7	8
$f(x)$	10	30	40	20

85. The function f is continuous on the closed interval $[2,8]$ and has values that are given in the table above. Using the subintervals $[2,5]$, $[5,7]$, and $[7,8]$, what is the trapezoidal approximation of $\int_2^8 f(x)\,dx$?

(A) 110 (B) 130 (C) 160 (D) 190 (E) 210



86. The base of a solid is a region in the first quadrant bounded by the x-axis, the y-axis, and the line $x+2y=8$, as shown in the figure above. If cross sections of the solid perpendicular to the x-axis are semicircles, what is the volume of the solid?

(A) 12.566 (B) 14.661 (C) 16.755 (D) 67.021 (E) 134.041

87. Which of the following is an equation of the line tangent to the graph of $f(x)=x^4+2x^2$ at the point where $f'(x)=1$?

(A) $y=8x-5$
(B) $y=x+7$
(C) $y=x+0.763$
(D) $y=x-0.122$
(E) $y=x-2.146$

88. Let $F(x)$ be an antiderivative of $\dfrac{(\ln x)^3}{x}$. If $F(1)=0$, then $F(9)=$

(A) 0.048 (B) 0.144 (C) 5.827 (D) 23.308 (E) 1,640.250

89. If g is a differentiable function such that $g(x) < 0$ for all real numbers x and if $f'(x) = \left(x^2 - 4\right)g(x)$, which of the following is true?

(A) f has a relative maximum at $x = -2$ and a relative minimum at $x = 2$.
(B) f has a relative minimum at $x = -2$ and a relative maximum at $x = 2$.
(C) f has relative minima at $x = -2$ and at $x = 2$.
(D) f has relative maxima at $x = -2$ and at $x = 2$.
(E) It cannot be determined if f has any relative extrema.

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90. If the base b of a triangle is increasing at a rate of 3 inches per minute while its height h is decreasing at a rate of 3 inches per minute, which of the following must be true about the area A of the triangle?

(A) A is always increasing.
(B) A is always decreasing.
(C) A is decreasing only when $b < h$.
(D) A is decreasing only when $b > h$.
(E) A remains constant.

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91. Let f be a function that is differentiable on the open interval $(1,10)$. If $f(2) = -5,\ f(5) = 5,$ and $f(9) = -5$, which of the following must be true?

I. f has at least 2 zeros.
II. The graph of f has at least one horizontal tangent.
III. For some $c,\ 2 < c < 5,\ f(c) = 3$.

(A) None
(B) I only
(C) I and II only
(D) I and III only
(E) I, II, and III

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92. If $0 \le k < \frac{\pi}{2}$ and the area under the curve $y = \cos x$ from $x = k$ to $x = \frac{\pi}{2}$ is 0.1, then $k =$

(A) 1.471 (B) 1.414 (C) 1.277 (D) 1.120 (E) 0.436

76. A From the graph it is clear that f is not continuous at $x = a$. All others are true.

77. C Parallel tangents will occur when the slopes of f and g are equal. $f'(x) = 6e^{2x}$ and $g'(x) = 18x^2$. The graphs of these derivatives reveal that they are equal only at $x = -0.391$.

78. B $A = \pi r^2 \Rightarrow \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$. However, $C = 2\pi r$ and $\frac{dr}{dt} = -0.1$. Thus $\frac{dA}{dt} = -0.1C$.

79. A The graph of the derivative would have to change from positive to negative. This is only true for the graph of f'.

80. B Look at the graph of $f'(x)$ on the interval (0,10) and count the number of x-intercepts in the interval.

81. D Only II is false since the graph of the absolute value function has a sharp corner at $x = 0$.

82. E Since F is an antiderivative of f, $\int_1^3 f(2x)\,dx = \frac{1}{2}F(2x)\Big|_1^3 = \frac{1}{2}\big(F(6) - F(2)\big)$

83. B $\lim\limits_{x\to a} \frac{x^2 - a^2}{x^4 - a^4} = \lim\limits_{x\to a} \frac{x^2 - a^2}{(x^2 - a^2)(x^2 + a^2)} = \lim\limits_{x\to a} \frac{1}{(x^2 + a^2)} = \frac{1}{2a^2}$

84. A A known solution to this differential equation is $y(t) = y(0)e^{kt}$. Use the fact that the population is $2y(0)$ when $t = 10$. Then
$2y(0) = y(0)e^{k(10)} \Rightarrow e^{10k} = 2 \Rightarrow k = (0.1)\ln 2 = 0.069$

85. C There are 3 trapezoids. $\frac{1}{2}\cdot 3\big(f(2) + f(5)\big) + \frac{1}{2}\cdot 2\big(f(5) + f(7)\big) + \frac{1}{2}\cdot 1\big(f(7) + f(8)\big)$

86. C Each cross section is a semicircle with a diameter of y. The volume would be given by
$$\int_0^8 \frac{1}{2}\pi\left(\frac{y}{2}\right)^2 dx = \frac{\pi}{8}\int_0^8 \left(\frac{8-x}{2}\right)^2 dx = 16.755$$

87. D Find the x for which $f'(x) = 1$. $f'(x) = 4x^3 + 4x = 1$ only for $x = 0.237$. Then $f(0.237) = 0.115$. So the equation is $y - 0.115 = x - 0.237$. This is equivalent to option (D).

88. C $F(9)-F(1)=\int_1^9 \frac{(\ln t)^3}{t}\,dt = 5.827$ using a calculator. Since $F(1)=0$, $F(9)=5.827$.

Or solve the differential equation with an initial condition by finding an antiderivative for $\frac{(\ln x)^3}{x}$. This is of the form $u^3 du$ where $u=\ln x$. Hence $F(x)=\frac{1}{4}(\ln x)^4 + C$ and since $F(1)=0$, $C=0$. Therefore $F(9)=\frac{1}{4}(\ln 9)^4 = 5.827$

89. B The graph of $y=x^2-4$ is a parabola that changes from positive to negative at $x=-2$ and from negative to positive at $x=2$. Since g is always negative, f' changes sign opposite to the way $y=x^2-4$ does. Thus f has a relative minimum at $x=-2$ and a relative maximum at $x=2$.

90. D The area of a triangle is given by $A=\frac{1}{2}bh$. Taking the derivative with respect to t of both sides of the equation yields $\frac{dA}{dt}=\frac{1}{2}\left(\frac{db}{dt}\cdot h + b\cdot\frac{dh}{dt}\right)$. Substitute the given rates to get $\frac{dA}{dt}=\frac{1}{2}(3h-3b)=\frac{3}{2}(h-b)$. The area will be decreasing whenever $\frac{dA}{dt}<0$. This is true whenever $b>h$.

91. E I. True. Apply the Intermediate Value Theorem to each of the intervals $[2,5]$ and $[5,9]$.

II. True. Apply the Mean Value Theorem to the interval $[2,9]$.

III. True. Apply the Intermediate Value Theorem to the interval $[2,5]$.

92. D $\int_k^{\frac{\pi}{2}} \cos x\,dx = 0.1 \Rightarrow \sin\left(\frac{\pi}{2}\right) - \sin k = 0.1 \Rightarrow \sin k = 0.9$. Therefore $k=\sin^{-1}(0.9)=1.120$.