1997 AP Calculus BC: Section I, Part A

50 Minutes—No Calculator

Note: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. $\int_0^1 \sqrt{x}\left(x+1\right) dx =$

(A) 0 (B) 1 (C) $\frac{16}{15}$ (D) $\frac{7}{5}$ (E) 2

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2. If $x = e^{2t}$ and $y = \sin(2t)$, then $\frac{dy}{dx} =$

(A) $4e^{2t}\cos(2t)$ (B) $\frac{e^{2t}}{\cos(2t)}$ (C) $\frac{\sin(2t)}{2e^{2t}}$ (D) $\frac{\cos(2t)}{2e^{2t}}$ (E) $\frac{\cos(2t)}{e^{2t}}$

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3. The function f given by $f(x) = 3x^5 - 4x^3 - 3x$ has a relative maximum at $x =$

(A) -1 (B) $-\frac{\sqrt{5}}{5}$ (C) 0 (D) $\frac{\sqrt{5}}{5}$ (E) 1

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4. $\frac{d}{dx}\left(xe^{\ln x^2}\right) =$

(A) $1+2x$ (B) $x+x^2$ (C) $3x^2$ (D) x^3 (E) x^2+x^3

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5. If $f(x) = (x-1)^{\frac{3}{2}} + \frac{e^{x-2}}{2}$, then $f'(2) =$

(A) 1 (B) $\frac{3}{2}$ (C) 2 (D) $\frac{7}{2}$ (E) $\frac{3+e}{2}$

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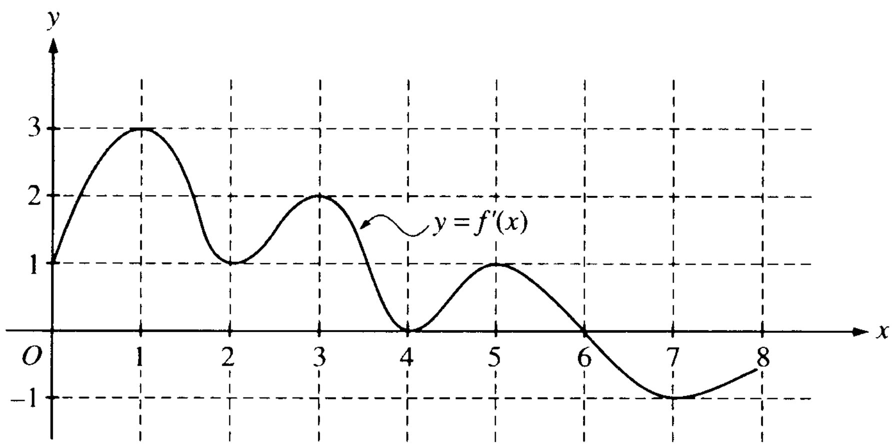
6. The line normal to the curve $y = \sqrt{16-x}$ at the point $\left(0,4\right)$ has slope

(A) 8 (B) 4 (C) $\frac{1}{8}$ (D) $-\frac{1}{8}$ (E) -8

Questions 7-9 refer to the graph and the information below.



The function f is defined on the closed interval $[0,8]$. The graph of its derivative f' is shown above.

7. The point $(3,5)$ is on the graph of $y = f(x)$. An equation of the line tangent to the graph of f at $(3,5)$ is

(A) $y = 2$

(B) $y = 5$

(C) $y - 5 = 2(x-3)$

(D) $y + 5 = 2(x-3)$

(E) $y + 5 = 2(x+3)$

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8. How many points of inflection does the graph of f have?

(A) Two
(B) Three
(C) Four
(D) Five
(E) Six

9. At what value of x does the absolute minimum of f occur?

(A) 0
(B) 2
(C) 4
(D) 6
(E) 8

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10. If $y = xy + x^2 + 1$, then when $x = -1$, $\frac{dy}{dx}$ is

(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) -1 (D) -2 (E) nonexistent

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11. $\int_1^\infty \frac{x}{(1+x^2)^2}\,dx$ is

(A) $-\frac{1}{2}$ (B) $-\frac{1}{4}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$ (E) divergent

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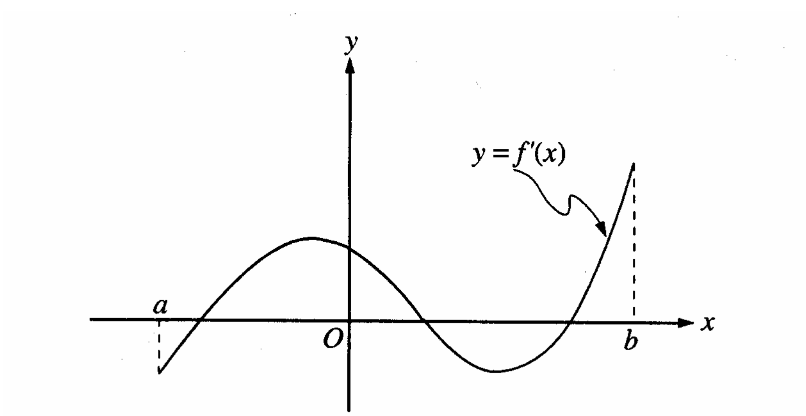

12. The graph of f', the derivative of f, is shown in the figure above. Which of the following describes all relative extrema of f on the open interval (a, b)?

(A) One relative maximum and two relative minima
(B) Two relative maxima and one relative minimum
(C) Three relative maxima and one relative minimum
(D) One relative maximum and three relative minima
(E) Three relative maxima and two relative minima

13. A particle moves along the x-axis so that its acceleration at any time t is $a(t) = 2t - 7$. If the initial velocity of the particle is 6, at what time t during the interval $0 \le t \le 4$ is the particle farthest to the right?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

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14. The sum of the infinite geometric series $\frac{3}{2} + \frac{9}{16} + \frac{27}{128} + \frac{81}{1{,}024} + \ldots$ is

(A) 1.60 (B) 2.35 (C) 2.40 (D) 2.45 (E) 2.50

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15. The length of the path described by the parametric equations $x = \cos^3 t$ and $y = \sin^3 t$, for $0 \le t \le \frac{\pi}{2}$, is given by

(A) $\int_0^{\frac{\pi}{2}} \sqrt{3\cos^2 t + 3\sin^2 t}\, dt$

(B) $\int_0^{\frac{\pi}{2}} \sqrt{-3\cos^2 t \sin t + 3\sin^2 t \cos t}\, dt$

(C) $\int_0^{\frac{\pi}{2}} \sqrt{9\cos^4 t + 9\sin^4 t}\, dt$

(D) $\int_0^{\frac{\pi}{2}} \sqrt{9\cos^4 t \sin^2 t + 9\sin^4 t \cos^2 t}\, dt$

(E) $\int_0^{\frac{\pi}{2}} \sqrt{\cos^6 t + \sin^6 t}\, dt$

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16. $\lim\limits_{h \to 0} \dfrac{e^h - 1}{2h}$ is

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) e (E) nonexistent

17. Let f be the function given by $f(x) = \ln(3-x)$. The third-degree Taylor polynomial for f about $x = 2$ is

(A) $-(x-2)+\frac{(x-2)^2}{2}-\frac{(x-2)^3}{3}$

(B) $-(x-2)-\frac{(x-2)^2}{2}-\frac{(x-2)^3}{3}$

(C) $(x-2)+(x-2)^2+(x-2)^3$

(D) $(x-2)+\frac{(x-2)^2}{2}+\frac{(x-2)^3}{3}$

(E) $(x-2)-\frac{(x-2)^2}{2}+\frac{(x-2)^3}{3}$

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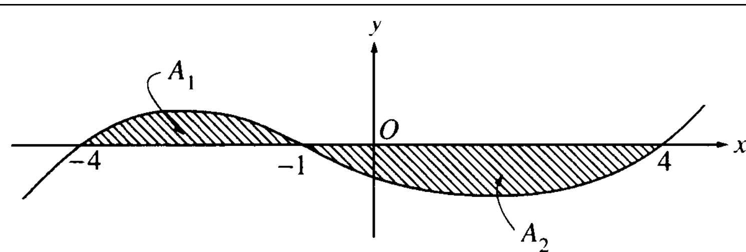
18. For what values of t does the curve given by the parametric equations $x = t^3 - t^2 - 1$ and $y = t^4 + 2t^2 - 8t$ have a vertical tangent?

(A) 0 only
(B) 1 only
(C) 0 and $\frac{2}{3}$ only
(D) 0, $\frac{2}{3}$, and 1
(E) No value



19. The graph of $y = f(x)$ is shown in the figure above. If A_1 and A_2 are positive numbers that represent the areas of the shaded regions, then in terms of A_1 and A_2,

$\int_{-4}^{4} f(x)\,dx - 2\int_{-1}^{4} f(x)\,dx =$

(A) A_1 (B) $A_1 - A_2$ (C) $2A_1 - A_2$ (D) $A_1 + A_2$ (E) $A_1 + 2A_2$

20. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n \cdot 3^n}$ converges?

(A) $-3 \le x \le 3$
(B) $-3 < x < 3$
(C) $-1 < x \le 5$
(D) $-1 \le x \le 5$
(E) $-1 \le x < 5$

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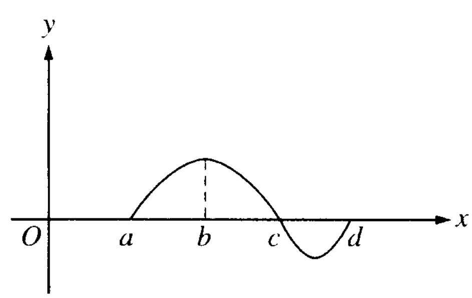
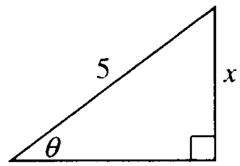
21. Which of the following is equal to the area of the region inside the polar curve $r = 2\cos\theta$ and outside the polar curve $r = \cos\theta$?

(A) $3\int_0^{\frac{\pi}{2}} \cos^2\theta \, d\theta$ (B) $3\int_0^{\pi} \cos^2\theta \, d\theta$ (C) $\frac{3}{2}\int_0^{\frac{\pi}{2}} \cos^2\theta \, d\theta$ (D) $3\int_0^{\frac{\pi}{2}} \cos\theta \, d\theta$ (E) $3\int_0^{\pi} \cos\theta \, d\theta$



22. The graph of f is shown in the figure above. If $g(x) = \int_a^x f(t)\,dt$, for what value of x does $g(x)$ have a maximum?

(A) a
(B) b
(C) c
(D) d
(E) It cannot be determined from the information given.



23. In the triangle shown above, if θ increases at a constant rate of 3 radians per minute, at what rate is x increasing in units per minute when x equals 3 units?

(A) 3 (B) $\frac{15}{4}$ (C) 4 (D) 9 (E) 12

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24. The Taylor series for $\sin x$ about $x = 0$ is $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots$. If f is a function such that $f'(x) = \sin\left(x^2\right)$, then the coefficient of x^7 in the Taylor series for $f(x)$ about $x = 0$ is

(A) $\frac{1}{7!}$ (B) $\frac{1}{7}$ (C) 0 (D) $-\frac{1}{42}$ (E) $-\frac{1}{7!}$

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25. The closed interval $[a, b]$ is partitioned into n equal subintervals, each of width Δx, by the numbers $x_0, x_1, \ldots, x_n$ where $a = x_0 < x_1 < x_2 < \cdots < x_{n-1} < x_n = b$. What is $\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{x_i}\,\Delta x$?

(A) $\frac{2}{3}\left(b^{\frac{3}{2}} - a^{\frac{3}{2}}\right)$

(B) $b^{\frac{3}{2}} - a^{\frac{3}{2}}$

(C) $\frac{3}{2}\left(b^{\frac{3}{2}} - a^{\frac{3}{2}}\right)$

(D) $b^{\frac{1}{2}} - a^{\frac{1}{2}}$

(E) $2\left(b^{\frac{1}{2}} - a^{\frac{1}{2}}\right)$

1997 Calculus BC Solutions: Part A

1. C $\int_0^1 \sqrt{x}(x+1)\,dx = \int_0^1 x^{\frac{3}{2}} + x^{\frac{1}{2}}\,dx = \frac{2}{5}x^{\frac{5}{2}} + \frac{2}{3}x^{\frac{3}{2}}\Big|_0^1 = \frac{16}{15}$

2. E $x = e^{2t}$, $y = \sin(2t)$; $\frac{dy}{dx} = \frac{2\cos(2t)}{2e^{2t}} = \frac{\cos(2t)}{e^{2t}}$

3. A $f(x) = 3x^5 - 4x^3 - 3x$; $f'(x) = 15x^4 - 12x^2 - 3 = 3(5x^2+1)(x^2-1) = 3(5x^2+1)(x+1)(x-1)$; f' changes from positive to negative only at $x = -1$.

4. C $e^{\ln x^2} = x^2$; so $xe^{\ln x^2} = x^3$ and $\frac{d}{dx}(x^3) = 3x^2$

5. C $f(x) = (x-1)^{\frac{3}{2}} + \frac{1}{2}e^{x-2}$; $f'(x) = \frac{3}{2}(x-1)^{\frac{1}{2}} + \frac{1}{2}e^{x-2}$; $f'(2) = \frac{3}{2} + \frac{1}{2} = 2$

6. A $y = (16-x)^{\frac{1}{2}}$; $y' = -\frac{1}{2}(16-x)^{-\frac{1}{2}}$; $y'(0) = -\frac{1}{8}$; The slope of the normal line is 8.

7. C The slope at $x = 3$ is 2. The equation of the tangent line is $y - 5 = 2(x-3)$.

8. E Points of inflection occur where f' changes from increasing to decreasing, or from decreasing to increasing. There are six such points.

9. A f increases for $0 \le x \le 6$ and decreases for $6 \le x \le 8$. By comparing areas it is clear that f increases more than it decreases, so the absolute minimum must occur at the left endpoint, $x = 0$.

10. B $y = xy + x^2 + 1$; $y' = xy' + y + 2x$; at $x = -1, y = 1$; $y' = -y' + 1 - 2 \Rightarrow y' = -\frac{1}{2}$

11. C $\int_1^\infty x(1+x^2)^{-2}\,dx = \lim_{L\to\infty} -\frac{1}{2}(1+x^2)^{-1}\Big|_1^L = \lim_{L\to\infty} \frac{1}{4} - \frac{1}{2(1+L^2)} = \frac{1}{4}$

12. A f' changes from positive to negative once and from negative to positive twice. Thus one relative maximum and two relative minimums.

13. B $a(t) = 2t - 7$ and $v(0) = 6$; so $v(t) = t^2 - 7t + 6 = (t-1)(t-6)$. Movement is right then left with the particle changing direction at $t = 1$, , therefore it will be farthest to the right at $t = 1$.

14. C Geometric Series. $r = \frac{3}{8} < 1 \Rightarrow$ convergence. $a = \frac{3}{2}$ so the sum will be $S = \dfrac{\frac{3}{2}}{1-\frac{3}{8}} = 2.4$

15. D $x = \cos^3 t,\ y = \sin^3 t$ for $0 \le t \le \frac{\pi}{2}$. $L = \int_0^{\pi/2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$$L = \int_0^{\pi/2} \sqrt{(-3\cos^2 t \sin t)^2 + (3\sin^2 t \cos t)^2}\, dt = \int_0^{\pi/2} \sqrt{9\cos^4 t \sin^2 t + 9\sin^4 t \cos^2 t}\, dt$$

16. B $\lim_{h\to 0} \frac{e^h - 1}{2h} = \frac{1}{2}\lim_{h\to 0} \frac{e^h - e^0}{h} = \frac{1}{2} f'(0)$, where $f(x) = e^x$ and $f'(0) = 1$. $\lim_{h\to 0} \frac{e^h - 1}{2h} = \frac{1}{2}$

17. B $f(x) = \ln(3-x);\ f'(x) = \frac{1}{x-3},\ f''(x) = -\frac{1}{(x-3)^2},\ f'''(x) = \frac{2}{(x-3)^3};$

$f(2) = 0,\ f'(2) = -1,\ f''(2) = -1,\ f'''(2) = -2;\ \ a_0 = 0,\ a_1 = -1,\ a_2 = -\frac{1}{2},\ a_3 = -\frac{1}{3}$

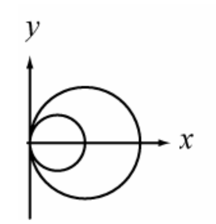
$f(x) \approx -(x-2) - \frac{(x-2)^2}{2} - \frac{(x-2)^3}{3}$

18. C $x = t^3 - t^2 - 1,\ y = t^4 + 2t^2 - 8t;\ \ \frac{dy}{dx} = \frac{4t^3 + 4t - 8}{3t^2 - 2t} = \frac{4t^3 + 4t - 8}{t(3t-2)}$. Vertical tangents at $t = 0, \frac{2}{3}$

19. D $\int_{-4}^{4} f(x)dx - 2\int_{-1}^{4} f(x)dx = (A_1 - A_2) - 2(-A_2) = A_1 + A_2$

20. E $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n \cdot 3^n}$. The endpoints of the interval of convergence are when $(x-2) = \pm 3;\ \ x = -1, 5$.
Check endpoints: $x = -1$ gives the alternating harmonic series which converges. $x = 5$ gives the harmonic series which diverges. Therefore the interval is $-1 \le x < 5$.

21. A Area $= 2 \cdot \frac{1}{2}\int_0^{\pi/2} ((2\cos\theta)^2 - \cos^2\theta)\, d\theta = \int_0^{\pi/2} 3\cos^2\theta\ d\theta$



22. C $g'(x) = f(x)$. The only critical value of g on (a,d) is at $x = c$. Since g' changes from positive to negative at $x = c$, the absolute maximum for g occurs at this relative maximum.

23. E $x = 5\sin\theta$; $\dfrac{dx}{dt} = 5\cos\theta \cdot \dfrac{d\theta}{dt}$; When $x = 3, \cos\theta = \dfrac{4}{5}$; $\dfrac{dx}{dt} = 5\left(\dfrac{4}{5}\right)(3) = 12$

24. D $f'(x) = \sin(x^2) = x^2 - \dfrac{(x^2)^3}{3!} + \cdots = x^2 - \dfrac{1}{6}x^6 + \cdots \Rightarrow f(x) = \dfrac{1}{3}x^3 - \dfrac{1}{42}x^7 + \cdots$ The coefficieint of x^7 is $-\dfrac{1}{42}$.

25. A This is the limit of a right Riemann sum of the function $f(x) = \sqrt{x}$ on the interval $[a, b]$, so

$$\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{x_i}\,\Delta x = \int_a^b \sqrt{x}\,dx = \frac{2}{3}x^{\frac{3}{2}}\bigg|_a^b = \frac{2}{3}\left(b^{\frac{3}{2}} - a^{\frac{3}{2}}\right)$$