50 Minutes—Graphing Calculator Required

Notes: (1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

76. For what integer k, $k > 1$, will both $\sum_{n=1}^{\infty} \frac{(-1)^{kn}}{n}$ and $\sum_{n=1}^{\infty} \left(\frac{k}{4}\right)^n$ converge?

(A) 6 (B) 5 (C) 4 (D) 3 (E) 2

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77. If f is a vector-valued function defined by $f(t) = \left(e^{-t}, \cos t\right)$, then $f''(t) =$

(A) $-e^{-t} + \sin t$
(B) $e^{-t} - \cos t$
(C) $\left(-e^{-t}, -\sin t\right)$
(D) $\left(e^{-t}, \cos t\right)$
(E) $\left(e^{-t}, -\cos t\right)$

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78. The radius of a circle is decreasing at a constant rate of 0.1 centimeter per second. In terms of the circumference C, what is the rate of change of the area of the circle, in square centimeters per second?

(A) $-(0.2)\pi C$

(B) $-(0.1)C$

(C) $-\frac{(0.1)C}{2\pi}$

(D) $(0.1)^2 C$

(E) $(0.1)^2 \pi C$

79. Let f be the function given by $f(x)=\frac{(x-1)(x^2-4)}{x^2-a}$. For what positive values of a is f continuous for all real numbers x?

(A) None
(B) 1 only
(C) 2 only
(D) 4 only
(E) 1 and 4 only

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80. Let R be the region enclosed by the graph of $y=1+\ln\left(\cos^4 x\right)$, the x-axis, and the lines $x=-\frac{2}{3}$ and $x=\frac{2}{3}$. The closest integer approximation of the area of R is

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

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81. If $\frac{dy}{dx}=\sqrt{1-y^2}$, then $\frac{d^2y}{dx^2}=$

(A) $-2y$ (B) $-y$ (C) $\frac{-y}{\sqrt{1-y^2}}$ (D) y (E) $\frac{1}{2}$

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82. If $f(x)=g(x)+7$ for $3\le x\le 5$, then $\int_3^5 [f(x)+g(x)]\,dx=$

(A) $2\int_3^5 g(x)\,dx+7$

(B) $2\int_3^5 g(x)\,dx+14$

(C) $2\int_3^5 g(x)\,dx+28$

(D) $\int_3^5 g(x)\,dx+7$

(E) $\int_3^5 g(x)\,dx+14$

83. The Taylor series for $\ln x$, centered at $x = 1$, is $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(x-1)^n}{n}$. Let f be the function given by the sum of the first three nonzero terms of this series. The maximum value of $\left|\ln x - f(x)\right|$ for $0.3 \le x \le 1.7$ is

(A) 0.030 (B) 0.039 (C) 0.145 (D) 0.153 (E) 0.529

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84. What are all values of x for which the series $\sum_{n=1}^{\infty}\frac{(x+2)^n}{\sqrt{n}}$ converges?

(A) $-3 < x < -1$ (B) $-3 \le x < -1$ (C) $-3 \le x \le -1$ (D) $-1 \le x < 1$ (E) $-1 \le x \le 1$

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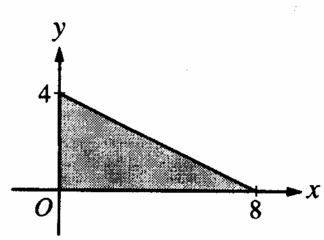
x	2	5	7	8
$f(x)$	10	30	40	20

85. The function f is continuous on the closed interval $[2,8]$ and has values that are given in the table above. Using the subintervals $[2,5]$, $[5,7]$, and $[7,8]$, what is the trapezoidal approximation of $\int_2^8 f(x)\,dx$?

(A) 110 (B) 130 (C) 160 (D) 190 (E) 210

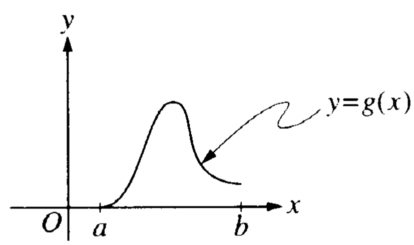


86. The base of a solid is a region in the first quadrant bounded by the x-axis, the y-axis, and the line $x + 2y = 8$, as shown in the figure above. If cross sections of the solid perpendicular to the x-axis are semicircles, what is the volume of the solid?

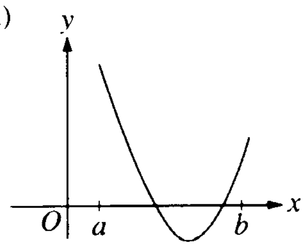
(A) 12.566 (B) 14.661 (C) 16.755 (D) 67.021 (E) 134.041

87. Which of the following is an equation of the line tangent to the graph of $f(x) = x^4 + 2x^2$ at the point where $f'(x) = 1$?

(A) $y = 8x - 5$
(B) $y = x + 7$
(C) $y = x + 0.763$
(D) $y = x - 0.122$
(E) $y = x - 2.146$



88. Let $g(x) = \int_a^x f(t)\,dt$, where $a \le x \le b$. The figure above shows the graph of g on $[a, b]$. Which of the following could be the graph of f on $[a, b]$?

(A)


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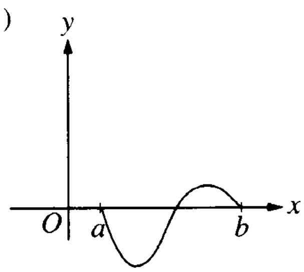

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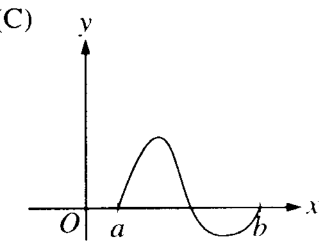

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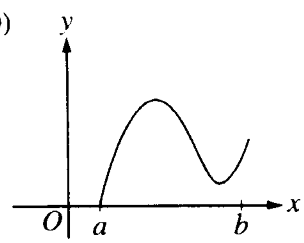
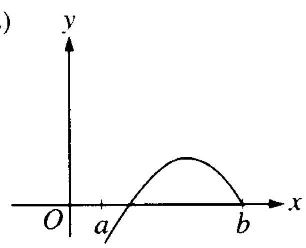


(E)


89. The graph of the function represented by the Maclaurin series

$1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots + \frac{(-1)^n x^n}{n!} + \ldots$ intersects the graph of $y = x^3$ at $x =$

(A) 0.773 (B) 0.865 (C) 0.929 (D) 1.000 (E) 1.857

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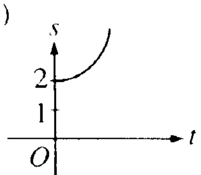
90. A particle starts from rest at the point $(2,0)$ and moves along the x-axis with a constant positive acceleration for time $t \geq 0$. Which of the following could be the graph of the distance $s(t)$ of the particle from the origin as a function of time t?

(A)


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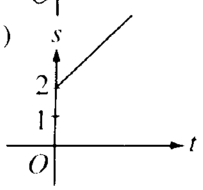
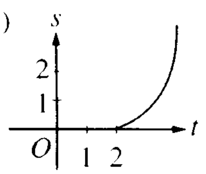
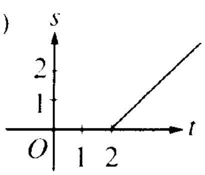
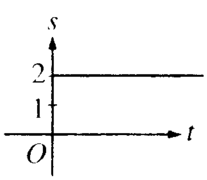


(C)


(D)


E)


t (sec)	0	2	4	6
$a(t)$ (ft/sec^2)	5	2	8	3

91. The data for the acceleration $a(t)$ of a car from 0 to 6 seconds are given in the table above. If the velocity at $t = 0$ is 11 feet per second, the approximate value of the velocity at $t = 6$, computed using a left-hand Riemann sum with three subintervals of equal length, is

(A) 26 ft/sec (B) 30 ft/sec (C) 37 ft/sec (D) 39 ft/sec (E) 41 ft/sec

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92. Let f be the function given by $f(x) = x^2 - 2x + 3$. The tangent line to the graph of f at $x = 2$ is used to approximate values of $f(x)$. Which of the following is the greatest value of x for which the error resulting from this tangent line approximation is less than 0.5?

(A) 2.4 (B) 2.5 (C) 2.6 (D) 2.7 (E) 2.8

76. D The first series is either the harmonic series or the alternating harmonic series depending on whether k is odd or even. It will converge if k is odd. The second series is geometric and will converge if $k < 4$.

77. E $f'(t) = \left(-e^{-t}, -\sin t\right)$; $f''(t) = \left(e^{-t}, -\cos t\right)$.

78. B $A = \pi r^2 \Rightarrow \dfrac{dA}{dt} = 2\pi r \dfrac{dr}{dt}$. However, $C = 2\pi r$ and $\dfrac{dr}{dt} = -0.1$. Thus $\dfrac{dA}{dt} = -0.1C$.

79. A None. For every positive value of a the denominator will be zero for some value of x.

80. B The area is given by $\int_{-\frac{2}{3}}^{\frac{2}{3}} \left(1 + \ln(\cos^4 x)\right) dx = 0.919$

81. B $\dfrac{dy}{dx} = \sqrt{1-y^2}$; $\dfrac{d^2y}{dx^2} = \dfrac{d}{dx}\left((1-y^2)^{\frac{1}{2}}\right) = \dfrac{1}{2}\left(1-y^2\right)^{-\frac{1}{2}} \cdot (-2y) \cdot \dfrac{dy}{dx} = -y$

82. B $\int_3^5 \left[f(x) + g(x)\right] dx = \int_3^5 \left[2g(x) + 7\right] dx = 2\int_3^5 g(x)\,dx + (7)(2) = 2\int_3^5 g(x)\,dx + 14$

83. C Use a calculator. The maximum for $\left|\ln x - \left(\dfrac{(x-1)}{1} - \dfrac{(x-1)^2}{2} + \dfrac{(x-1)^3}{3}\right)\right|$ on the interval $0.3 \le x \le 1.7$ occurs at $x = 0.3$.

84. B You may use the ratio test. However, the series will converge if the numerator is $(-1)^n$ and diverge if the numerator is 1^n. Any value of x for which $|x+2| > 1$ in the numerator will make the series diverge. Hence the interval is $-3 \le x < -1$.

85. C There are 3 trapezoids. $\dfrac{1}{2} \cdot 3\left(f(2) + f(5)\right) + \dfrac{1}{2} \cdot 2\left(f(5) + f(7)\right) + \dfrac{1}{2} \cdot 1\left(f(7) + f(8)\right)$

86. C Each cross section is a semicircle with a diameter of y. The volume would be given by

$$\int_0^8 \frac{1}{2}\pi\left(\frac{y}{2}\right)^2 dx = \frac{\pi}{8}\int_0^8 \left(\frac{8-x}{2}\right)^2 dx = 16.755$$

87. D Find the x for which $f'(x)=1$. $f'(x)=4x^3+4x=1$ only for $x=0.237$. Then $f(0.237)=0.115$. So the equation is $y-0.115=x-0.237$. This is equivalent to option (D).

88. C From the given information, f is the derivative of g. We want a graph for f that represents the slopes of the graph g. The slope of g is zero at a and b. Also the slope of g changes from positive to negative at one point between a and b. This is true only for figure (C).

89. A The series is the Maclaurin expansion of e^{-x}. Use the calculator to solve $e^{-x}=x^3$.

90. A Constant acceleration means linear velocity which in turn leads to quadratic position. Only the graph in (A) is quadratic with initial $s=2$.

91. E $v(t)=11+\int_0^t a(x)\,dx \approx 11+\left[2\cdot 5+2\cdot 2+2\cdot 8\right]=41$ ft/sec.

92. D $f'(x)=2x-2,\ f'(2)=2,$ and $f(2)=3$, so an equation for the tangent line is $y=2x-1$. The difference between the function and the tangent line is represented by $(x-2)^2$. Solve $(x-2)^2<0.5$. This inequality is satisfied for all x such that $2-\sqrt{0.5}<x<2+\sqrt{0.5}$. This is the same as $1.293<x<2.707$. Thus the largest value in the list that satisfies the inequality is 2.7.