90 Minutes—No Calculator

Note: In this examination, $\ln x$ denotes the natural logarithm of x (that is, logarithm to the base e).

1. The asymptotes of the graph of the parametric equations $x = \frac{1}{t}$, $y = \frac{t}{t+1}$ are

(A) $x = 0,\ y = 0$ (B) $x = 0$ only (C) $x = -1,\ y = 0$
(D) $x = -1$ only (E) $x = 0,\ y = 1$

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2. What are the coordinates of the inflection point on the graph of $y = (x+1)\arctan x$?

(A) $(-1,0)$ (B) $(0,0)$ (C) $(0,1)$ (D) $\left(1,\frac{\pi}{4}\right)$ (E) $\left(1,\frac{\pi}{2}\right)$

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3. The Mean Value Theorem guarantees the existence of a special point on the graph of $y = \sqrt{x}$ between $(0,0)$ and $(4,2)$. What are the coordinates of this point?

(A) $(2,1)$
(B) $(1,1)$
(C) $(2,\sqrt{2})$
(D) $\left(\frac{1}{2},\frac{1}{\sqrt{2}}\right)$
(E) None of the above

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4. $\int_0^8 \frac{dx}{\sqrt{1+x}} =$

(A) 1 (B) $\frac{3}{2}$ (C) 2 (D) 4 (E) 6

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5. If $3x^2 + 2xy + y^2 = 2$, then the value of $\frac{dy}{dx}$ at $x = 1$ is

(A) –2 (B) 0 (C) 2 (D) 4 (E) not defined

6. What is $\lim_{h\to 0} \dfrac{8\left(\frac{1}{2}+h\right)^8 - 8\left(\frac{1}{2}\right)^8}{h}$?

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) The limit does not exist.

(E) It cannot be determined from the information given.

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7. For what value of k will $x + \frac{k}{x}$ have a relative maximum at $x = -2$?

(A) –4 (B) –2 (C) 2 (D) 4 (E) None of these

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8. If $h(x) = f^2(x) - g^2(x)$, $f'(x) = -g(x)$, and $g'(x) = f(x)$, then $h'(x) =$

(A) 0 (B) 1 (C) $-4f(x)g(x)$

(D) $\left(-g(x)\right)^2 - \left(f(x)\right)^2$ (E) $-2\left(-g(x) + f(x)\right)$

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9. The area of the closed region bounded by the polar graph of $r = \sqrt{3+\cos\theta}$ is given by the integral

(A) $\int_0^{2\pi} \sqrt{3+\cos\theta}\, d\theta$ (B) $\int_0^{\pi} \sqrt{3+\cos\theta}\, d\theta$ (C) $2\int_0^{\pi/2} \left(3+\cos\theta\right) d\theta$

(D) $\int_0^{\pi} \left(3+\cos\theta\right) d\theta$ (E) $2\int_0^{\pi/2} \sqrt{3+\cos\theta}\, d\theta$

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10. $\int_0^1 \frac{x^2}{x^2+1}\, dx =$

(A) $\frac{4-\pi}{4}$ (B) $\ln 2$ (C) 0 (D) $\frac{1}{2}\ln 2$ (E) $\frac{4+\pi}{4}$

11. The point <u>on the curve</u> $x^2 + 2y = 0$ that is nearest the point $\left(0, -\frac{1}{2}\right)$ occurs where y is

(A) $\frac{1}{2}$
(B) 0
(C) $-\frac{1}{2}$
(D) -1
(E) none of the above

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12. If $F(x) = \int_0^x e^{-t^2}\,dt$, then $F'(x) =$

(A) $2xe^{-x^2}$ (B) $-2xe^{-x^2}$ (C) $\frac{e^{-x^2+1}}{-x^2+1} - e$
(D) $e^{-x^2} - 1$ (E) e^{-x^2}

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13. The region bounded by the x-axis and the part of the graph of $y = \cos x$ between $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$ is separated into two regions by the line $x = k$. If the area of the region for $-\frac{\pi}{2} \le x \le k$ is three times the area of the region for $k \le x \le \frac{\pi}{2}$, then $k =$

(A) $\arcsin\left(\frac{1}{4}\right)$ (B) $\arcsin\left(\frac{1}{3}\right)$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{4}$ (E) $\frac{\pi}{3}$

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14. If $y = x^2 + 2$ and $u = 2x - 1$, then $\frac{dy}{du} =$

(A) $\frac{2x^2 - 2x + 4}{(2x-1)^2}$ (B) $6x^2 - 2x + 4$ (C) x^2
(D) x (E) $\frac{1}{x}$

15. If $f'(x)$ and $g'(x)$ exist and $f'(x) > g'(x)$ for all real x, then the graph of $y = f(x)$ and the graph of $y = g(x)$

(A) intersect exactly once.
(B) intersect no more than once.
(C) do not intersect.
(D) could intersect more than once.
(E) have a common tangent at each point of intersection.

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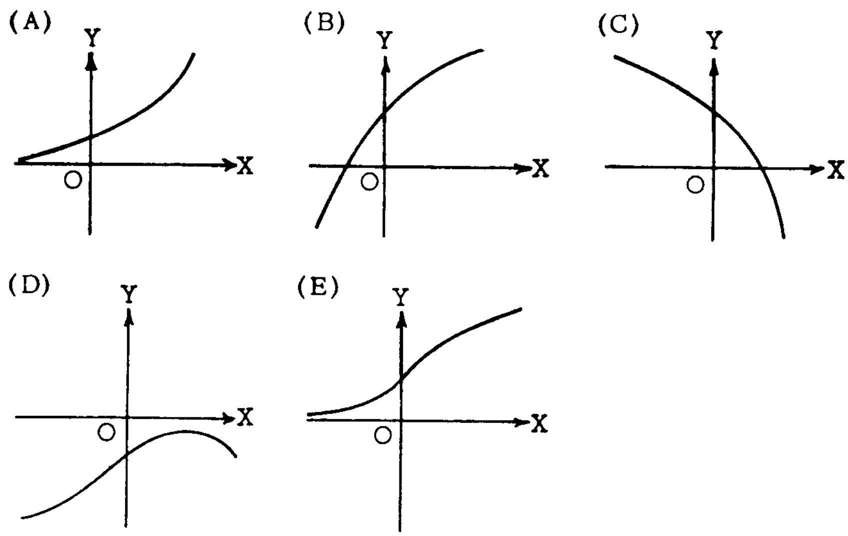
16. If y is a function x such that $y' > 0$ for all x and $y'' < 0$ for all x, which of the following could be part of the graph of $y = f(x)$?



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17. The graph of $y = 5x^4 - x^5$ has a point of inflection at

(A) $(0,0)$ only
(B) $(3,162)$ only
(C) $(4,256)$ only
(D) $(0,0)$ and $(3,162)$
(E) $(0,0)$ and $(4,256)$

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18. If $f(x) = 2 + |x - 3|$ for all x, then the value of the derivative $f'(x)$ at $x = 3$ is

(A) -1
(B) 0
(C) 1
(D) 2
(E) nonexistent

19. A point moves on the x-axis in such a way that its velocity at time t $(t > 0)$ is given by $v = \frac{\ln t}{t}$. At what value of t does v attain its maximum?

(A) 1 (B) $e^{\frac{1}{2}}$ (C) e (D) $e^{\frac{3}{2}}$

(E) There is no maximum value for v.

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20. An equation for a tangent to the graph of $y = \arcsin\frac{x}{2}$ at the origin is

(A) $x - 2y = 0$ (B) $x - y = 0$ (C) $x = 0$

(D) $y = 0$ (E) $\pi x - 2y = 0$

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21. At $x = 0$, which of the following is true of the function f defined by $f(x) = x^2 + e^{-2x}$?

(A) f is increasing.
(B) f is decreasing.
(C) f is discontinuous.
(D) f has a relative minimum.
(E) f has a relative maximum.

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22. If $f(x) = \int_0^x \frac{1}{\sqrt{t^3 + 2}}\,dt$, which of the following is FALSE?

(A) $f(0) = 0$

(B) f is continuous at x for all $x \geq 0$.

(C) $f(1) > 0$

(D) $f'(1) = \frac{1}{\sqrt{3}}$

(E) $f(-1) > 0$

23. If the graph of $y = f(x)$ contains the point $(0, 2)$, $\frac{dy}{dx} = \frac{-x}{ye^{x^2}}$ and $f(x) > 0$ for all x, then $f(x) =$

(A) $3+e^{-x^2}$ (B) $\sqrt{3}+e^{-x}$ (C) $1+e^{-x}$

(D) $\sqrt{3+e^{-x^2}}$ (E) $\sqrt{3+e^{x^2}}$

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24. If $\sin x = e^y$, $0 < x < \pi$, what is $\frac{dy}{dx}$ in terms of x ?

(A) $-\tan x$ (B) $-\cot x$ (C) $\cot x$ (D) $\tan x$ (E) $\csc x$

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25. A region in the plane is bounded by the graph of $y = \frac{1}{x}$, the x-axis, the line $x = m$, and the line $x = 2m$, $m > 0$. The area of this region

(A) is independent of m .

(B) increases as m increases.

(C) decreases as m increases.

(D) decreases as m increases when $m < \frac{1}{2}$; increases as m increases when $m > \frac{1}{2}$.

(E) increases as m increases when $m < \frac{1}{2}$; decreases as m increases when $m > \frac{1}{2}$.

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26. $\int_0^1 \sqrt{x^2 - 2x + 1}\, dx$ is

(A) -1

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) 1

(E) none of the above

27. If $\frac{dy}{dx} = \tan x$, then $y =$

(A) $\frac{1}{2}\tan^2 x + C$ (B) $\sec^2 x + C$ (C) $\ln|\sec x| + C$
(D) $\ln|\cos x| + C$ (E) $\sec x \tan x + C$

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28. What is $\lim_{x\to 0} \frac{e^{2x}-1}{\tan x}$?

(A) -1 (B) 0 (C) 1 (D) 2 (E) The limit does not exist.

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29. $\int_0^1 \left(4 - x^2\right)^{-\frac{3}{2}} dx =$

(A) $\frac{2-\sqrt{3}}{3}$ (B) $\frac{2\sqrt{3}-3}{4}$ (C) $\frac{\sqrt{3}}{12}$ (D) $\frac{\sqrt{3}}{3}$ (E) $\frac{\sqrt{3}}{2}$

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30. $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}$ is the Taylor series about zero for which of the following functions?

(A) $\sin x$ (B) $\cos x$ (C) e^x (D) e^{-x} (E) $\ln(1+x)$

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31. If $f'(x) = -f(x)$ and $f(1) = 1$, then $f(x) =$

(A) $\frac{1}{2}e^{-2x+2}$ (B) e^{-x-1} (C) e^{1-x} (D) e^{-x} (E) $-e^x$

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32. For what values of x does the series $1 + 2^x + 3^x + 4^x + \cdots + n^x + \cdots$ converge?

(A) No values of x (B) $x < -1$ (C) $x \geq -1$ (D) $x > -1$ (E) All values of x

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33. What is the average (mean) value of $3t^3 - t^2$ over the interval $-1 \leq t \leq 2$?

(A) $\frac{11}{4}$ (B) $\frac{7}{2}$ (C) 8 (D) $\frac{33}{4}$ (E) 16

34. Which of the following is an equation of a curve that intersects at right angles every curve of the family $y = \frac{1}{x} + k$ (where k takes all real values)?

(A) $y = -x$ (B) $y = -x^2$ (C) $y = -\frac{1}{3}x^3$ (D) $y = \frac{1}{3}x^3$ (E) $y = \ln x$

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35. At $t = 0$ a particle starts at rest and moves along a line in such a way that at time t its acceleration is $24t^2$ feet per second per second. Through how many feet does the particle move during the first 2 seconds?

(A) 32 (B) 48 (C) 64 (D) 96 (E) 192

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36. The approximate value of $y = \sqrt{4 + \sin x}$ at $x = 0.12$, obtained from the tangent to the graph at $x = 0$, is

(A) 2.00 (B) 2.03 (C) 2.06 (D) 2.12 (E) 2.24

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37. Of the following choices of δ, which is the <u>largest</u> that could be used successfully with an arbitrary ε in an epsilon-delta proof of $\lim_{x \to 2} (1 - 3x) = -5$?

(A) $\delta = 3\varepsilon$ (B) $\delta = \varepsilon$ (C) $\delta = \frac{\varepsilon}{2}$ (D) $\delta = \frac{\varepsilon}{4}$ (E) $\delta = \frac{\varepsilon}{5}$

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38. If $f(x) = \left(x^2 + 1\right)^{(2-3x)}$, then $f'(1) =$

(A) $-\frac{1}{2}\ln(8e)$ (B) $-\ln(8e)$ (C) $-\frac{3}{2}\ln(2)$ (D) $-\frac{1}{2}$ (E) $\frac{1}{8}$

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39. If $y = \tan u$, $u = v - \frac{1}{v}$, and $v = \ln x$, what is the value of $\frac{dy}{dx}$ at $x = e$?

(A) 0 (B) $\frac{1}{e}$ (C) 1 (D) $\frac{2}{e}$ (E) $\sec^2 e$

40. If n is a non-negative integer, then $\int_0^1 x^n \, dx = \int_0^1 (1-x)^n \, dx$ for

(A) no n (B) n even, only (C) n odd, only
(D) nonzero n, only (E) all n

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41. If $\begin{cases} f(x) = 8 - x^2 & \text{for } -2 \le x \le 2, \\ f(x) = x^2 & \text{elsewhere}, \end{cases}$ then $\int_{-1}^{3} f(x)\,dx$ is a number between

(A) 0 and 8 (B) 8 and 16 (C) 16 and 24 (D) 24 and 32 (E) 32 and 40

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42. If $\int x^2 \cos x \, dx = f(x) - \int 2x \sin x \, dx$, then $f(x) =$

(A) $2\sin x + 2x\cos x + C$

(B) $x^2 \sin x + C$

(C) $2x\cos x - x^2 \sin x + C$

(D) $4\cos x - 2x\sin x + C$

(E) $\left(2 - x^2\right)\cos x - 4\sin x + C$

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43. Which of the following integrals gives the length of the graph of $y = \tan x$ between $x = a$ and $x = b$, where $0 < a < b < \dfrac{\pi}{2}$?

(A) $\int_a^b \sqrt{x^2 + \tan^2 x}\, dx$

(B) $\int_a^b \sqrt{x + \tan x}\, dx$

(C) $\int_a^b \sqrt{1 + \sec^2 x}\, dx$

(D) $\int_a^b \sqrt{1 + \tan^2 x}\, dx$

(E) $\int_a^b \sqrt{1 + \sec^4 x}\, dx$

44. If $f''(x) - f'(x) - 2f(x) = 0$, $f'(0) = -2$, and $f(0) = 2$, then $f(1) =$

(A) $e^2 + e^{-1}$ (B) 1 C) 0 (D) e^2 (E) $2e^{-1}$

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45. The complete interval of convergence of the series $\sum_{k=1}^{\infty} \frac{(x+1)^k}{k^2}$ is

(A) $0 < x < 2$ (B) $0 \le x \le 2$ (C) $-2 < x \le 0$

(D) $-2 \le x < 0$ (E) $-2 \le x \le 0$

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1. C For horizontal asymptotes consider the limit as $x \to \pm\infty$: $t \to 0 \Rightarrow y = 0$ is an asymptote
For vertical asymptotes consider the limit as $y \to \pm\infty$: $t \to -1 \Rightarrow x = -1$ is an asymptote

2. E $y = (x+1)\tan^{-1} x$, $y' = \dfrac{x+1}{1+x^2} + \tan^{-1} x$

$$y'' = \frac{\left(1+x^2\right)(1) - (x+1)(2x)}{\left(1+x^2\right)^2} + \frac{1}{1+x^2} = \frac{2-2x}{\left(1+x^2\right)^2}$$

y'' changes sign at $x = 1$ only. The point of inflection is $\left(1, \pi/2\right)$

3. B $y = \sqrt{x}$, $y' = \dfrac{1}{2\sqrt{x}}$. By the Mean Value Theorem we have $\dfrac{1}{2\sqrt{c}} = \dfrac{2}{4} \Rightarrow c = 1$.

The point is $(1,1)$.

4. D $\displaystyle\int_0^8 \frac{dx}{\sqrt{1+x}} dx = 2\sqrt{1+x}\Big|_0^8 = 2(3-1) = 4$

5. E Using implicit differentiation, $6x + 2xy' + 2y + 2y \cdot y' = 0$. Therefore $y' = \dfrac{-2y-6x}{2x+2y}$.
When $x = 1$, $3 + 2y + y^2 = 2 \Rightarrow 0 = y^2 + 2y + 1 = (y+1)^2 \Rightarrow y = -1$
Therefore $2x + 2y = 0$ and so $\dfrac{dy}{dx}$ is not defined at $x = 1$.

6. B This is the derivative of $f(x) = 8x^8$ at $x = \dfrac{1}{2}$.

$$f'\left(\frac{1}{2}\right) = 64\left(\frac{1}{2}\right)^7 = \frac{1}{2}$$

7. D With $f(x) = x + \dfrac{k}{x}$, we need $0 = f'(-2) = 1 - \dfrac{k}{4}$ and so $k = 4$. Since $f''(-2) < 0$ for $k = 4$, f does have a relative maximum at $x = -2$.

8. C $h'(x) = 2f(x) \cdot f'(x) - 2g(x) \cdot g'(x) = 2f(x) \cdot \left(-g(x)\right) - 2g(x) \cdot f(x) = -4f(x) \cdot g(x)$

9. D $A=\frac{1}{2}\int_0^{2\pi}\left(\sqrt{3+\cos\theta}\right)^2 d\theta = 2\cdot\frac{1}{2}\int_0^{\pi}\left(\sqrt{3+\cos\theta}\right)^2 d\theta = \int_0^{\pi}(3+\cos\theta)\,d\theta$

10. A $\int_0^1 \frac{x^2}{x^2+1}dx = \int_0^1 \frac{x^2+1-1}{x^2+1}dx = \int_0^1\left(\frac{x^2+1}{x^2+1}-\frac{1}{x^2+1}\right)dx = \left(x-\tan^{-1}x\right)\Big|_0^1 = 1-\frac{\pi}{4}=\frac{4-\pi}{4}$

11. B Let L be the distance from $\left(x,-\frac{x^2}{2}\right)$ and $\left(0,-\frac{1}{2}\right)$.

$$L^2=(x-0)^2+\left(\frac{x^2}{2}-\frac{1}{2}\right)^2$$

$$2L\cdot\frac{dL}{dx}=2x+2\left(\frac{x^2}{2}-\frac{1}{2}\right)(x)$$

$$\frac{dL}{dx}=\frac{2x+2\left(\frac{x^2}{2}-\frac{1}{2}\right)(x)}{2L}=\frac{2x+x^3-x}{2L}=\frac{x^3+x}{2L}=\frac{x\left(x^2+1\right)}{2L}$$

$\frac{dL}{dx}<0$ for all $x<0$ and $\frac{dL}{dx}>0$ for all $x>0$, so the minimum distance occurs at $x=0$.

The nearest point is the origin.

12. E By the Fundamental Theorem of Calculus, if $F(x)=\int_0^x e^{-t^2}dt$ then $F'(x)=e^{-x^2}$.

13. C $\int_{-\pi/2}^{k}\cos x\,dx = 3\int_k^{\pi/2}\cos x\,dx$; $\sin k-\sin\left(-\frac{\pi}{2}\right)=3\left(\sin\frac{\pi}{2}-\sin k\right)$

$\sin k+1=3-3\sin k$; $4\sin k=2 \Rightarrow k=\pi/6$

14. D $y=x^2+2$ and $u=2x-1$, $\frac{dy}{du}=\frac{dy}{dx}\cdot\frac{dx}{du}=(2x)\left(\frac{1}{2}\right)=x$

15. B The graphs do not need to intersect (eg. $f(x)=-e^{-x}$ and $g(x)=e^{-x}$). The graphs could intersect (e.g. $f(x)=2x$ and $g(x)=x$). However, if they do intersect, they will intersect no more than once because $f(x)$ grows faster than $g(x)$.

16. B $y' > 0 \Rightarrow y$ is increasing; $y'' < 0 \Rightarrow$ the graph is concave down. Only B meets these conditions.

17. B $y' = 20x^3 - 5x^4$, $y'' = 60x^2 - 20x^3 = 20x^2(3-x)$. The only sign change in y'' is at $x = 3$. The only point of inflection is $(3,162)$.

18. E There is no derivative at the vertex which is located at $x = 3$.

19. C $\frac{dv}{dt} = \frac{1-\ln t}{t^2} > 0$ for $0 < t < e$ and $\frac{dv}{dt} < 0$ for $t > e$, thus v has its maximum at $t = e$.

20. A $y(0) = 0$ and $y'(0) = \left.\frac{1/2}{\sqrt{1-\frac{x^2}{4}}}\right|_{x=0} = \left.\frac{1}{\sqrt{4-x^2}}\right|_{x=0} = \frac{1}{2}$. The tangent line is

$y = \frac{1}{2}x \Rightarrow x - 2y = 0$.

21. B $f'(x) = 2x - 2e^{-2x}$, $f'(0) = -2$, so f is decreasing

22. E $f(x) = \int_0^x \frac{1}{\sqrt{t^3+2}}\,dt$, $f(-1) = \int_0^{-1} \frac{1}{\sqrt{t^3+2}}\,dt = -\int_{-1}^{0} \frac{1}{\sqrt{t^3+2}}\,dt < 0$

$f(-1) < 0$ so E is false.

23. D $\frac{dy}{dx} = \frac{-xe^{-x^2}}{y} \Rightarrow 2y\,dy = -2xe^{-x^2}\,dx \Rightarrow y^2 = e^{-x^2} + C$

$4 = 1 + C \Rightarrow C = 3$; $y^2 = e^{-x^2} + 3 \Rightarrow y = \sqrt{e^{-x^2} + 3}$

24. C $y = \ln \sin x$, $y' = \frac{\cos x}{\sin x} = \cot x$

25. A $\int_m^{2m} \frac{1}{x}\,dx = \ln x \Big|_m^{2m} = \ln(2m) - \ln(m) = \ln 2$ so the area is independent of m.

26. C $\int_0^1 \sqrt{x^2 - 2x + 1}\, dx = \int_0^1 |x - 1|\, dx = \int_0^1 -(x-1)\, dx = -\frac{1}{2}(x-1)^2 \Big|_0^1 = \frac{1}{2}$

Alternatively, the graph of the region is a right triangle with vertices at (0,0), (0,1), and (1,0). The area is $\frac{1}{2}$.

27. C $\int \tan x\, dx = \int \frac{\sin x}{\cos x} dx = -\ln|\cos x| + C = \ln|\sec x| + C$

28. D Use L'Hôpital's Rule: $\lim_{x \to 0} \frac{e^{2x} - 1}{\tan x} = \lim_{x \to 0} \frac{2e^{2x}}{\sec^2 x} = 2$

29. C Make the subsitution $x = 2\sin\theta \Rightarrow dx = 2\cos\theta\, d\theta$.

$\int_0^1 \left(4 - x^2\right)^{-3/2} dx = \int_0^{\pi/6} \frac{2\cos\theta}{8\cos^3\theta} d\theta = \frac{1}{4}\int_0^{\pi/6} \sec^2\theta\, d\theta = \frac{1}{4}\tan\theta \Big|_0^{\pi/6} = \frac{1}{4} \cdot \frac{\sqrt{3}}{3} = \frac{\sqrt{3}}{12}$

30. D Substitute $-x$ for x in $\sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x$ to get $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!} = e^{-x}$

31. C $\frac{dy}{dx} = -y \Rightarrow y = ce^{-x}$ and $1 = ce^{-1} \Rightarrow c = e$; $y = e \cdot e^{-x} = e^{1-x}$

32. B $1 + 2^x + 3^x + 4^x + \cdots + n^x + \cdots = \sum_{n=1}^{\infty} \frac{1}{n^p}$ where $p = -x$. This is a *p*-series and is convergent if $p > 1 \Rightarrow -x > 1 \Rightarrow x < -1$.

33. A $\frac{1}{3}\int_{-1}^{2} 3t^3 - t^2\, dt = \frac{1}{3}\left(\frac{3}{4}t^4 - \frac{1}{3}t^3\right)\Big|_{-1}^{2} = \frac{1}{3}\left(\left(12 - \frac{8}{3}\right) - \left(\frac{3}{4} + \frac{1}{3}\right)\right) = \frac{11}{4}$

34. D $y' = -\frac{1}{x^2}$, so the desired curve satisfies $y' = x^2 \Rightarrow y = \frac{1}{3}x^3 + C$

35. A $a(t) = 24t^2$, $v(t) = 8t^3 + C$ and $v(0) = 0 \Rightarrow C = 0$. The particle is always moving to the right, so distance = $\int_0^2 8t^3 dt = 2t^4 \Big|_0^2 = 32$.

36. B $y = \sqrt{4 + \sin x}$, $y(0) = 2$, $y'(0) = \dfrac{\cos 0}{2\sqrt{4 + \sin 0}} = \dfrac{1}{4}$. The linear approximation to y is

$L(x) = 2 + \frac{1}{4}x$. $L(1.2) = 2 + \frac{1}{4}(1.2) = 2.03$

37. D This item uses the formal definition of a limit and is no longer part of the AP Course Description. Need to have $\left|(1-3x)-(-5)\right| < \varepsilon$ whenever $0 < \left|x-2\right| < \delta$.
$\left|(1-3x)-(-5)\right| = \left|6-3x\right| = 3\left|x-2\right| < \varepsilon$ if $\left|x-2\right| < \varepsilon/3$.
Thus we can use any $\delta < \varepsilon/3$. Of the five choices, the largest satisfying this condition is $\delta = \varepsilon/4$.

38. A Note $f(1) = \frac{1}{2}$. Take the natural logarithm of each side of the equation and then differentiate.

$$\ln f(x) = (2-3x)\ln\left(x^2+1\right); \quad \frac{f'(x)}{f(x)} = (2-3x)\cdot\frac{2x}{x^2+1} - 3\ln\left(x^2+1\right)$$

$$f'(1) = f(1)\left((-1)\cdot\frac{2}{2} - 3\ln(2)\right) \Rightarrow f'(1) = \frac{1}{2}\left(-1-3\ln 2\right) = -\frac{1}{2}\left(\ln e + \ln 2^3\right) = -\frac{1}{2}\ln 8e$$

39. D $x = e \Rightarrow v = 1,\ u = 0,\ y = 0;\ \dfrac{dy}{dx} = \dfrac{dy}{du}\cdot\dfrac{du}{dv}\cdot\dfrac{dv}{dx} = \left(\sec^2 u\right)\left(1+\dfrac{1}{v^2}\right)\left(\dfrac{1}{x}\right) = (1)(2)\left(e^{-1}\right) = \dfrac{2}{e}$

40. E One solution technique is to evaluate each integral and note that the value is $\dfrac{1}{n+1}$ for each.

Another technique is to use the substitution $u = 1-x$; $\int_0^1 (1-x)^n\,dx = \int_1^0 u^n\,(-du) = \int_0^1 u^n\,du$.

Integrals do not depend on the variable that is used and so $\int_0^1 u^n\,du$ is the same as $\int_0^1 x^n\,dx$.

41. D $\int_{-1}^{3} f(x)\,dx = \int_{-1}^{2}\left(8-x^2\right)dx + \int_2^3 x^2\,dx = \left(8x - \frac{1}{3}x^3\right)\Big|_{-1}^{2} + \frac{1}{3}x^3\Big|_2^3 = 27\tfrac{1}{3}$

42. B Use the technique of antiderivatives by parts to evaluate $\int x^2 \cos x\,dx$

$u = x^2 \qquad dv = \cos x\,dx$

$du = 2x\,dx \qquad v = \sin x$

$f(x) - \int 2x\sin x\,dx = \int x^2 \cos x\,dx = x^2 \sin x - \int 2x\sin x\,dx + C$

$f(x) = x^2 \sin x + C$

43. E $L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx = \int_a^b \sqrt{1 + \left(\sec^2 x\right)^2}\,dx = \int_a^b \sqrt{1 + \sec^4 x}\,dx$

44. E $y'' - y' - 2y = 0$, $y'(0) = -2$, $y(0) = 2$; the characteristic equation is $r^2 - r - 2 = 0$.

The solutions are $r = -1$, $r = 2$ so the general solution to the differential equation is

$y = c_1 e^{-x} + c_2 e^{2x}$ with $y' = -c_1 e^{-x} + 2c_2 e^{2x}$. Using the initial conditions we have the system:

$2 = c_1 + c_2$ and $-2 = -c_1 + 2c_2 \Rightarrow c_2 = 0$, $c_1 = 2$. The solution is $f(x) = 2e^{-x} \Rightarrow f(1) = 2e^{-1}$.

45. E The ratio test shows that the series is convergent for any value of x that makes $|x+1| < 1$. The solutions to $|x+1| = 1$ are the endpoints of the interval of convergence. Test $x = -2$ and $x = 0$ in the series. The resulting series are $\sum_{k=1}^{\infty} \frac{(-1)^k}{k^2}$ and $\sum_{k=1}^{\infty} \frac{1}{k^2}$ which are both convergent. The interval is $-2 \le x \le 0$.