1973 AP Calculus BC: Section I

90 Minutes—No Calculator

Note: In this examination, ln x denotes the natural logarithm of x (that is, logarithm to the base e).

1. If $f(x)=e^{1/x}$, then $f'(x)=$

(A) $-\frac{e^{1/x}}{x^2}$ (B) $-e^{1/x}$ (C) $\frac{e^{1/x}}{x}$ (D) $\frac{e^{1/x}}{x^2}$ (E) $\frac{1}{x}e^{(1/x)-1}$

---

2. $\int_0^3 (x+1)^{1/2}\,dx=$

(A) $\frac{21}{2}$ (B) 7 (C) $\frac{16}{3}$ (D) $\frac{14}{3}$ (E) $-\frac{1}{4}$

---

3. If $f(x)=x+\frac{1}{x}$, then the set of values for which f increases is

(A) $(-\infty,-1]\cup[1,\infty)$ (B) $[-1,1]$ (C) $(-\infty,\infty)$
(D) $(0,\infty)$ (E) $(-\infty,0)\cup(0,\infty)$

---

4. For what non-negative value of b is the line given by $y=-\frac{1}{3}x+b$ normal to the curve $y=x^3$?

(A) 0 (B) 1 (C) $\frac{4}{3}$ (D) $\frac{10}{3}$ (E) $\frac{10\sqrt{3}}{3}$

---

5. $\int_{-1}^{2}\frac{|x|}{x}dx$ is

(A) –3 (B) 1 (C) 2 (D) 3 (E) nonexistent

---

6. If $f(x)=\frac{x-1}{x+1}$ for all $x\neq-1$, then $f'(1)=$

(A) –1 (B) $-\frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$ (E) 1

7. If $y = \ln\left(x^2 + y^2\right)$, then the value of $\dfrac{dy}{dx}$ at the point $(1,0)$ is

(A) 0 (B) $\dfrac{1}{2}$ (C) 1 (D) 2 (E) undefined

---

8. If $y = \sin x$ and $y^{(n)}$ means "the *n*th derivative of *y* with respect to *x*," then the smallest positive integer *n* for which $y^{(n)} = y$ is

(A) 2 (B) 4 (C) 5 (D) 6 (E) 8

---

9. If $y = \cos^2 3x$, then $\dfrac{dy}{dx} =$

(A) $-6\sin 3x\cos 3x$ (B) $-2\cos 3x$ (C) $2\cos 3x$

(D) $6\cos 3x$ (E) $2\sin 3x\cos 3x$

---

10. The length of the curve $y = \ln\sec x$ from $x = 0$ to $x = b$, where $0 < b < \dfrac{\pi}{2}$, may be expressed by which of the following integrals?

(A) $\int_0^b \sec x\,dx$

(B) $\int_0^b \sec^2 x\,dx$

(C) $\int_0^b (\sec x\tan x)\,dx$

(D) $\int_0^b \sqrt{1+(\ln\sec x)^2}\,dx$

(E) $\int_0^b \sqrt{1+\left(\sec^2 x\tan^2 x\right)}\,dx$

---

11. Let $y = x\sqrt{1+x^2}$. When $x = 0$ and $dx = 2$, the value of dy is

(A) –2 (B) –1 (C) 0 (D) 1 (E) 2

12. If n is a known positive integer, for what value of k is $\int_1^k x^{n-1}\,dx = \frac{1}{n}$?

(A) 0 (B) $\left(\frac{2}{n}\right)^{1/n}$ (C) $\left(\frac{2n-1}{n}\right)^{1/n}$

(D) $2^{1/n}$ (E) 2^n

---

13. The acceleration α of a body moving in a straight line is given in terms of time t by $\alpha = 8 - 6t$. If the velocity of the body is 25 at $t = 1$ and if $s(t)$ is the distance of the body from the origin at time t, what is $s(4) - s(2)$?

(A) 20 (B) 24 (C) 28 (D) 32 (E) 42

---

14. If $x = t^2 - 1$ and $y = 2e^t$, then $\frac{dy}{dx} =$

(A) $\frac{e^t}{t}$ (B) $\frac{2e^t}{t}$ (C) $\frac{e^{|t|}}{t^2}$ (D) $\frac{4e^t}{2t-1}$ (E) e^t

---

15. The area of the region bounded by the lines $x = 0$, $x = 2$, and $y = 0$ and the curve $y = e^{x/2}$ is

(A) $\frac{e-1}{2}$ (B) $e-1$ (C) $2(e-1)$ (D) $2e-1$ (E) $2e$

---

16. A series expansion of $\frac{\sin t}{t}$ is

(A) $1 - \frac{t^2}{3!} + \frac{t^4}{5!} - \frac{t^6}{7!} + \cdots$

(B) $\frac{1}{t} - \frac{t}{2!} + \frac{t^3}{4!} - \frac{t^5}{6!} + \cdots$

(C) $1 + \frac{t^2}{3!} + \frac{t^4}{5!} + \frac{t^6}{7!} + \cdots$

(D) $\frac{1}{t} + \frac{t}{2!} + \frac{t^3}{4!} + \frac{t^5}{6!} + \cdots$

(E) $t - \frac{t^3}{3!} + \frac{t^5}{5!} - \frac{t^7}{7!} + \cdots$

17. The number of bacteria in a culture is growing at a rate of $3{,}000e^{2t/5}$ per unit of time t. At $t = 0$, the number of bacteria present was 7,500. Find the number present at $t = 5$.

(A) $1{,}200e^2$ (B) $3{,}000e^2$ (C) $7{,}500e^2$ (D) $7{,}500e^5$ (E) $\frac{15{,}000}{7}e^7$

---

18. Let g be a continuous function on the closed interval $[0,1]$. Let $g(0) = 1$ and $g(1) = 0$. Which of the following is NOT necessarily true?

(A) There exists a number h in $[0,1]$ such that $g(h) \ge g(x)$ for all x in $[0,1]$.

(B) For all a and b in $[0,1]$, if $a = b$, then $g(a) = g(b)$.

(C) There exists a number h in $[0,1]$ such that $g(h) = \frac{1}{2}$.

(D) There exists a number h in $[0,1]$ such that $g(h) = \frac{3}{2}$.

(E) For all h in the open interval $(0,1)$, $\lim_{x \to h} g(x) = g(h)$.

---

19. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{1}{n^2}$ II. $\sum_{n=1}^{\infty} \frac{1}{n}$ III. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

(A) I only (B) III only (C) I and II only (D) I and III only (E) I, II, and III

---

20. $\int x\sqrt{4 - x^2}\, dx =$

(A) $\frac{\left(4 - x^2\right)^{3/2}}{3} + C$ (B) $-\left(4 - x^2\right)^{3/2} + C$ (C) $\frac{x^2\left(4 - x^2\right)^{3/2}}{3} + C$

(D) $-\frac{x^2\left(4 - x^2\right)^{3/2}}{3} + C$ (E) $-\frac{\left(4 - x^2\right)^{3/2}}{3} + C$

---

21. $\int_0^1 (x+1)e^{x^2+2x}\, dx =$

(A) $\frac{e^3}{2}$ (B) $\frac{e^3 - 1}{2}$ (C) $\frac{e^4 - e}{2}$ (D) $e^3 - 1$ (E) $e^4 - e$

22. A particle moves on the curve $y = \ln x$ so that the x-component has velocity $x'(t) = t + 1$ for $t \geq 0$. At time $t = 0$, the particle is at the point $(1, 0)$. At time $t = 1$, the particle is at the point

(A) $(2, \ln 2)$
(B) $(e^2, 2)$
(C) $\left(\frac{5}{2}, \ln\frac{5}{2}\right)$
(D) $(3, \ln 3)$
(E) $\left(\frac{3}{2}, \ln\frac{3}{2}\right)$

---

23. $\lim_{h \to 0} \frac{1}{h} \ln\left(\frac{2+h}{2}\right)$ is

(A) e^2
(B) 1
(C) $\frac{1}{2}$
(D) 0
(E) nonexistent

---

24. Let $f(x) = 3x + 1$ for all real x and let $\varepsilon > 0$. For which of the following choices of δ is $|f(x) - 7| < \varepsilon$ whenever $|x - 2| < \delta$?

(A) $\frac{\varepsilon}{4}$
(B) $\frac{\varepsilon}{2}$
(C) $\frac{\varepsilon}{\varepsilon + 1}$
(D) $\frac{\varepsilon + 1}{\varepsilon}$
(E) 3ε

---

25. $\int_0^{\pi/4} \tan^2 x \, dx =$

(A) $\frac{\pi}{4} - 1$
(B) $1 - \frac{\pi}{4}$
(C) $\frac{1}{3}$
(D) $\sqrt{2} - 1$
(E) $\frac{\pi}{4} + 1$

---

26. Which of the following is true about the graph of $y = \ln\left|x^2 - 1\right|$ in the interval $(-1, 1)$?

(A) It is increasing.
(B) It attains a relative minimum at $(0, 0)$.
(C) It has a range of all real numbers.
(D) It is concave down.
(E) It has an asymptote of $x = 0$.

---

27. If $f(x) = \frac{1}{3}x^3 - 4x^2 + 12x - 5$ and the domain is the set of all x such that $0 \leq x \leq 9$, then the absolute maximum value of the function f occurs when x is

(A) 0
(B) 2
(C) 4
(D) 6
(E) 9

28. If the substitution $\sqrt{x} = \sin y$ is made in the integrand of $\int_0^{1/2} \frac{\sqrt{x}}{\sqrt{1-x}}\,dx$, the resulting integral is

(A) $\int_0^{1/2} \sin^2 y\,dy$ (B) $2\int_0^{1/2} \frac{\sin^2 y}{\cos y}\,dy$ (C) $2\int_0^{\pi/4} \sin^2 y\,dy$

(D) $\int_0^{\pi/4} \sin^2 y\,dy$ (E) $2\int_0^{\pi/6} \sin^2 y\,dy$

---

29. If $y'' = 2y'$ and if $y = y' = e$ when $x = 0$, then when $x = 1$, $y =$

(A) $\frac{e}{2}\left(e^2+1\right)$ (B) e (C) $\frac{e^3}{2}$ (D) $\frac{e}{2}$ (E) $\frac{\left(e^3-e\right)}{2}$

---

30. $\int_1^2 \frac{x-4}{x^2}\,dx$

(A) $-\frac{1}{2}$ (B) $\ln 2 - 2$ (C) $\ln 2$ (D) 2 (E) $\ln 2 + 2$

---

31. If $f(x) = \ln(\ln x)$, then $f'(x) =$

(A) $\frac{1}{x}$ (B) $\frac{1}{\ln x}$ (C) $\frac{\ln x}{x}$ (D) x (E) $\frac{1}{x \ln x}$

---

32. If $y = x^{\ln x}$, then y' is

(A) $\frac{x^{\ln x}\ln x}{x^2}$

(B) $x^{1/x}\ln x$

(C) $\frac{2x^{\ln x}\ln x}{x}$

(D) $\frac{x^{\ln x}\ln x}{x}$

(E) None of the above

33. Suppose that f is an odd function; i.e., $f(-x) = -f(x)$ for all x. Suppose that $f'(x_0)$ exists. Which of the following must necessarily be equal to $f'(-x_0)$?

(A) $f'(x_0)$

(B) $-f'(x_0)$

(C) $\dfrac{1}{f'(x_0)}$

(D) $-\dfrac{1}{f'(x_0)}$

(E) None of the above

---

34. The average (mean) value of $\sqrt{x}$ over the interval $0 \le x \le 2$ is

(A) $\frac{1}{3}\sqrt{2}$ (B) $\frac{1}{2}\sqrt{2}$ (C) $\frac{2}{3}\sqrt{2}$ (D) 1 (E) $\frac{4}{3}\sqrt{2}$

---

35. The region in the first quadrant bounded by the graph of $y = \sec x$, $x = \dfrac{\pi}{4}$, and the axes is rotated about the x-axis. What is the volume of the solid generated?

(A) $\dfrac{\pi^2}{4}$ (B) $\pi - 1$ (C) π (D) 2π (E) $\dfrac{8\pi}{3}$

---

36. $\displaystyle\int_0^1 \frac{x+1}{x^2+2x-3}\,dx$ is

(A) $-\ln\sqrt{3}$ (B) $-\dfrac{\ln\sqrt{3}}{2}$ (C) $\dfrac{1-\ln\sqrt{3}}{2}$ (D) $\ln\sqrt{3}$ (E) divergent

---

37. $\displaystyle\lim_{x\to 0} \frac{1-\cos^2(2x)}{x^2} =$

(A) –2 (B) 0 (C) 1 (D) 2 (E) 4

---

38. If $\displaystyle\int_1^2 f(x-c)\,dx = 5$ where c is a constant, then $\displaystyle\int_{1-c}^{2-c} f(x)\,dx =$

(A) $5+c$ (B) 5 (C) $5-c$ (D) $c-5$ (E) –5

39. Let f and g be differentiable functions such that

$f(1) = 2$, $f'(1) = 3$, $f'(2) = -4$,

$g(1) = 2$, $g'(1) = -3$, $g'(2) = 5$.

If $h(x) = f\left(g(x)\right)$, then $h'(1) =$

(A) –9 (B) –4 (C) 0 (D) 12 (E) 15

---

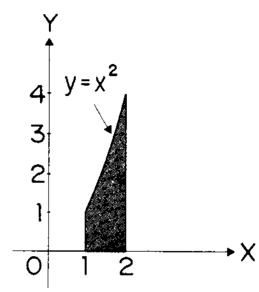
40. The area of the region enclosed by the polar curve $r = 1 - \cos\theta$ is

(A) $\frac{3}{4}\pi$ (B) π (C) $\frac{3}{2}\pi$ (D) 2π (E) 3π

---

41. Given $f(x) = \begin{cases} x+1 & \text{for } x < 0, \\ \cos \pi x & \text{for } x \geq 0, \end{cases}$ $\int_{-1}^{1} f(x)\,dx =$

(A) $\frac{1}{2} + \frac{1}{\pi}$ (B) $-\frac{1}{2}$ (C) $\frac{1}{2} - \frac{1}{\pi}$ (D) $\frac{1}{2}$ (E) $-\frac{1}{2} + \pi$



42. Calculate the approximate area of the shaded region in the figure by the trapezoidal rule, using divisions at $x = \frac{4}{3}$ and $x = \frac{5}{3}$.

(A) $\frac{50}{27}$ (B) $\frac{251}{108}$ (C) $\frac{7}{3}$ (D) $\frac{127}{54}$ (E) $\frac{77}{27}$

43. $\int \arcsin x \, dx =$

(A) $\sin x - \int \frac{x\,dx}{\sqrt{1-x^2}}$

(B) $\frac{(\arcsin x)^2}{2} + C$

(C) $\arcsin x + \int \frac{dx}{\sqrt{1-x^2}}$

(D) $x \arccos x - \int \frac{x\,dx}{\sqrt{1-x^2}}$

(E) $x \arcsin x - \int \frac{x\,dx}{\sqrt{1-x^2}}$

---

44. If f is the solution of $x\,f'(x) - f(x) = x$ such that $f(-1) = 1$, then $f\left(e^{-1}\right) =$

(A) $-2e^{-1}$ (B) 0 C) e^{-1} (D) $-e^{-1}$ (E) $2e^{-2}$

---

45. Suppose $g'(x) < 0$ for all $x \ge 0$ and $F(x) = \int_0^x t\,g'(t)\,dt$ for all $x \ge 0$. Which of the following statements is FALSE?

(A) F takes on negative values.

(B) F is continuous for all $x > 0$.

(C) $F(x) = x\,g(x) - \int_0^x g(t)\,dt$

(D) $F'(x)$ exists for all $x > 0$.

(E) F is an increasing function.

1973 Calculus BC Solutions

1. A $f'(x) = e^{\frac{1}{x}} \cdot \dfrac{d\left(\frac{1}{x}\right)}{dx} = e^{\frac{1}{x}}\left(-\dfrac{1}{x^2}\right) = -\dfrac{e^{\frac{1}{x}}}{x^2}$

2. D $\int_0^3 (x+1)^{\frac{1}{2}}\,dx = \dfrac{2}{3}(x+1)^{\frac{3}{2}}\Big|_0^3 = \dfrac{2}{3}\left(4^{\frac{3}{2}} - 1^{\frac{3}{2}}\right) = \dfrac{2}{3}(8-1) = \dfrac{14}{3}$

3. A $f'(x) = 1 - \dfrac{1}{x^2} = \dfrac{(x+1)(x-1)}{x^2}$. $f'(x) > 0$ for $x < -1$ and for $x > 1$.

   f is increasing for $x \le -1$ and for $x \ge 1$.

4. C The slopes will be negative reciprocals at the point of intersection.

   $3x^2 = 3 \Rightarrow x = \pm 1$ and $x \ge 0$, thus $x = 1$ and the y values must be the same at $x = 1$.

   $-\dfrac{1}{3} + b = 1 \Rightarrow b = \dfrac{4}{3}$

5. B $\int_{-1}^{2} \dfrac{|x|}{x}\,dx = \int_{-1}^{0} -1\,dx + \int_0^2 dx = -1 + 2 = 1$

6. D $f'(x) = \dfrac{(1)(x+1) - (x-1)(1)}{(x+1)^2}$, $f'(1) = \dfrac{2}{4} = \dfrac{1}{2}$

7. D $\dfrac{dy}{dx} = \dfrac{2x + 2y \cdot \frac{dy}{dx}}{x^2 + y^2}$ at $(1,0) \Rightarrow y' = \dfrac{2}{1} = 2$

8. B $y = \sin x$, $y' = \cos x$, $y'' = -\sin x$, $y''' = -\cos x$, $y^{(4)} = \sin x$

9. A $y' = 2\cos 3x \cdot \dfrac{d}{dx}(\cos 3x) = 2\cos 3x \cdot (-\sin 3x) \cdot \dfrac{d}{dx}(3x) = 2\cos 3x \cdot (-\sin 3x) \cdot 3$

   $y' = -6 \sin 3x \cos 3x$

10. A $L=\int_0^b \sqrt{1+(y')^2}\,dx=\int_0^b \sqrt{1+\left(\frac{\sec x\tan x}{\sec x}\right)^2}\,dx$

$=\int_0^b \sqrt{1+(\tan x)^2}\,dx=\int_0^b \sqrt{\sec^2 x}\,dx=\int_0^b \sec x\,dx$

11. E $dy=\left(x\cdot\frac{1}{2}\left(1+x^2\right)^{-\frac{1}{2}}(2x)+\left(1+x^2\right)^{\frac{1}{2}}\right)dx$; $dy=(0+1)(2)=2$

12. D $\frac{1}{n}=\int_1^k x^{n-1}\,dx=\left.\frac{x^n}{n}\right|_1^k\Rightarrow\frac{1}{n}=\frac{k^n}{n}-\frac{1}{n}$; $\frac{k^n}{n}=\frac{2}{n}\Rightarrow k=2^{\frac{1}{n}}$

13. D $v(t)=8t-3t^2+C$ and $v(1)=25\Rightarrow C=20$ so $v(t)=8t-3t^2+20$.

$s(4)-s(2)=\int_2^4 v(t)\,dt=\left.\left(4t^2-t^3+20t\right)\right|_2^4=32$

14. A $\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{2e^t}{2t}=\frac{e^t}{t}$

15. C Area $=\int_0^2 e^{\frac{1}{2}x}\,dx=\left.2e^{\frac{1}{2}x}\right|_0^2=2(e-1)$

16. A $\sin t=t-\frac{t^3}{3!}+\frac{t^5}{5!}-\frac{t^7}{7!}+\cdots\Rightarrow\frac{\sin t}{t}=1-\frac{t^2}{3!}+\frac{t^4}{5!}-\frac{t^6}{7!}+\cdots$

17. C $\frac{dN}{dt}=3000e^{\frac{2}{5}t}$, $N=7500e^{\frac{2}{5}t}+C$ and $N(0)=7500\Rightarrow C=0$

$N=7500e^{\frac{2}{5}t}$, $N(5)=7500e^2$

18. D D could be false, consider $g(x)=1-x$ on $[0,1]$. A is true by the Extreme Value Theorem, B is true because g is a function, C is true by the Intermediate Value Theorem, and E is true because g is continuous.

19. D I is a convergent p-series, $p = 2 > 1$
II is the Harmonic series and is known to be divergent,
III is convergent by the Alternating Series Test.

20. E $\int x\sqrt{4-x^2}\,dx = -\frac{1}{2}\int \left(4-x^2\right)^{\frac{1}{2}}\left(-2x\,dx\right) = -\frac{1}{2}\cdot\frac{2}{3}\left(4-x^2\right)^{\frac{3}{2}} + C = -\frac{1}{3}\left(4-x^2\right)^{\frac{3}{2}} + C$

21. B $\int_0^1 (x+1)e^{x^2+2x}\,dx = \frac{1}{2}\int_0^1 e^{x^2+2x}\left((2x+2)\,dx\right) = \frac{1}{2}\left(e^{x^2+2x}\right)\Big|_0^1 = \frac{1}{2}\left(e^3 - e^0\right) = \frac{e^3-1}{2}$

22. C $x'(t) = t+1 \Rightarrow x(t) = \frac{1}{2}(t+1)^2 + C$ and $x(0) = 1 \Rightarrow C = \frac{1}{2} \Rightarrow x(t) = \frac{1}{2}(t+1)^2 + \frac{1}{2}$

$x(1) = \frac{5}{2},\; y(1) = \ln\frac{5}{2};\qquad \left(\frac{5}{2},\, \ln\frac{5}{2}\right)$

23. C $\lim_{h\to 0}\frac{\ln(2+h)-\ln 2}{h} = f'(2)$ where $f(x) = \ln x$; $\quad f'(x) = \frac{1}{x} \Rightarrow f'(2) = \frac{1}{2}$

24. A This item uses the formal definition of a limit and is no longer part of the AP Course Description. $\left|\, f(x) - 7\,\right| = \left|(3x+1)-7\right| = \left|3x-6\right| = 3\left|x-2\right| < \varepsilon$ whenever $\left|x-2\right| < \frac{\varepsilon}{3}$.

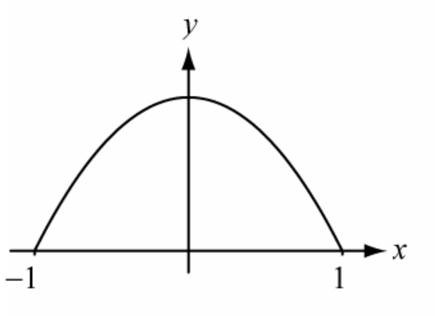
Any $\delta < \frac{\varepsilon}{3}$ will be sufficient and $\frac{\varepsilon}{4} < \frac{\varepsilon}{3}$, thus the answer is $\frac{\varepsilon}{4}$.

25. B $\int_0^{\frac{\pi}{4}} \tan^2 x\,dx = \int_0^{\frac{\pi}{4}}\left(\sec^2 x - 1\right)dx = \left(\tan x - x\right)\Big|_0^{\pi/4} = 1 - \frac{\pi}{4}$

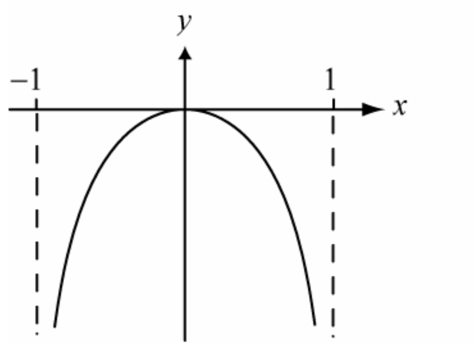
26. D For x in the interval (–1, 1), $g(x) = \left|x^2-1\right| = -(x^2-1)$ and so $y = \ln g(x) = \ln(-(x^2-1))$.
Therefore

$$y' = \frac{2x}{x^2-1},\quad y'' = \frac{\left(x^2-1\right)(2)-(2x)(2x)}{\left(x^2-1\right)^2} = \frac{-2x^2-2}{\left(x^2-1\right)^2} < 0$$

Alternative graphical solution: Consider the graphs of $g(x) = \left|x^2 - 1\right|$ and $\ln g(x)$.



$g(x) = \left|x^2 - 1\right|$



$\ln\left|x^2 - 1\right|$

concave down

27. E $\quad f'(x) = x^2 - 8x + 12 = (x-2)(x-6)$; the candidates are: $x = 0, 2, 6, 9$

x	0	2	6	9
$f(x)$	−5	$17/3$	−5	22

the maximum is at $x = 9$

28. C $\quad x = \sin^2 y \Rightarrow dx = 2\sin y\cos y\,dy$; when $x = 0,\ y = 0$ and when $x = \frac{1}{2},\ y = \frac{\pi}{4}$

$$\int_0^{\frac{1}{2}} \frac{\sqrt{x}}{\sqrt{1-x}}\,dx = \int_0^{\frac{\pi}{4}} \frac{\sin y}{\sqrt{1-\sin^2 y}} \cdot 2\sin y\cos y\,dy = \int_0^{\frac{\pi}{4}} 2\sin^2 y\,dy$$

29. A $\quad$ Let $z = y'$. Then $z = e$ when $x = 0$. Thus $y'' = 2y' \Rightarrow z' = 2z$. Solve this differential equation.

$z = Ce^{2x};\ e = Ce^0 \Rightarrow C = e \Rightarrow y' = z = e^{2x+1}$. Solve this differential equation.

$$y = \frac{1}{2}e^{2x+1} + K;\ e = \frac{1}{2}e^1 + K \Rightarrow K = \frac{1}{2}e;\ y = \frac{1}{2}e^{2x+1} + \frac{1}{2}e,\ y(1) = \frac{1}{2}e^3 + \frac{1}{2}e = \frac{1}{2}e\left(e^2+1\right)$$

Alternative Solution: $y'' = 2y' \Rightarrow y' = Ce^{2x} = e\cdot e^{2x}$. Therefore $y'(1) = e^3$.

$y'(1) - y'(0) = \int_0^1 y''(x)dx = \int_0^1 2y'(x)dx = 2y(1) - 2y(0)$ and so

$$y(1) = \frac{y'(1) - y'(0) + 2y(0)}{2} = \frac{e^3 + e}{2}.$$

30. B $\int_1^2 \frac{x-4}{x^2}dx = \int_1^2 \left(\frac{1}{x} - 4x^{-2}\right)dx = \left(\ln x + \frac{4}{x}\right)\Big|_1^2 = (\ln 2 + 2) - (\ln 1 + 4) = \ln 2 - 2$

31. E $f'(x) = \dfrac{\frac{d}{dx}(\ln x)}{\ln x} = \dfrac{\frac{1}{x}}{\ln x} = \dfrac{1}{x \ln x}$

32. C Take the log of each side of the equation and differentiate. $\ln y = \ln x^{\ln x} = \ln x \cdot \ln x = (\ln x)^2$

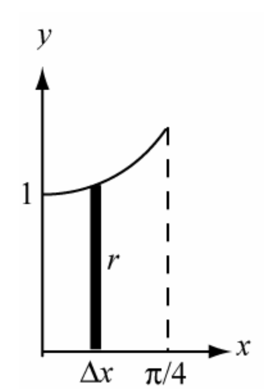
$$\frac{y'}{y} = 2\ln x \cdot \frac{d}{dx}(\ln x) = \frac{2}{x}\ln x \Rightarrow y' = x^{\ln x}\left(\frac{2}{x}\ln x\right)$$

33. A $f(-x) = -f(x) \Rightarrow f'(-x)\cdot(-1) = -f'(x) \Rightarrow f'(-x) = -f'(x)$ thus $f'(-x_0) = -f'(x_0)$.

34. C $\frac{1}{2}\int_0^2 \sqrt{x}\,dx = \frac{1}{2}\cdot\frac{2}{3}x^{\frac{3}{2}}\Big|_0^2 = \frac{1}{3}\cdot 2^{\frac{3}{2}} = \frac{2}{3}\sqrt{2}$

35. C Washers: $\sum \pi r^2 \Delta x$ where $r = y = \sec x$.

Volume $= \pi\int_0^{\frac{\pi}{4}} \sec^2 x\,dx = \pi \tan x\Big|_0^{\frac{\pi}{4}} = \pi\left(\tan\frac{\pi}{4} - \tan 0\right) = \pi$



36. E $\int_0^1 \frac{x+1}{x^2+2x-3}dx = \frac{1}{2}\lim_{L\to 1^-}\int_0^L \frac{2x+2}{x^2+2x-3}dx = \frac{1}{2}\lim_{L\to 1^-} \ln\left|x^2+2x-3\right|\Big|_0^L$

$$= \frac{1}{2}\lim_{L\to 1^-}\left(\ln\left|L^2+2L-3\right| - \ln\left|-3\right|\right) = -\infty. \text{ Divergent}$$

37. E $\lim_{x\to 0}\frac{1-\cos^2 2x}{x^2} = \lim_{x\to 0}\frac{\sin^2 2x}{x^2} = \lim_{x\to 0}\frac{\sin 2x}{2x}\cdot\frac{\sin 2x}{2x}\cdot 4 = 1\cdot 1\cdot 4 = 4$

38. B Let $z = x - c$. $5 = \int_1^2 f(x-c)\,dx = \int_{1-c}^{2-c} f(z)\,dz$

39. D $h'(x) = f'(g(x))\cdot g'(x)$; $h'(1) = f'(g(1))\cdot g'(1) = f'(2)\cdot g'(1) = (-4)(-3) = 12$

40. C Area $=\frac{1}{2}\int_0^{2\pi}(1-\cos\theta)^2 d\theta=\int_0^{\pi}\left(1-2\cos\theta+\cos^2\theta\right)d\theta$; $\cos^2\theta=\frac{1}{2}(1+\cos 2\theta)$

$$\text{Area}=\int_0^{\pi}\left(1-2\cos\theta+\frac{1}{2}(1+\cos 2\theta)\right)d\theta=\left(\frac{3}{2}\theta-2\sin\theta+\frac{1}{4}\sin 2\theta\right)\Bigg|_0^{\pi}=\frac{3}{2}\pi$$

41. D $\int_{-1}^{1} f(x)\,dx=\int_{-1}^{0}(x+1)\,dx+\int_0^1\cos(\pi x)\,dx$

$$=\frac{1}{2}(x+1)^2\Bigg|_{-1}^{0}+\frac{1}{\pi}\sin(\pi x)\Bigg|_0^1=\frac{1}{2}+\frac{1}{\pi}(\sin\pi-\sin 0)=\frac{1}{2}$$

42. D $\Delta x=\frac{1}{3}$; $T=\frac{1}{2}\cdot\frac{1}{3}\left(1^2+2\left(\frac{4}{3}\right)^2+2\left(\frac{5}{3}\right)^2+2^2\right)=\frac{127}{54}$

43. E Use the technique of antiderivatives by part:

$u=\sin^{-1}x \qquad dv=dx$

$du=\frac{dx}{\sqrt{1-x^2}} \qquad v=x$

$$\int\sin^{-1}x\,dx=x\sin^{-1}x-\int\frac{x}{\sqrt{1-x^2}}dx$$

44. A Multiply both sides of $x=xf'(x)-f(x)$ by $\frac{1}{x^2}$. Then $\frac{1}{x}=\frac{xf'(x)-f(x)}{x^2}=\frac{d}{dx}\left(\frac{f(x)}{x}\right)$.

Thus we have $\frac{f(x)}{x}=\ln|x|+C\Rightarrow f(x)=x\left(\ln|x|+C\right)=x\left(\ln|x|-1\right)$ since $f(-1)=1$.

Therefore $f(e^{-1})=e^{-1}\left(\ln\left|e^{-1}\right|-1\right)=e^{-1}(-1-1)=-2e^{-1}$

This was most likely the solution students were expected to produce while solving this problem on the 1973 multiple-choice exam. However, the problem itself is not well-defined. A solution to an initial value problem should be a function that is differentiable on an interval containing the initial point. In this problem that would be the domain $x<0$ since the solution requires the choice of the branch of the logarithm function with $x<0$. Thus one cannot ask about the value of the function at $x=e^{-1}$.

45. E $F'(x)=xg'(x)$ with $x\ge 0$ and $g'(x)<0\Rightarrow F'(x)<0\Rightarrow F$ is not increasing.