90 Minutes—Scientific Calculator

Notes: (1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. The area of the region enclosed by the graphs of $y = x^2$ and $y = x$ is

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{5}{6}$ (E) 1

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2. If $f(x) = 2x^2 + 1$, then $\lim_{x \to 0} \frac{f(x) - f(0)}{x^2}$ is

(A) 0 (B) 1 (C) 2 (D) 4 (E) nonexistent

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3. If p is a polynomial of degree n, $n > 0$, what is the degree of the polynomial $Q(x) = \int_0^x p(t)dt$?

(A) 0 (B) 1 (C) $n-1$ (D) n (E) $n+1$

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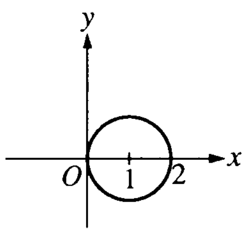
4. A particle moves along the curve $xy = 10$. If $x = 2$ and $\frac{dy}{dt} = 3$, what is the value of $\frac{dx}{dt}$?

(A) $-\frac{5}{2}$ (B) $-\frac{6}{5}$ (C) 0 (D) $\frac{4}{5}$ (E) $\frac{6}{5}$

5. Which of the following represents the graph of the polar curve $r = 2\sec\theta$?

(A)


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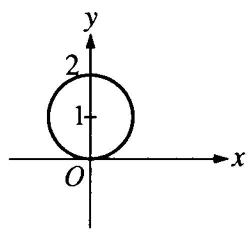

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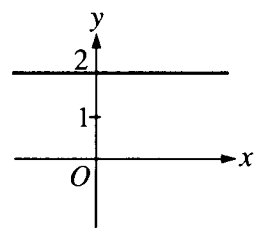

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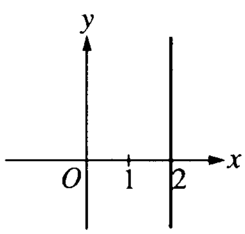
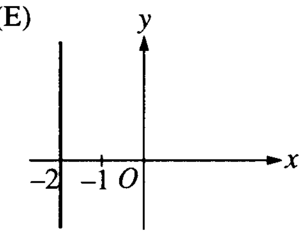


(E)


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6. If $x = t^2 + 1$ and $y = t^3$, then $\dfrac{d^2y}{dx^2} =$

(A) $\dfrac{3}{4t}$ (B) $\dfrac{3}{2t}$ (C) $3t$ (D) $6t$ (E) $\dfrac{3}{2}$

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7. $\int_0^1 x^3 e^{x^4}\,dx =$

(A) $\dfrac{1}{4}(e-1)$ (B) $\dfrac{1}{4}e$ (C) $e-1$ (D) e (E) $4(e-1)$

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8. If $f(x) = \ln(e^{2x})$, then $f'(x) =$

(A) 1 (B) 2 (C) $2x$ (D) e^{-2x} (E) $2e^{-2x}$

9. If $f(x) = 1 + x^{\frac{2}{3}}$, which of the following is NOT true?

(A) f is continuous for all real numbers.

(B) f has a minimum at $x = 0$.

(C) f is increasing for $x > 0$.

(D) $f'(x)$ exists for all x.

(E) $f''(x)$ is negative for $x > 0$.

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10. Which of the following functions are continuous at $x = 1$?

I. $\ln x$

II. e^x

III. $\ln(e^x - 1)$

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, and III

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11. $\int_4^{\infty} \frac{-2x}{\sqrt[3]{9-x^2}}\,dx$ is

(A) $7^{\frac{2}{3}}$ (B) $\frac{3}{2}\left(7^{\frac{2}{3}}\right)$ (C) $9^{\frac{2}{3}} + 7^{\frac{2}{3}}$ (D) $\frac{3}{2}\left(9^{\frac{2}{3}} + 7^{\frac{2}{3}}\right)$ (E) nonexistent

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12. The position of a particle moving along the x-axis is $x(t) = \sin(2t) - \cos(3t)$ for time $t \geq 0$. When $t = \pi$, the acceleration of the particle is

(A) 9 (B) $\frac{1}{9}$ (C) 0 (D) $-\frac{1}{9}$ (E) –9

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13. If $\frac{dy}{dx} = x^2 y$, then y could be

(A) $3\ln\left(\frac{x}{3}\right)$ (B) $e^{\frac{x^3}{3}} + 7$ (C) $2e^{\frac{x^3}{3}}$ (D) $3e^{2x}$ (E) $\frac{x^3}{3} + 1$

14. The <u>derivative</u> of f is $x^4(x-2)(x+3)$. At how many points will the graph of f have a relative maximum?

(A) None (B) One (C) Two (D) Three (E) Four

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15. If $f(x) = e^{\tan^2 x}$, then $f'(x) =$

(A) $e^{\tan^2 x}$

(B) $\sec^2 x\, e^{\tan^2 x}$

(C) $\tan^2 x\, e^{\tan^2 x - 1}$

(D) $2\tan x \sec^2 x\, e^{\tan^2 x}$

(E) $2\tan x\, e^{\tan^2 x}$

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16. Which of the following series diverge?

I. $\sum_{k=3}^{\infty} \frac{2}{k^2+1}$

II. $\sum_{k=1}^{\infty} \left(\frac{6}{7}\right)^k$

III. $\sum_{k=2}^{\infty} \frac{(-1)^k}{k}$

(A) None (B) II only (C) III only (D) I and III (E) II and III

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17. The slope of the line tangent to the graph of $\ln(xy) = x$ at the point where $x = 1$ is

(A) 0 (B) 1 (C) e (D) e^2 (E) $1-e$

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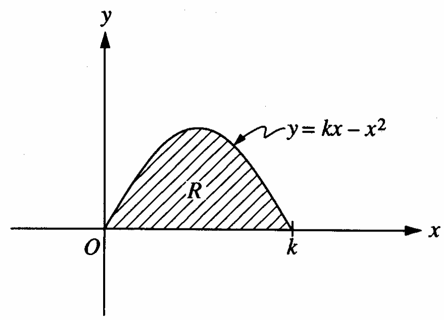
18. If $e^{f(x)} = 1 + x^2$, then $f'(x) =$

(A) $\frac{1}{1+x^2}$ (B) $\frac{2x}{1+x^2}$ (C) $2x(1+x^2)$ (D) $2x\left(e^{1+x^2}\right)$ (E) $2x\ln(1+x^2)$



19. The shaded region R, shown in the figure above, is rotated about the y-axis to form a solid whose volume is 10 cubic units. Of the following, which best approximates k ?

(A) 1.51 (B) 2.09 (C) 2.49 (D) 4.18 (E) 4.77

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20. A particle moves along the x-axis so that at any time $t \geq 0$ the acceleration of the particle is $a(t) = e^{-2t}$. If at $t = 0$ the velocity of the particle is $\frac{5}{2}$ and its position is $\frac{17}{4}$, then its position at any time $t > 0$ is $x(t) =$

(A) $-\frac{e^{-2t}}{2} + 3$

(B) $\frac{e^{-2t}}{4} + 4$

(C) $4e^{-2t} + \frac{9}{2}t + \frac{1}{4}$

(D) $\frac{e^{-2t}}{2} + 3t + \frac{15}{4}$

(E) $\frac{e^{-2t}}{4} + 3t + 4$

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21. The value of the derivative of $y = \frac{\sqrt[3]{x^2 + 8}}{\sqrt[4]{2x + 1}}$ at $x = 0$ is

(A) -1 (B) $-\frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$ (E) 1

22. If $f(x) = x^2 e^x$, then the graph of f is decreasing for all x such that

(A) $x < -2$ (B) $-2 < x < 0$ (C) $x > -2$ (D) $x < 0$ (E) $x > 0$

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23. The length of the curve determined by the equations $x = t^2$ and $y = t$ from $t = 0$ to $t = 4$ is

(A) $\int_0^4 \sqrt{4t+1}\, dt$

(B) $2\int_0^4 \sqrt{t^2+1}\, dt$

(C) $\int_0^4 \sqrt{2t^2+1}\, dt$

(D) $\int_0^4 \sqrt{4t^2+1}\, dt$

(E) $2\pi\int_0^4 \sqrt{4t^2+1}\, dt$

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24. Let f and g be functions that are differentiable for all real numbers, with $g(x) \neq 0$ for $x \neq 0$. If $\lim_{x\to 0} f(x) = \lim_{x\to 0} g(x) = 0$ and $\lim_{x\to 0} \frac{f'(x)}{g'(x)}$ exists, then $\lim_{x\to 0} \frac{f(x)}{g(x)}$ is

(A) 0

(B) $\frac{f'(x)}{g'(x)}$

(C) $\lim_{x\to 0} \frac{f'(x)}{g'(x)}$

(D) $\frac{f'(x)g(x) - f(x)g'(x)}{\left(f(x)\right)^2}$

(E) nonexistent

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25. Consider the curve in the xy-plane represented by $x = e^t$ and $y = te^{-t}$ for $t \geq 0$. The slope of the line tangent to the curve at the point where $x = 3$ is

(A) 20.086 (B) 0.342 (C) –0.005 (D) –0.011 (E) –0.033

26. If $y = \arctan(e^{2x})$, then $\dfrac{dy}{dx} =$

(A) $\dfrac{2e^{2x}}{\sqrt{1-e^{4x}}}$ (B) $\dfrac{2e^{2x}}{1+e^{4x}}$ (C) $\dfrac{e^{2x}}{1+e^{4x}}$ (D) $\dfrac{1}{\sqrt{1-e^{4x}}}$ (E) $\dfrac{1}{1+e^{4x}}$

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27. The interval of convergence of $\displaystyle\sum_{n=0}^{\infty} \frac{(x-1)^n}{3^n}$ is

(A) $-3 < x \le 3$ (B) $-3 \le x \le 3$ (C) $-2 < x < 4$

(D) $-2 \le x < 4$ (E) $0 \le x \le 2$

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28. If a particle moves in the *xy*-plane so that at time $t > 0$ its position vector is $\left(\ln(t^2 + 2t), 2t^2\right)$, then at time $t = 2$, its velocity vector is

(A) $\left(\dfrac{3}{4}, 8\right)$ (B) $\left(\dfrac{3}{4}, 4\right)$ (C) $\left(\dfrac{1}{8}, 8\right)$ (D) $\left(\dfrac{1}{8}, 4\right)$ (E) $\left(-\dfrac{5}{16}, 4\right)$

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29. $\int x \sec^2 x\, dx =$

(A) $x \tan x + C$ (B) $\dfrac{x^2}{2} \tan x + C$ (C) $\sec^2 x + 2\sec^2 x \tan x + C$

(D) $x \tan x - \ln|\cos x| + C$ (E) $x \tan x + \ln|\cos x| + C$

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30. What is the volume of the solid generated by rotating about the *x*-axis the region enclosed by the curve $y = \sec x$ and the lines $x = 0$, $y = 0$, and $x = \dfrac{\pi}{3}$?

(A) $\dfrac{\pi}{\sqrt{3}}$

(B) π

(C) $\pi\sqrt{3}$

(D) $\dfrac{8\pi}{3}$

(E) $\pi \ln\left(\dfrac{1}{2} + \sqrt{3}\right)$

31. If $s_n = \left(\frac{(5+n)^{100}}{5^{n+1}}\right)\left(\frac{5^n}{(4+n)^{100}}\right)$, to what number does the sequence $\{s_n\}$ converge?

(A) $\frac{1}{5}$ (B) 1 (C) $\frac{5}{4}$ (D) $\left(\frac{5}{4}\right)^{100}$ (E) The sequence does not converge.

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32. If $\int_a^b f(x)dx = 5$ and $\int_a^b g(x)dx = -1$, which of the following must be true?

I. $f(x) > g(x)$ for $a \le x \le b$

II. $\int_a^b \left(f(x) + g(x)\right) dx = 4$

III. $\int_a^b \left(f(x)g(x)\right) dx = -5$

(A) I only (B) II only (C) III only (D) II and III only (E) I, II, and III

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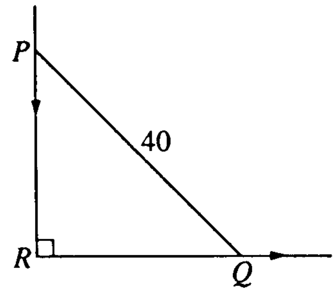
33. Which of the following is equal to $\int_0^{\pi} \sin x\, dx$?

(A) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x\, dx$ (B) $\int_0^{\pi} \cos x\, dx$ (C) $\int_{-\pi}^{0} \sin x\, dx$

(D) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin x\, dx$ (E) $\int_{\pi}^{2\pi} \sin x\, dx$



34. In the figure above, PQ represents a 40-foot ladder with end P against a vertical wall and end Q on level ground. If the ladder is slipping down the wall, what is the distance RQ at the instant when Q is moving along the ground $\frac{3}{4}$ as fast as P is moving down the wall?

(A) $\frac{6}{5}\sqrt{10}$ (B) $\frac{8}{5}\sqrt{10}$ (C) $\frac{80}{\sqrt{7}}$ (D) 24 (E) 32

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35. If F and f are differentiable functions such that $F(x) = \int_0^x f(t)dt$, and if $F(a) = -2$ and $F(b) = -2$ where $a < b$, which of the following must be true?

(A) $f(x) = 0$ for some x such that $a < x < b$.

(B) $f(x) > 0$ for all x such that $a < x < b$.

(C) $f(x) < 0$ for all x such that $a < x < b$.

(D) $F(x) \le 0$ for all x such that $a < x < b$.

(E) $F(x) = 0$ for some x such that $a < x < b$.

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36. Consider all right circular cylinders for which the sum of the height and circumference is 30 centimeters. What is the radius of the one with maximum volume?

(A) 3 cm (B) 10 cm (C) 20 cm (D) $\frac{30}{\pi^2}$ cm (E) $\frac{10}{\pi}$ cm

37. If $f(x)=\begin{cases} x & \text{for } x\le 1 \\ \dfrac{1}{x} & \text{for } x>1, \end{cases}$ then $\int_0^e f(x)dx =$

(A) 0 (B) $\frac{3}{2}$ (C) 2 (D) e (E) $e+\frac{1}{2}$

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38. During a certain epidemic, the number of people that are infected at any time increases at a rate proportional to the number of people that are infected at that time. If 1,000 people are infected when the epidemic is first discovered, and 1,200 are infected 7 days later, how many people are infected 12 days after the epidemic is first discovered?

(A) 343 (B) 1,343 (C) 1,367 (D) 1,400 (E) 2,057

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39. If $\frac{dy}{dx}=\frac{1}{x}$, then the average rate of change of y with respect to x on the closed interval $[1,4]$ is

(A) $-\frac{1}{4}$ (B) $\frac{1}{2}\ln 2$ (C) $\frac{2}{3}\ln 2$ (D) $\frac{2}{5}$ (E) 2

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40. Let R be the region in the first quadrant enclosed by the x-axis and the graph of $y=\ln(1+2x-x^2)$. If Simpson's Rule with 2 subintervals is used to approximate the area of R, the approximation is

(A) 0.462 (B) 0.693 (C) 0.924 (D) 0.986 (E) 1.850

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41. Let $f(x)=\int_{-2}^{x^2-3x} e^{t^2}\,dt$. At what value of x is $f(x)$ a minimum?

(A) For no value of x (B) $\frac{1}{2}$ (C) $\frac{3}{2}$ (D) 2 (E) 3

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42. $\lim_{x\to 0}(1+2x)^{\csc x} =$

(A) 0 (B) 1 (C) 2 (D) e (E) e^2

43. The coefficient of x^6 in the Taylor series expansion about $x = 0$ for $f(x) = \sin\left(x^2\right)$ is

(A) $-\frac{1}{6}$ (B) 0 (C) $\frac{1}{120}$ (D) $\frac{1}{6}$ (E) 1

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44. If f is continuous on the interval $[a,b]$, then there exists c such that $a < c < b$ and $\int_a^b f(x)dx =$

(A) $\frac{f(c)}{b-a}$ (B) $\frac{f(b)-f(a)}{b-a}$ (C) $f(b)-f(a)$ (D) $f'(c)(b-a)$ (E) $f(c)(b-a)$

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45. If $f(x) = \sum_{k=1}^{\infty}\left(\sin^2 x\right)^k$, then $f(1)$ is

(A) 0.369 (B) 0.585 (C) 2.400 (D) 2.426 (E) 3.426

1. A $\int_0^1 (x-x^2)\,dx = \left(\frac{1}{2}x^2 - \frac{1}{3}x^3\right)\Big|_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$

2. C $\lim_{x\to 0} \frac{2x^2+1-1}{x^2} = 2$

3. E $Q'(x) = p(x) \Rightarrow$ degree of Q is $n+1$

4. B If $x=2$ then $y=5$. $x\frac{dy}{dt} + y\frac{dx}{dt} = 0$; $2(3) + 5\cdot\frac{dx}{dt} = 0 \Rightarrow \frac{dx}{dt} = -\frac{6}{5}$

5. D $r = 2\sec\theta$; $r\cos\theta = 2 \Rightarrow x = 2$. This is a vertical line through the point $(2,0)$.

6. A $\frac{dx}{dt} = 2t,\ \frac{dy}{dt} = 3t^2$ thus $\frac{dy}{dx} = \frac{3t^2}{2t} = \frac{3}{2}t;\ \frac{d^2y}{dx^2} = \frac{d\left(\frac{dy}{dx}\right)}{dx} = \frac{\frac{d\left(\frac{dy}{dx}\right)}{dt}}{\frac{dx}{dt}} = \frac{\frac{3}{2}}{2t} = \frac{3}{4t}$

7. A $\int_0^1 x^3 e^{x^4}\,dx = \frac{1}{4}\int_0^1 e^{x^4}(4x^3dx) = \frac{1}{4}e^{x^4}\Big|_0^1 = \frac{1}{4}(e-1)$

8. B $f(x) = \ln e^{2x} = 2x,\ f'(x) = 2$

9. D $f'(x) = \frac{2}{3}\cdot\frac{1}{x^{1/3}}$. This does not exist at $x=0$. D is false, all others are true.

10. E I. $\ln x$ is continuous for $x>0$
II. e^x is continuous for all x
III. $\ln(e^x-1)$ is continuous for $x>0$.

11. E $\int_4^\infty \frac{-2x}{\sqrt[3]{9-x^2}}\,dx = \lim_{b\to\infty} \frac{3}{2}\left(9-x^2\right)^{2/3}\Big|_4^b$. This limit diverges. Another way to see this without doing the integration is to observe that the denominator behaves like $x^{2/3}$ which has a smaller degree than the degree of the numerator. This would imply that the integral will diverge.

12. E $v(t)=2\cos 2t+3\sin 3t,\ a(t)=-4\sin 2t+9\cos 3t,\ a(\pi)=-9$.

13. C $\dfrac{dy}{y}=x^2dx,\ \ln|y|=\dfrac{1}{3}x^3+C_1,\ y=Ce^{\frac{1}{3}x^3}$. Only C is of this form.

14. B The only place that $f'(x)$ changes sign from positive to negative is at $x=-3$.

15. D $f'(x)=e^{\tan^2 x}\cdot\dfrac{d\left(\tan^2 x\right)}{dx}=2\tan x\cdot\sec^2 x\cdot e^{\tan^2 x}$

16. A I. Compare with p-series, $p=2$

II. Geometric series with $r=\dfrac{6}{7}$

III. Alternating harmonic series

17. A Using implicit differentiation, $\dfrac{y+xy'}{xy}=1$. When $x=1$, $\dfrac{y+y'}{y}=1\Rightarrow y'=0$.

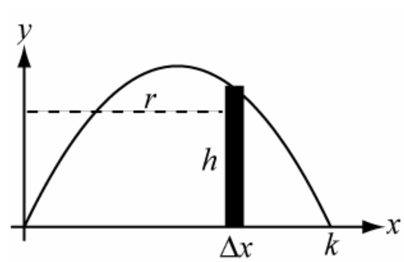
Alternatively, $xy=e^x$, $y=\dfrac{e^x}{x}$, $y'=\dfrac{xe^x-e^x}{x^2}=\dfrac{e^x(x-1)}{x^2}$. $y'(1)=0$

18. B $f'(x)\cdot e^{f(x)}=2x\Rightarrow f'(x)=\dfrac{2x}{e^{f(x)}}=\dfrac{2x}{1+x^2}$

19. B Use cylindrical shells which is no longer part of the AP Course Description. Each shell is of the form $2\pi rh\Delta x$ where $r=x$ and $h=kx-x^2$. Solve the equation

$$10=2\pi\int_0^k x(kx-x^2)\,dx=2\pi\left(\frac{kx^3}{3}-\frac{x^4}{4}\right)\Bigg|_0^k=2\pi\cdot\frac{k^4}{12}.$$

$k=\sqrt[4]{\dfrac{60}{\pi}}\approx 2.0905$.



20. E $v(t)=-\dfrac{1}{2}e^{-2t}+3$ and $x(t)=\dfrac{1}{4}e^{-2t}+3t+4$

21. A Use logarithms.

$\ln y = \frac{1}{3}\ln\left(x^2+8\right) - \frac{1}{4}\ln(2x+1)$; $\frac{y'}{y} = \frac{2x}{3\left(x^2+8\right)} - \frac{2}{4(2x+1)}$; at $(0,2)$, $y' = -1$.

22. B $f'(x) = x^2e^x + 2xe^x = xe^x(x+2)$; $f'(x) < 0$ for $-2 < x < 0$

23. D $L = \int_0^4 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \int_0^4 \sqrt{4t^2+1}\, dt$

24. C This is L'Hôpital's Rule.

25. D At $t = 3$, slope $= \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-te^{-t} + e^{-t}}{e^t} = \left.\frac{1-t}{e^{2t}}\right|_{t=3} = -\frac{2}{e^6} = -0.005$

26. B $\frac{2e^{2x}}{1+(e^{2x})^2} = \frac{2e^{2x}}{1+e^{4x}}$

27. C This is a geometric series with $r = \frac{x-1}{3}$. Convergence for $-1 < r < 1$. Thus the series is convergent for $-2 < x < 4$.

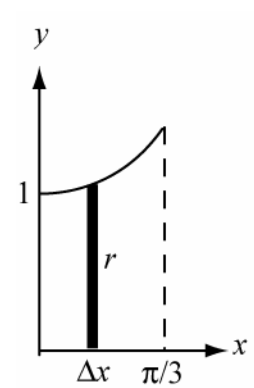
28. A $v = \left(\frac{2t+2}{t^2+2t}, 4t\right)$, $v(2) = \left(\frac{6}{8}, 8\right) = \left(\frac{3}{4}, 8\right)$

29. E Use the technique of antiderivatives by parts: $u = x$ and $dv = \sec^2 x\, dx$

$\int x\sec^2 x\, dx = x\tan x - \int \tan x\, dx = x\tan x + \ln\left|\cos x\right| + C$

30. C Each slice is a disk with radius $r = \sec x$ and width Δx.

Volume $= \pi\int_0^{\pi/3} \sec^2 x\, dx = \pi \tan x\Big|_0^{\pi/3} = \pi\sqrt{3}$



31. A $s_n = \frac{1}{5}\left(\frac{5+n}{4+n}\right)^{100}$, $\lim_{n\to\infty} s_n = \frac{1}{5}\cdot 1 = \frac{1}{5}$

32. B Only II is true. To see that neither I nor III must be true, let $f(x)=1$ and let $g(x)=x^2-\frac{128}{15}$ on the interval [0, 5].

33. A The value of this integral is 2. Option A is also 2 and none of the others have a value of 2. Visualizing the graphs of $y=\sin x$ and $y=\cos x$ is a useful approach to the problem.

34. E Let $y = PR$ and x=RQ.
$x^2+y^2=40^2$, $2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$, $x\cdot\frac{3}{4}\left(-\frac{dy}{dt}\right)+y\frac{dy}{dt}=0 \Rightarrow y=\frac{3}{4}x$.
Substitute into $x^2+y^2=40^2$. $x^2+\frac{9}{16}x^2=40^2$, $\frac{25}{16}x^2=40^2$, $x=32$

35. A Apply the Mean Value Theorem to *F*. $F'(c)=\frac{F(b)-F(a)}{b-a}=\frac{0}{a}=0$. This means that there is number in the interval (a,b) for which F' is zero. However, $F'(x)=f(x)$. So, $f(x)=0$ for some number in the interval (a,b).

36. E $v=\pi r^2 h$ and $h+2\pi r=30 \Rightarrow v=2\pi\left(15r^2-\pi r^3\right)$ for $0<r<\frac{15}{\pi}$; $\frac{dv}{dr}=6\pi r\left(10-\pi r\right)$. The maximum volume is when $r=\frac{10}{\pi}$ because $\frac{dv}{dr}>0$ on $\left(0,\frac{10}{\pi}\right)$ and $\frac{dv}{dr}<0$ on $\left(\frac{10}{\pi},\frac{15}{\pi}\right)$.

37. B $\int_0^e f(x)\,dx=\int_0^1 x\,dx+\int_1^e \frac{1}{x}\,dx=\frac{1}{2}+\ln e=\frac{3}{2}$

38. C $\frac{dN}{dt}=kN \Rightarrow N=Ce^{kt}$. $N(0)=1000 \Rightarrow C=1000$. $N(7)=1200 \Rightarrow k=\frac{1}{7}\ln(1.2)$. Therefore $N(12)=1000e^{\frac{12}{7}\ln(1.2)}\approx 1367$.

39. C Want $\frac{y(4)-y(1)}{4-1}$ where $y(x)=\ln|x|+C$. This gives $\frac{\ln 4-\ln 1}{3}=\frac{1}{3}\ln 4=\frac{1}{3}\ln 2^2=\frac{2}{3}\ln 2$.

40. C The interval is $[0,2]$, $x_0=0,\ x_1=1,\ x_2=2$. $S=\frac{1}{3}\left(0+4\ln 2+0\right)=\frac{4}{3}\ln 2$. Note that Simpson's rule is no longer part of the BC Course Description.

41. C $f'(x)=(2x-3)e^{(x^2-3x)^2}$; $f'<0$ for $x<\dfrac{3}{2}$ and $f'>0$ for $x>\dfrac{3}{2}$.

Thus f has its absolute minimum at $x=\dfrac{3}{2}$.

42. E Suppose $\lim\limits_{x\to 0}\ln\left((1+2x)^{\csc x}\right)=A$. The answer to the given question is e^A.

Use L'Hôpital's Rule: $\lim\limits_{x\to 0}\ln\left((1+2x)^{\csc x}\right)=\lim\limits_{x\to 0}\dfrac{\ln(1+2x)}{\sin x}=\lim\limits_{x\to 0}\dfrac{2}{1+2x}\cdot\dfrac{1}{\cos x}=2$.

43. A $\sin x=x-\dfrac{x^3}{3!}+\dfrac{x^5}{5!}-\cdots\Rightarrow\ \sin x^2=x^2-\dfrac{(x^2)^3}{3!}+\dfrac{(x^2)^5}{5!}-\cdots=x^2-\dfrac{x^6}{3!}+\dfrac{x^{10}}{5!}-\cdots$

44. E By the Intermediate Value Theorem there is a c satisfying $a<c<b$ such that $f(c)$ is equal to the average value of f on the interval $[a,b]$. But the average value is also given by $\dfrac{1}{b-a}\int_a^b f(x)\,dx$. Equating the two gives option E.

Alternatively, let $F(t)=\int_a^t f(x)\,dx$. By the Mean Value Theorem, there is a c satisfying $a<c<b$ such that $\dfrac{F(b)-F(a)}{b-a}=F'(c)$. But $F(b)-F(a)=\int_a^b f(x)\,dx$, and $F'(c)=f(c)$ by the Fundamental Theorem of Calculus. This again gives option E as the answer. This result is called the Mean Value Theorem for Integrals.

45. D This is an infinite geometric series with a first term of $\sin^2 x$ and a ratio of $\sin^2 x$.

The series converges to $\dfrac{\sin^2 x}{1-\sin^2 x}=\tan^2 x$ for $x\neq(2k+1)\dfrac{\pi}{2}$, k an integer. The answer is therefore $\tan^2 1=2.426$.