AP® Precalculus Exam

SECTION II: Free Response, Questions

DO NOT OPEN THIS BOOKLET OR BREAK THE SEALS ON PART B UNTIL YOU ARE TOLD TO DO SO.

Instructions

The questions for Section II are printed in this booklet. Do not break the seals on Part B until you are told to do so. You may use the pages in this orange booklet for scratch work, but you must write your answers in the separate Section II: Free Response booklet. **No credit will be given for any work written in this orange booklet**. In the Free Response booklet, write your solution to each part of each question in the space provided for that part. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be scored.

- Manage your time carefully. As you begin each part, you may wish to look over both questions for that part before starting to work on them. You are encouraged to use the allotted time to respond to all parts of all questions.

- Show all of your work. Your work will be scored on the correctness and completeness of your responses, including your supporting work and answers. Answers without supporting work may not receive credit in cases where supporting work is requested.

- During Part A, work only on questions 1 and 2. You are expected to use your graphing calculator for tasks such as producing graphs and tables, evaluating functions, solving equations, and performing computations.

- **For Part A, your calculator must be in radian mode**. Avoid rounding intermediate computations on the way to the final result. Unless otherwise specified, any decimal approximations reported in your work should be accurate to three places after the decimal point.

- For Part A, it may be helpful to use your graphing calculator to store information such as computed values for constants, functions you are working with, solutions to equations, and any intermediate values. Computations with the graphing calculator that use the stored information help to maintain as much precision as possible and ensure the desired accuracy in final answers.

- During Part B, questions 3 and 4, no calculator is allowed. Carefully read the instructions provided with the questions. You may continue to work on questions 1 and 2 without the use of a calculator.

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

PRECALCULUS

SECTION II, Part A

Time—30 minutes

2 Questions

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

x	1	2	4	8	16	32
$f(x)$	0	5	10	15	20	25

1. Let f be an increasing function defined for $x > 0$. The table gives values for $f(x)$ at selected values of x. The function g is given by $g(x) = 0.25x^3 - 9.5x^2 + 110x - 399$.

   (A) (i) The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(8)$ as a decimal approximation, or indicate that it is not defined.

   (ii) Find the value of $f^{-1}(20)$, or indicate that it is not defined.

   (B) (i) Find all values of x, as decimal approximations, for which $g(x) = -45$, or indicate that there are no such values.

   (ii) Determine the end behavior of g as x increases without bound. Express your answer using the mathematical notation of a limit.

   (C) (i) Use the table of values of $f(x)$ to determine if f is best modeled by a linear, quadratic, exponential, or logarithmic function.

   (ii) Give a reason for your answer based on the relationship between the change in the output values of f and the change in the input values of f.

Write your responses to this question only on the designated pages in the separate Free Response booklet.
Write your solution to each part in the space provided for that part.

GO ON TO THE NEXT PAGE.

2. A student won \$500 in an art contest. At first, the student kept the money in a desk. After 10 months, the student deposited the money in a savings account that earned interest. Six months after depositing the money ($t = 6$), the amount in the account is \$508.67. Twelve months after depositing the money ($t = 12$), the amount in the account is \$517.50.

The amount of money the student has can be modeled by the piecewise function M given by

$$M(t) = \begin{cases} 500 & \text{for } -10 \leq t < 0 \\ ab^{(t/12)} & \text{for } t \geq 0 \end{cases},$$

where $M(t)$ is the amount, in dollars, at time t months since the \$500 was deposited into the savings account. A negative value for t represents the number of months before the student deposited the \$500 into the savings account.

(A)   (i) Use the given data to write two equations that can be used to find the values for constants a and b in the expression for $M(t)$.

    (ii) Find the values for a and b as decimal approximations.

(B)   (i) Use the given data to find the average rate of change of the amount of money the student has, in dollars per month, from $t = -2$ to $t = 12$ months. Express your answer as a decimal approximation. Show the computations that lead to your answer.

    (ii) Use $M(12)$ and the average rate of change found in (i) to estimate the amount of money, in dollars, the student has when $t = 20$ months. Show the work that leads to your answer.

    (iii) Let $A(t)$ be the estimate of the amount of money, in dollars, the student has at time t months using the average rate of change found in (i). In (ii), $A(20)$ was computed. If $A(t)$ is used to estimate values for $M(t)$ for $t > 12$, the error in the estimates will increase as t increases. Explain why this is true.

(C)   The student plans to close the account when the amount of money in the account reaches \$565. Explain how this information can be used to determine the domain limitations for the model M.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

END OF PART A

IF YOU FINISH BEFORE TIME IS CALLED,

YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

PRECALCULUS

SECTION II, Part B

Time—30 minutes

2 Questions

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.
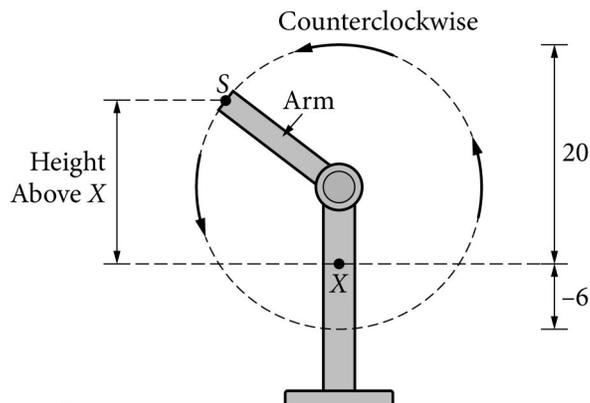
DO NOT BREAK THE SEALS UNTIL YOU ARE TOLD TO DO SO.
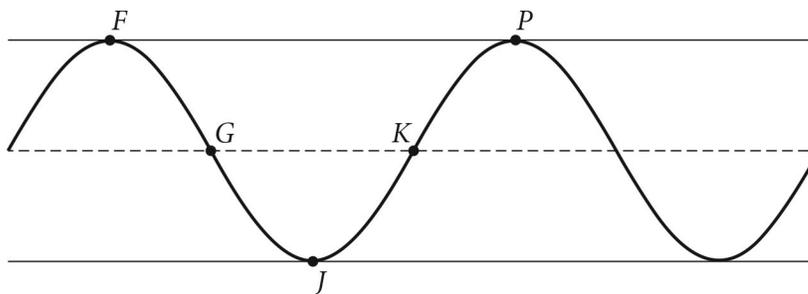
NO CALCULATOR ALLOWED

Note: Figure not drawn to scale.

3. The figure shows a robotic arm rotating in a circular counterclockwise direction that completes one rotation every 2 seconds. Point S is on the tip of the arm, and point X does not move. As the arm rotates at a constant speed, the height of S above X periodically increases and decreases. At time $t = 0$ seconds, S is at its lowest position, 6 inches directly below X. At its highest position, S is 20 inches directly above X.

The sinusoidal function h models the height of S above X, in inches, as a function of time t, in seconds. A positive value of $h(t)$ indicates S is above X; a negative value of $h(t)$ indicates S is below X.

(A) The graph of h and its dashed midline for two full cycles is shown. Five points, F, G, J, K, and P, are labeled on the graph. No scale is indicated, and no axes are presented.

Determine possible coordinates $\left(t, h(t)\right)$ for the five points: F, G, J, K, and P.

(B) The function h can be written in the form $h(t) = a\cos\left(b(t+c)\right) + d$. Find values of constants a, b, c, and d.

GO ON TO THE NEXT PAGE.

NO CALCULATOR ALLOWED

(C) Refer to the graph of h in part (A). The t-coordinate of K is t_1, and the t-coordinate of P is t_2.

(i) On the interval (t_1, t_2), which of the following is true about h?

    a. h is positive and increasing.

    b. h is positive and decreasing.

    c. h is negative and increasing.

    d. h is negative and decreasing.

(ii) Describe how the rate of change of h is changing on the interval (t_1, t_2).

Write your responses to this question only on the designated pages in the separate Free Response booklet.

Write your solution to each part in the space provided for that part.

NO CALCULATOR ALLOWED

4. **Directions:**

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.

- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\dfrac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.

- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\dfrac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.

- For each part of the question, show the work that leads to your answers.

(A) The functions g and h are given by

$$g(x) = \log_5(4x - 2)$$
$$h(x) = \sin^{-1}(8x).$$

(i) Solve $g(x) = 3$ for values of x in the domain of g.

(ii) Solve $h(x) = \dfrac{\pi}{4}$ for values of x in the domain of h.

(B) The functions j and k are given by

$$j(x) = (\sec x)(\cot x)$$
$$k(x) = \frac{\left(16^{3x}\right) \cdot 4^x}{2}.$$

(i) Rewrite $j(x)$ as an expression involving $\sin x$ and no other trigonometric functions.

(ii) Rewrite $k(x)$ as an expression of the form $4^{(ax+b)}$, where a and b are constants.

(C) The function m is given by

$$m(x) = \sqrt{3}\,\tan\left(x + \frac{\pi}{2}\right).$$

Find all values in the domain of m that yield an output value of 1.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

STOP
END OF EXAM

Answer QUESTION 1 part (A) on this page.

x	1	2	4	8	16	32
$f(x)$	0	5	10	15	20	25

Response for question 1(A)

(i)

(ii)

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Answer QUESTION 1 parts (B) and (C) on this page.

Response for question 1(B)

(i)

(ii)

Response for question 1(C)

(i)

(ii)

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Answer QUESTION 2 part (A) on this page.

Response for question 2(A)

(i)

(ii)

Answer QUESTION 2 parts (B) and (C) on this page.

Response for question 2(B)

(i)

(ii)

(iii)

Response for question 2(C)

Answer QUESTION 3 part (A) on this page.

Response for question 3(A)
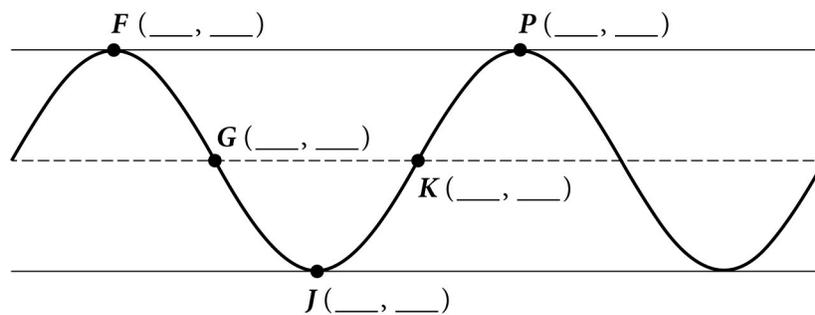
F (___, ___)   P (___, ___)

G (___, ___)

K (___, ___)

J (___, ___)

Answer QUESTION 3 parts (B) and (C) on this page.

Response for question 3(B)

$a =$ _____

$b =$ _____

$c =$ _____

$d =$ _____

Response for question 3(C)

(i)

- -

(ii)

Answer QUESTION 4 part (A) on this page.

Response for question 4(A)

(i)

(ii)

Answer QUESTION 4 parts (B) and (C) on this page.

Response for question 4(B)

(i)

(ii)

Response for question 4(C)