AP® Calculus AB Exam

SECTION II: Free Response

2019

DO NOT OPEN THIS BOOKLET OR BREAK THE SEALS ON PART B UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour and 30 minutes
Number of Questions
6
Percent of Total Score
50%
Writing Instrument
Either pencil or pen with black or dark blue ink
Weight
The questions are weighted equally, but the parts of a question are not necessarily given equal weight.

Part A

Number of Questions
2
Time
30 minutes
Electronic Device
Graphing calculator required
Percent of Section II Score
33.33%

Part B

Number of Questions
4
Time
1 hour
Electronic Device
None allowed
Percent of Section II Score
66.67%

IMPORTANT Identification Information

PLEASE PRINT WITH PEN:

1. First two letters of your last name ☐☐
   First letter of your first name ☐
2. Date of birth
   ☐☐ Month ☐☐ Day ☐☐☐☐ Year
3. Six-digit school code
   ☐☐☐☐☐☐
4. Unless I check the box below, I grant the College Board the unlimited right to use, reproduce, and publish my free-response materials, both written and oral, for educational research and instructional purposes. My name and the name of my school will not be used in any way in connection with my free-response materials. I understand that I am free to mark "No" with no effect on my score or its reporting.
   No, I do not grant the College Board these rights. ☐

Instructions

The questions for Section II are printed in this booklet. Do not break the seals on Part B until you are told to do so. Write your solution to each part of each question in the space provided. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be scored.

Manage your time carefully. During Part A, work only on the questions in Part A. You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. During Part B, you may continue to work on the questions in Part A without the use of a calculator.

As you begin each part, you may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions.

- Show all of your work, even though a question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X^2, X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

Form I
Form Code 4PBP4-S

66

CALCULUS AB

SECTION II, Part A

Time—30 minutes

Number of questions—2

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

t (hours)	2	5	9	11	12
$L(t)$ (cars per hour)	15	40	24	68	18

1. The rate at which cars enter a parking lot is modeled by $E(t) = 30 + 5(t-2)(t-5)e^{-0.2t}$. The rate at which cars leave the parking lot is modeled by the differentiable function L. Selected values of $L(t)$ are given in the table above. Both $E(t)$ and $L(t)$ are measured in cars per hour, and time t is measured in hours after 5 A.M. ($t = 0$). Both functions are defined for $0 \le t \le 12$.

(a) What is the rate of change of $E(t)$ at time $t = 7$? Indicate units of measure.

(b) How many cars enter the parking lot from time $t = 0$ to time $t = 12$? Give your answer to the nearest whole number.

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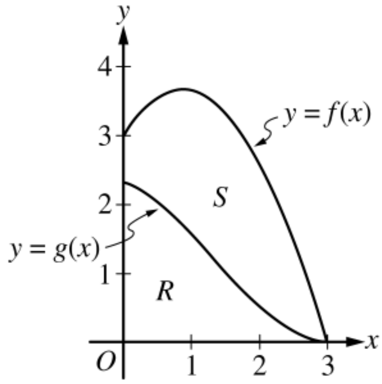
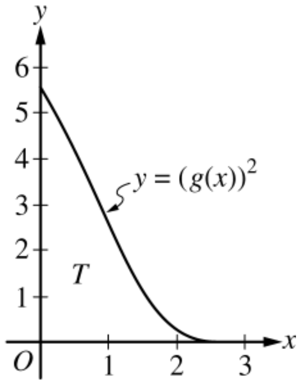
Continue question 1 on page 5.

(c) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate $\int_2^{12} L(t)\,dt$. Using correct units, explain the meaning of $\int_2^{12} L(t)\,dt$ in the context of this problem.

(d) For $0 \le t < 6$, 5 dollars are collected from each car entering the parking lot. For $6 \le t \le 12$, 8 dollars are collected from each car entering the parking lot. How many dollars are collected from the cars entering the parking lot from time $t = 0$ to time $t = 12$? Give your answer to the nearest whole dollar.





2. The function f is defined by $f(x) = 3(1+x)^{0.5}\cos\left(\frac{\pi x}{6}\right)$ for $0 \le x \le 3$. The function g is continuous and decreasing for $0 \le x \le 3$ with $g(3) = 0$.

The figure above on the left shows the graphs of f and g and the regions R and S. R is the region bounded by the graph of g and the x- and y-axes. Region R has area 3.24125. S is the region bounded by the y-axis and the graphs of f and g.

The figure above on the right shows the graph of $y = (g(x))^2$ and the region T. T is the region bounded by the graph of $y = (g(x))^2$ and the x- and y-axes. Region T has area 5.32021.

(a) Find the area of region S.

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Continue question 2 on page 7.

(b) Find the volume of the solid generated when region S is revolved about the horizontal line $y = -3$.

(c) Region S is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a rectangle whose height is 7 times the length of its base in region S. Write, but do not evaluate, an integral expression for the volume of this solid.

END OF PART A

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS AB

SECTION II, Part B

Time—1 hour

Number of questions—4

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.

DO NOT BREAK THE SEALS UNTIL YOU ARE TOLD TO DO SO.

NO CALCULATOR ALLOWED

$$f(x) = \begin{cases} \sqrt{9 - x^2} & \text{for } -3 \le x \le 0 \\ -x + 3\cos\left(\dfrac{\pi x}{2}\right) & \text{for } 0 < x \le 4 \end{cases}$$

3. Let f be the function defined above.

(a) Find the average rate of change of f on the interval $-3 \le x \le 4$.

(b) Write an equation for the line tangent to the graph of f at $x = 3$.

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Continue question 3 on page 15.

NO CALCULATOR ALLOWED

(c) Find the average value of f on the interval $-3 \le x \le 4$.

(d) Must there be a value of x at which $f(x)$ attains an absolute maximum on the closed interval $-3 \le x \le 4$? Justify your answer.

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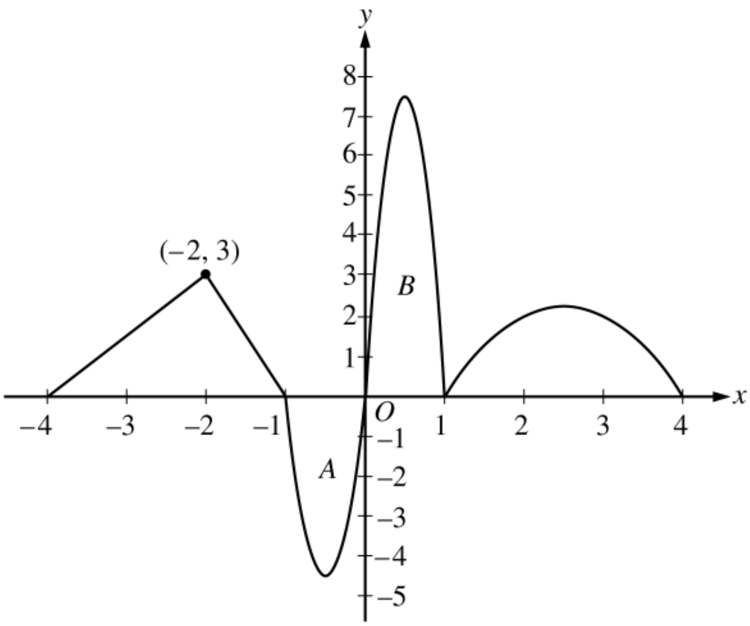
NO CALCULATOR ALLOWED



Graph of f

4. The continuous function f is defined for $-4 \le x \le 4$. The graph of f, shown above, consists of two line segments and portions of three parabolas. The graph has horizontal tangents at $x = -\frac{1}{2}$, $x = \frac{1}{2}$, and $x = \frac{5}{2}$. It is known that $f(x) = -x^2 + 5x - 4$ for $1 \le x \le 4$. The areas of regions A and B bounded by the graph of f and the x-axis are 3 and 5, respectively. Let g be the function defined by $g(x) = \int_{-4}^{x} f(t)\,dt$.

(a) Find $g(0)$ and $g(4)$.

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Continue question 4 on page 17.

NO CALCULATOR ALLOWED

(b) Find the absolute minimum value of g on the closed interval $[-4, 4]$. Justify your answer.

(c) Find all intervals on which the graph of g is concave down. Give a reason for your answer.

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NO CALCULATOR ALLOWED

t (hours)	0	1	2	3	4
$B(t)$ (miles per hour)	1	8	1.5	–5	11

5. Brandon and Chloe ride their bikes for 4 hours along a flat, straight road. Brandon's velocity, in miles per hour, at time t hours is given by a differentiable function B for $0 \le t \le 4$. Values of $B(t)$ for selected times t are given in the table above. Chloe's velocity, in miles per hour, at time t hours is given by the piecewise function C defined by

$$C(t) = \begin{cases} te^{4-t^2} & \text{for } 0 \le t \le 2 \\ 12 - 3t - t^2 & \text{for } 2 < t \le 4. \end{cases}$$

(a) How many miles did Chloe travel from time $t = 0$ to time $t = 2$?

(b) At time $t = 3$, is Chloe's speed increasing or decreasing? Give a reason for your answer.

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Continue question 5 on page 19.

NO CALCULATOR ALLOWED

(c) Is there a time t, for $0 \le t \le 4$, at which Brandon's acceleration is equal to 2.5 miles per hour per hour? Justify your answer.

(d) Is there a time t, for $0 \le t \le 2$, at which Brandon's velocity is equal to Chloe's velocity? Justify your answer.

NO CALCULATOR ALLOWED

6. Consider the curve defined by $2x^2 + 3y^2 - 4xy = 36$.

(a) Show that $\frac{dy}{dx} = \frac{2y - 2x}{3y - 2x}$.

(b) Find the slope of the line tangent to the curve at each point on the curve where $x = 6$.

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Continue question 6 on page 21.

NO CALCULATOR ALLOWED

(c) Find the positive value of x at which the curve has a vertical tangent line. Show the work that leads to your answer.

(d) Let x and y be functions of time t that are related by the equation $2x^2 + 3y^2 - 4xy = 36$. At time $t = 1$, the value of x is 2, the value of y is -2, and the value of $\frac{dy}{dt}$ is 4. Find the value of $\frac{dx}{dt}$ at time $t = 1$.

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