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## **Section II: Free-Response Questions**

This is the free-response section of the 2012 AP exam. It includes cover material and other administrative instructions to help familiarize students with the mechanics of the exam. (Note that future exams may differ in look from the following content.)

# AP<sup>®</sup> Calculus AB Exam

## SECTION II: Free Response

2012

DO NOT OPEN THIS BOOKLET OR BREAK THE SEALS ON PART B UNTIL YOU ARE TOLD TO DO SO.

### At a Glance

**Total Time**

1 hour, 30 minutes

**Number of Questions**

6

**Percent of Total Score**

50%

**Writing Instrument**

Either pencil or pen with black or dark blue ink

**Weight**

The questions are weighted equally, but the parts of a question are not necessarily given equal weight.

### Part A

**Number of Questions**

2

**Time**

30 minutes

**Electronic Device**

Graphing calculator required

**Percent of Section II Score**

33.3%

### Part B

**Number of Questions**

4

**Time**

60 minutes

**Electronic Device**

None allowed

**Percent of Section II Score**

66.6%

### IMPORTANT Identification Information

PLEASE PRINT WITH PEN:

1. First two letters of your last name   
First letter of your first name
2. Date of birth  
    
Month Day Year
3. Six-digit school code
4. Unless I check the box below, I grant the College Board the unlimited right to use, reproduce, and publish my free-response materials, both written and oral, for educational research and instructional purposes. My name and the name of my school will not be used in any way in connection with my free-response materials. I understand that I am free to mark "No" with no effect on my score or its reporting.  
No, I do not grant the College Board  these rights.

### Instructions

The questions for Section II are printed in this booklet. Do not break the seals on Part B until you are told to do so. Write your solution to each part of each question in the space provided. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be scored.

Manage your time carefully. During the timed portion for Part A, work only on the questions in Part A. You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. During the timed portion for Part B, you may continue to work on the questions in Part A without the use of a calculator.

For each part of Section II, you may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions.

- Show all of your work. Clearly label any functions, graphs, tables, or other objects that you use. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit. Justifications require that you give mathematical (noncalculator) reasons.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example,  $\int_1^5 x^2 dx$  may not be written as `fnInt(X^2, X, 1, 5)`.
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function  $f$  is assumed to be the set of all real numbers  $x$  for which  $f(x)$  is a real number.



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Form I  
Form Code 4IBP-S2

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**CALCULUS AB**  
**SECTION II, Part A**  
**Time—30 minutes**  
**Number of problems—2**

**A graphing calculator is required for these problems.**

$t$ (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

1. The temperature of water in a tub at time  $t$  is modeled by a strictly increasing, twice-differentiable function  $W$ , where  $W(t)$  is measured in degrees Fahrenheit and  $t$  is measured in minutes. At time  $t = 0$ , the temperature of the water is  $55^\circ\text{F}$ . The water is heated for 30 minutes, beginning at time  $t = 0$ . Values of  $W(t)$  at selected times  $t$  for the first 20 minutes are given in the table above.
- (a) Use the data in the table to estimate  $W'(12)$ . Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

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- (b) Use the data in the table to evaluate  $\int_0^{20} W'(t) dt$ . Using correct units, interpret the meaning of  $\int_0^{20} W'(t) dt$  in the context of this problem.

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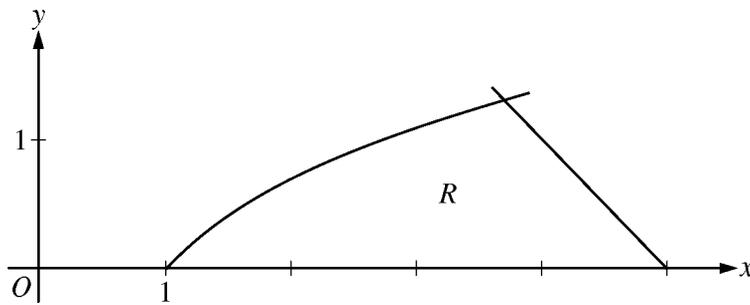
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- (c) For  $0 \leq t \leq 20$ , the average temperature of the water in the tub is  $\frac{1}{20} \int_0^{20} W(t) dt$ . Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate  $\frac{1}{20} \int_0^{20} W(t) dt$ . Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

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- (d) For  $20 \leq t \leq 25$ , the function  $W$  that models the water temperature has first derivative given by  $W'(t) = 0.4\sqrt{t} \cos(0.06t)$ . Based on the model, what is the temperature of the water at time  $t = 25$ ?

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2. Let  $R$  be the region in the first quadrant bounded by the  $x$ -axis and the graphs of  $y = \ln x$  and  $y = 5 - x$ , as shown in the figure above.
- (a) Find the area of  $R$ .

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- (b) Region  $R$  is the base of a solid. For the solid, each cross section perpendicular to the  $x$ -axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

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- (c) The horizontal line  $y = k$  divides  $R$  into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of  $k$ .

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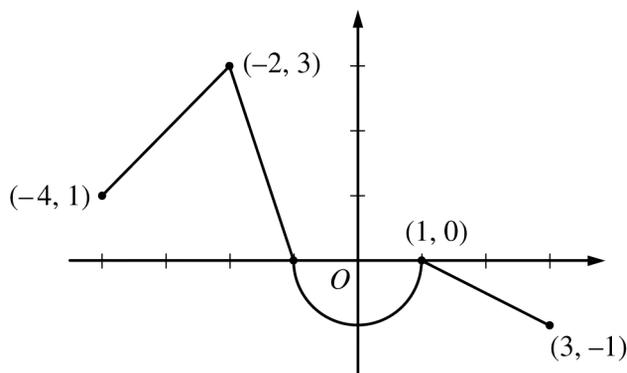
**CALCULUS AB**  
**SECTION II, Part B**  
**Time—60 minutes**  
**Number of problems—4**

**No calculator is allowed for these problems.**

**DO NOT BREAK THE SEALS UNTIL YOU ARE TOLD TO DO SO.**

**GO ON TO THE NEXT PAGE.**

## NO CALCULATOR ALLOWED

Graph of  $f$ 

3. Let  $f$  be the continuous function defined on  $[-4, 3]$  whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let  $g$  be the function given by  $g(x) = \int_1^x f(t) dt$ .

(a) Find the values of  $g(2)$  and  $g(-2)$ .

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(b) For each of  $g'(-3)$  and  $g''(-3)$ , find the value or state that it does not exist.

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- (c) Find the  $x$ -coordinate of each point at which the graph of  $g$  has a horizontal tangent line. For each of these points, determine whether  $g$  has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.

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- (d) For  $-4 < x < 3$ , find all values of  $x$  for which the graph of  $g$  has a point of inflection. Explain your reasoning.

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4. The function  $f$  is defined by  $f(x) = \sqrt{25 - x^2}$  for  $-5 \leq x \leq 5$ .

(a) Find  $f'(x)$ .

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(b) Write an equation for the line tangent to the graph of  $f$  at  $x = -3$ .

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NO CALCULATOR ALLOWED

(c) Let  $g$  be the function defined by  $g(x) = \begin{cases} f(x) & \text{for } -5 \leq x \leq -3 \\ x + 7 & \text{for } -3 < x \leq 5. \end{cases}$

Is  $g$  continuous at  $x = -3$ ? Use the definition of continuity to explain your answer.

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(d) Find the value of  $\int_0^5 x\sqrt{25 - x^2} dx$ .

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## NO CALCULATOR ALLOWED

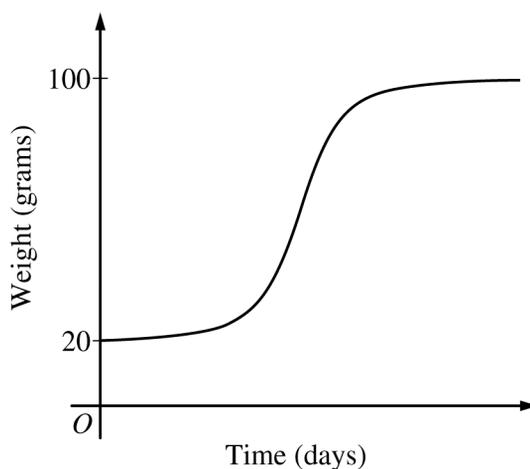
5. The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time  $t = 0$ , when the bird is first weighed, its weight is 20 grams. If  $B(t)$  is the weight of the bird, in grams, at time  $t$  days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let  $y = B(t)$  be the solution to the differential equation above with initial condition  $B(0) = 20$ .

- (a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.

- (b) Find  $\frac{d^2B}{dt^2}$  in terms of  $B$ . Use  $\frac{d^2B}{dt^2}$  to explain why the graph of  $B$  cannot resemble the following graph.



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- (c) Use separation of variables to find  $y = B(t)$ , the particular solution to the differential equation with initial condition  $B(0) = 20$ .

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## NO CALCULATOR ALLOWED

6. For  $0 \leq t \leq 12$ , a particle moves along the  $x$ -axis. The velocity of the particle at time  $t$  is given by

$$v(t) = \cos\left(\frac{\pi}{6}t\right). \text{ The particle is at position } x = -2 \text{ at time } t = 0.$$

(a) For  $0 \leq t \leq 12$ , when is the particle moving to the left?

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(b) Write, but do not evaluate, an integral expression that gives the total distance traveled by the particle from time  $t = 0$  to time  $t = 6$ .

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- (c) Find the acceleration of the particle at time  $t$ . Is the speed of the particle increasing, decreasing, or neither at time  $t = 4$ ? Explain your reasoning.

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- (d) Find the position of the particle at time  $t = 4$ .

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**STOP**

**END OF EXAM**

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**THE FOLLOWING INSTRUCTIONS APPLY TO THE COVERS OF THE SECTION II BOOKLET.**

- **MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE FRONT AND BACK COVERS OF THE SECTION II BOOKLET.**
- **CHECK TO SEE THAT YOUR AP NUMBER LABEL APPEARS IN THE BOX ON THE COVER.**
- **MAKE SURE YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON ALL AP EXAMS YOU HAVE TAKEN THIS YEAR.**