AP® Calculus BC Exam

SECTION II: Free Response

DO NOT OPEN THIS BOOKLET OR BREAK THE SEALS ON PART B UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour and 30 minutes
Number of Questions
6
Percent of Total Score
50%
Writing Instrument
Either pencil or pen with black or dark blue ink
Weight
The questions are weighted equally, but the parts of a question are not necessarily given equal weight.

Part A

Number of Questions
2
Time
30 minutes
Electronic Device
Graphing calculator required
Percent of Section II Score
33.33%

Part B

Number of Questions
4
Time
1 hour
Electronic Device
None allowed
Percent of Section II Score
66.67%

Instructions

The questions for Section II are printed in this booklet. Do not break the seals on Part B until you are told to do so. Write your solution to each part of each question in the space provided. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be scored.

Manage your time carefully. During Part A, work only on the questions in Part A. You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. During Part B, you may continue to work on the questions in Part A without the use of a calculator.

As you begin each part, you may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions.

- Show all of your work, even though a question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_{1}^{5} x^2 \, dx$ may not be written as fnInt(X², X, 1, 5).

- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

CALCULUS BC

SECTION II, Part A

Time—30 minutes

Number of questions—2

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

t (hours)	2	5	9	11	12
$L(t)$ (cars per hour)	15	40	24	68	18

1. The rate at which cars enter a parking lot is modeled by $E(t) = 30 + 5(t-2)(t-5)e^{-0.2t}$. The rate at which cars leave the parking lot is modeled by the differentiable function L. Selected values of $L(t)$ are given in the table above. Both $E(t)$ and $L(t)$ are measured in cars per hour, and time t is measured in hours after 5 A.M. $(t = 0)$. Both functions are defined for $0 \le t \le 12$.

    (a) What is the rate of change of $E(t)$ at time $t = 7$? Indicate units of measure.

    (b) How many cars enter the parking lot from time $t = 0$ to time $t = 12$? Give your answer to the nearest whole number.

(c) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate $\int_{2}^{12} L(t)\, dt$. Using correct units, explain the meaning of $\int_{2}^{12} L(t)\, dt$ in the context of this problem.

(d) For $0 \leq t < 6$, 5 dollars are collected from each car entering the parking lot. For $6 \leq t \leq 12$, 8 dollars are collected from each car entering the parking lot. How many dollars are collected from the cars entering the parking lot from time $t = 0$ to time $t = 12$? Give your answer to the nearest whole dollar.

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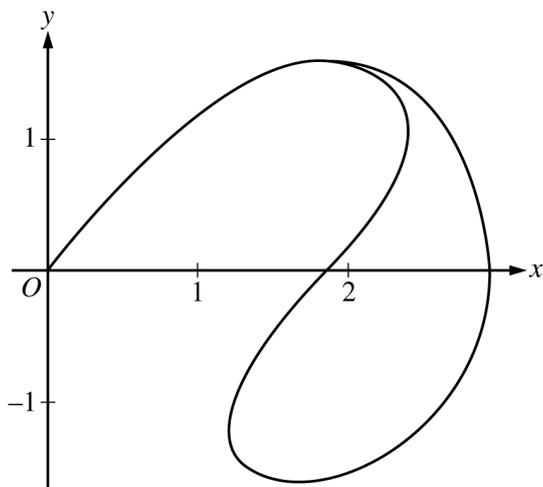
2. A laser is a device that produces a beam of light. A design, shown above, is etched onto a flat piece of metal using a moving laser. The position of the laser at time t seconds is represented by $(x(t), y(t))$ in the xy-plane. Both x and y are measured in centimeters, and t is measured in seconds. The laser starts at position $(0, 0)$ at time $t = 0$, and the design takes 3.1 seconds to complete. For $0 \leq t \leq 3.1$, $\dfrac{dx}{dt} = 3\cos\left(t^2\right)$ and $\dfrac{dy}{dt} = 4\cos(2.5t)$.

(a) Find the speed of the laser at time $t = 3$ seconds.

(b) Find the total distance traveled by the laser from time $t = 1$ to time $t = 3$ seconds.

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(c) The laser is farthest to the right at time $t = 1.253$ seconds. Find the x-coordinate of the laser's rightmost position.

(d) What is the difference between the y-coordinates of the laser's highest position and lowest position for $0 \leq t \leq 3.1$? Justify your answer.

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END OF PART A

**IF YOU FINISH BEFORE TIME IS CALLED,
YOU MAY CHECK YOUR WORK ON PART A ONLY.**

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS BC

SECTION II, Part B

Time—1 hour

Number of questions—4

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.

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$$f(x) = \begin{cases} \sqrt{9 - x^2} & \text{for } -3 \le x \le 0 \\ -x + 3\cos\left(\dfrac{\pi x}{2}\right) & \text{for } 0 < x \le 4 \end{cases}$$

3. Let f be the function defined above.

(a) Find the average rate of change of f on the interval $-3 \le x \le 4$.

(b) Write an equation for the line tangent to the graph of f at $x = 3$.

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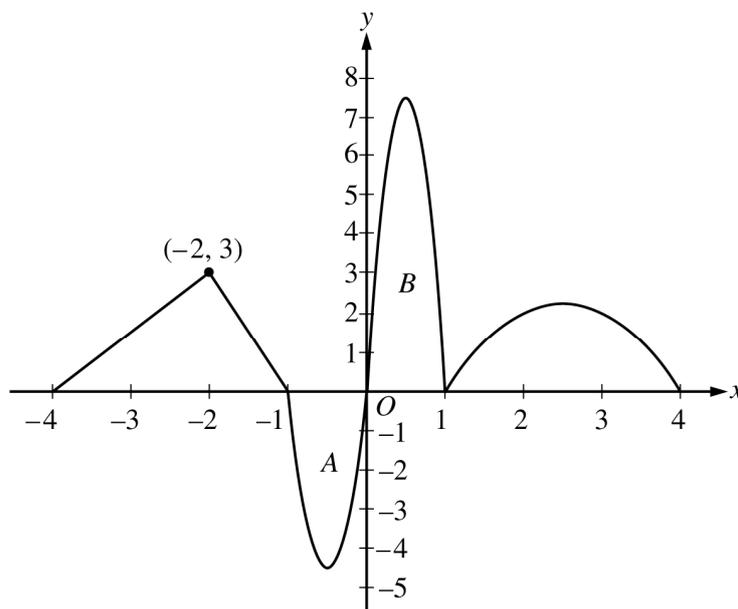-14-

Continue question 3 on page 15.

(c) Find the average value of f on the interval $-3 \le x \le 4$.

(d) Must there be a value of x at which $f(x)$ attains an absolute maximum on the closed interval $-3 \le x \le 4$? Justify your answer.

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NO CALCULATOR ALLOWED

(−2, 3)

B

A

Graph of *f*

4. The continuous function *f* is defined for $-4 \le x \le 4$. The graph of *f*, shown above, consists of two line segments and portions of three parabolas. The graph has horizontal tangents at $x = -\frac{1}{2}$, $x = \frac{1}{2}$, and $x = \frac{5}{2}$. It is known that $f(x) = -x^2 + 5x - 4$ for $1 \le x \le 4$. The areas of regions *A* and *B* bounded by the graph of *f* and the *x*-axis are 3 and 5, respectively. Let *g* be the function defined by $g(x) = \int_{-4}^{x} f(t) \, dt$.

(a) Find $g(0)$ and $g(4)$.

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(b) Find the absolute minimum value of g on the closed interval $[-4, 4]$. Justify your answer.

(c) Find all intervals on which the graph of g is concave down. Give a reason for your answer.

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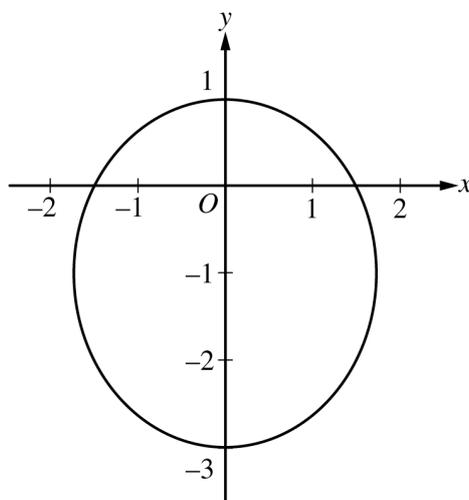Do not write beyond this border.

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5. The graph of the curve C, given by $4x^2 + 3y^2 + 6y = 9$, is shown in the figure above.

(a) Show that $\dfrac{dy}{dx} = \dfrac{-4x}{3(y+1)}$.

(b) Using the information from part (a), find $\dfrac{d^2y}{dx^2}$ in terms of x and y.

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NO CALCULATOR ALLOWED

(c) In polar coordinates, the curve C is given by $r = \dfrac{3}{2 + \sin \theta}$ for $0 \le \theta \le 2\pi$. Find $\dfrac{dr}{d\theta}$.

As θ increases, on what intervals is the distance between the origin and the point (r, θ) increasing?

Give a reason for your answer.

(d) Let S be the region inside curve C, as defined in part (c), but outside the curve $r = 2$. Write, but do not evaluate, an integral expression for the area of S.

6. Consider the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-3)^n}{5^n \cdot n^p}$, where p is a constant and $p > 0$.

(a) For $p = 3$ and $x = 8$, does the series converge absolutely, converge conditionally, or diverge? Explain your reasoning.

(b) For $p = 1$ and $x = 8$, does the series converge absolutely, converge conditionally, or diverge? Explain your reasoning.

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(c) When $x = -2$, for what values of p does the series converge? Explain your reasoning.

(d) When $p = 1$ and $x = 3.1$, the series converges to a value S. Use the first two terms of the series to

approximate S. Use the alternating series error bound to show that this approximation differs from S by

less than $\dfrac{1}{300,000}$.

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STOP

END OF EXAM

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THE FOLLOWING INSTRUCTIONS APPLY TO THE COVERS OF THE
SECTION II BOOKLET.

- MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION
  INFORMATION AS REQUESTED ON THE FRONT AND BACK
  COVERS OF THE SECTION II BOOKLET.

- CHECK TO SEE THAT YOUR AP NUMBER LABEL APPEARS IN
  THE BOX ON THE FRONT COVER.

- MAKE SURE YOU HAVE USED THE SAME SET OF AP
  NUMBER LABELS ON ALL AP EXAMS YOU HAVE TAKEN
  THIS YEAR.