AP® Calculus BC Exam

SECTION II: Free Response

2018

DO NOT OPEN THIS BOOKLET OR BREAK THE SEALS ON PART B UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour and 30 minutes
Number of Questions
6
Percent of Total Score
50%
Writing Instrument
Either pencil or pen with black or dark blue ink
Weight
The questions are weighted equally, but the parts of a question are not necessarily given equal weight.

Part A

Number of Questions
2
Time
30 minutes
Electronic Device
Graphing calculator required
Percent of Section II Score
33.33%

Part B

Number of Questions
4
Time
1 hour
Electronic Device
None allowed
Percent of Section II Score
66.67%

IMPORTANT Identification Information

PLEASE PRINT WITH PEN:

1. First two letters of your last name ☐☐
   First letter of your first name ☐
2. Date of birth
   ☐☐ Month ☐☐ Day ☐☐☐☐ Year
3. Six-digit school code
   ☐☐☐☐☐☐
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   No, I do not grant the College Board these rights. ☐

Instructions

The questions for Section II are printed in this booklet. Do not break the seals on Part B until you are told to do so. Write your solution to each part of each question in the space provided. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be scored.

Manage your time carefully. During Part A, work only on the questions in Part A. You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. During Part B, you may continue to work on the questions in Part A without the use of a calculator.

As you begin each part, you may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions.

- Show all of your work, even though a question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X^2, X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

Form I
Form Code 4OBP4-S

CALCULUS BC

SECTION II, Part A

Time—30 minutes

Number of questions—2

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

GO ON TO THE NEXT PAGE.

t (minutes)	0	1	5	6	8
$g(t)$ (cubic feet per minute)	12.8	15.1	20.5	18.3	22.7

1. Grain is being added to a silo. At time $t = 0$, the silo is empty. The rate at which grain is being added is modeled by the differentiable function g, where $g(t)$ is measured in cubic feet per minute for $0 \le t \le 8$ minutes. Selected values of $g(t)$ are given in the table above.

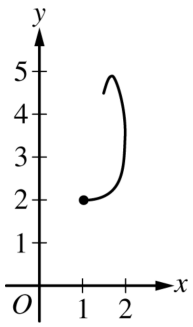
   (a) Using the data in the table, approximate $g'(3)$. Using correct units, interpret the meaning of $g'(3)$ in the context of the problem.

   (b) Write an integral expression that represents the total amount of grain added to the silo from time $t = 0$ to time $t = 8$. Use a right Riemann sum with the four subintervals indicated by the data in the table to approximate the integral.

Continue question 1 on page 5.

(c) The grain in the silo is spoiling at a rate modeled by $w(t) = 32 \cdot \sqrt{\sin\left(\frac{\pi t}{74}\right)}$, where $w(t)$ is measured in cubic feet per minute for $0 \le t \le 8$ minutes. Using the result from part (b), approximate the amount of unspoiled grain remaining in the silo at time $t = 8$.

(d) Based on the model in part (c), is the amount of unspoiled grain in the silo increasing or decreasing at time $t = 6$? Show the work that leads to your answer.



2. A particle moving in the xy-plane has position $(x(t), y(t))$ at time $t \geq 0$, where $\frac{dx}{dt} = \cos\left(t^2\right)$ and $\frac{dy}{dt} = e^t \sin\left(t^2\right)$. At time $t = 0$, the particle is at position $(1, 2)$. The figure above shows the path of the particle for $0 \leq t \leq 2$.

(a) Find the position of the particle at time $t = 2$.

(b) Find the slope of the line tangent to the particle's path at time $t = 2$.

Continue question 2 on page 7.

(c) Find the speed of the particle at time $t = 2$. Find the acceleration vector of the particle at time $t = 2$.

(d) Consider a rectangle with vertices at points $(0, 0)$, $(x(t), 0)$, $(x(t), y(t))$, and $(0, y(t))$ at time $t \geq 0$. For $0 \leq t \leq 2$, at what time t is the perimeter of the rectangle a maximum? Justify your answer.

Do not write beyond this border.

END OF PART A

**IF YOU FINISH BEFORE TIME IS CALLED,
YOU MAY CHECK YOUR WORK ON PART A ONLY.**

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS BC

SECTION II, Part B

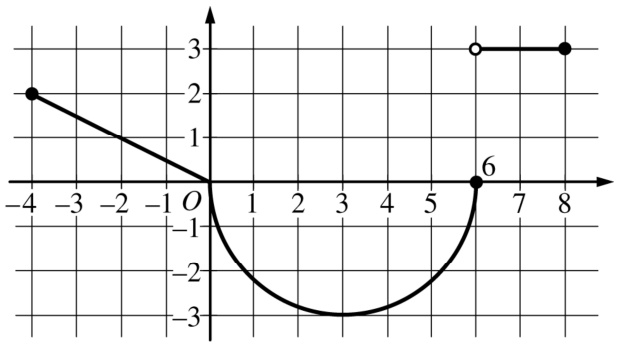
Time—1 hour

Number of questions—4

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.

DO NOT BREAK THE SEALS UNTIL YOU ARE TOLD TO DO SO.

NO CALCULATOR ALLOWED



Graph of g

3. The function g is defined on the closed interval $[-4, 8]$. The graph of g consists of two linear pieces and a semicircle, as shown in the figure above. Let f be the function defined by $f(x) = 3x + \int_0^x g(t)\,dt$.

(a) Find $f(7)$ and $f'(7)$.

(b) Find the value of x in the closed interval $[-4, 3]$ at which f attains its maximum value. Justify your answer.

Continue question 3 on page 15.

(c) For each of $\lim_{x \to 0^-} g'(x)$ and $\lim_{x \to 0^+} g'(x)$, find the value or state that it does not exist.

(d) Find $\lim_{x \to -2} \dfrac{f(x) + 7}{e^{3x+6} - 1}$.

NO CALCULATOR ALLOWED

4. Let g be the function that satisfies $g(0) = 0$ and whose derivative satisfies $g'(x) = 2|x|$.

(a) Find expressions for $g(x)$ and $g''(x)$.

(b) Find the x-coordinate, if any, of each point of inflection of the graph of $y = g(x)$. Explain your reasoning.

Continue question 4 on page 17.

NO CALCULATOR ALLOWED

(c) Let $h(x) = \int_0^x \sqrt{1 + 4t^2}\, dt$. For $x \geq 0$, $h(x)$ is the length of the graph of g from $t = 0$ to $t = x$. Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $h(4)$.

(d) Find the value of $\int_{\pi/2}^{\pi} g'(x) \cos x\, dx$.

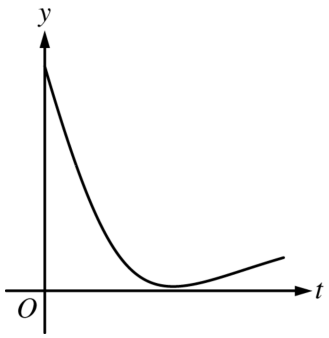
5 5 5 5 5 5 5 5 5 5

NO CALCULATOR ALLOWED

5. During a chemical reaction, the function $y = f(t)$ models the amount of a substance present, in grams, at time t seconds. At the start of the reaction $(t = 0)$, there are 10 grams of the substance present. The function $y = f(t)$ satisfies the differential equation $\frac{dy}{dt} = -0.02y^2$.

(a) Use the line tangent to the graph of $y = f(t)$ at $t = 0$ to approximate the amount of the substance remaining at time $t = 2$ seconds.

(b) Using the given differential equation, determine whether the graph of f could resemble the following graph. Give a reason for your answer.



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Continue question 5 on page 19.

(c) Find an expression for $y = f(t)$ by solving the differential equation $\frac{dy}{dt} = -0.02y^2$ with the initial condition $f(0) = 10$.

(d) Determine whether the amount of the substance is changing at an increasing or a decreasing rate. Explain your reasoning.

NO CALCULATOR ALLOWED

x	$f(x)$	$f'(x)$	$f''(x)$	$f'''(x)$	$f^{(4)}(x)$
0	4	5	–1	$-\frac{15}{2}$	23
1	8	3	–2	$\frac{3}{2}$	$\frac{2}{5}$

6. Let f be a function having derivatives of all orders for all real numbers. Selected values of f and its first four derivatives are shown in the table above.

(a) Write the second-degree Taylor polynomial for f about $x = 0$ and use it to approximate $f(0.2)$.

(b) Let g be a function such that $g(x) = f\left(x^3\right)$. Write the fifth-degree Taylor polynomial for g', the derivative of g, about $x = 0$.

Continue question 6 on page 21.

(c) Write the third-degree Taylor polynomial for f about $x = 1$.

(d) It is known that $\left| f^{(4)}(x) \right| \le 300$ for $0 \le x \le 1.125$. The third-degree Taylor polynomial for f about $x = 1$, found in part (c), is used to approximate $f(1.1)$. Use the Lagrange error bound along with the information about $f^{(4)}(x)$ to find an upper bound on the error of the approximation.

STOP

END OF EXAM

THE FOLLOWING INSTRUCTIONS APPLY TO THE COVERS OF THE SECTION II BOOKLET.

- **MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE FRONT AND BACK COVERS OF THE SECTION II BOOKLET.**
- **CHECK TO SEE THAT YOUR AP NUMBER LABEL APPEARS IN THE BOX ON THE FRONT COVER.**
- **MAKE SURE YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON ALL AP EXAMS YOU HAVE TAKEN THIS YEAR.**